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Resolução Exercícios:

1)

a) $\int_1^5 \int_2^{y/2} 6x^2 y \, dx \, dy = \int_1^5 ? \, dy$

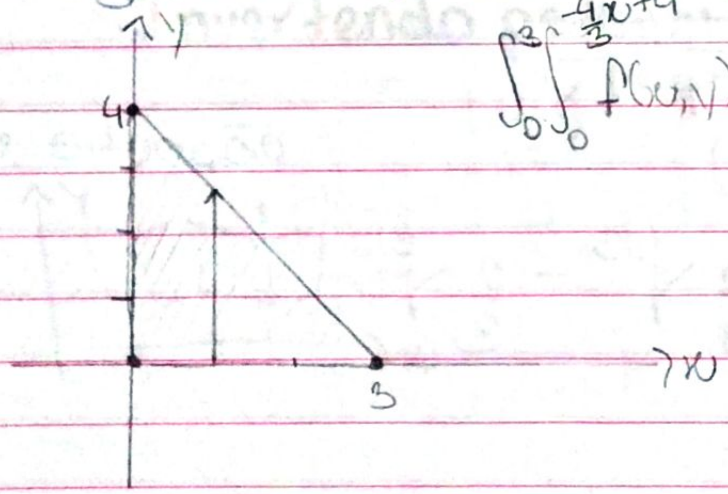
$\int_2^{y/2} 6x^2 y \, dx = \frac{\cancel{6}^{2} x^3}{\cancel{3}} y \Big|_2^{y/2} = 2\left(\frac{y}{2}\right)^3 y - 2 \cdot 2^3 y$

$\Rightarrow \frac{2y^4}{8} - 16y = \frac{y^4}{4} - 16y$

$\Rightarrow \int_1^5 \frac{1}{4} y^4 - 16y \, dy$ //

2) (0,0), (3,0) e (0,4)

a) Região do tipo 1.



$\int_0^3 \int_0^{-\frac{4}{3}x+4} f(x,y) \, dy \, dx$

encontrar a equação da reta

$\overset{x_1\, y_1}{(3,0)}$ e $\overset{x_2\, y_2}{(0,4)}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{4-0}{0-3} = \frac{4}{-3} = -\frac{4}{3}$

$y = mx + n$

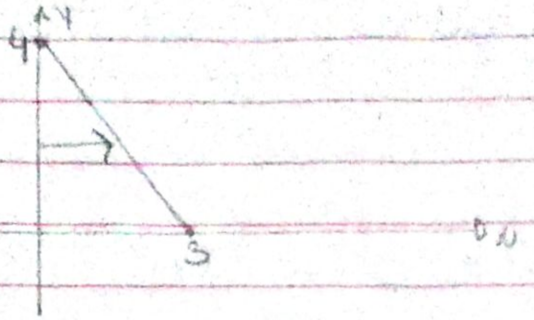
$4 = -\frac{4}{3} 0 + n$

$n = 4$

$y = -\frac{4}{3}x + 4$

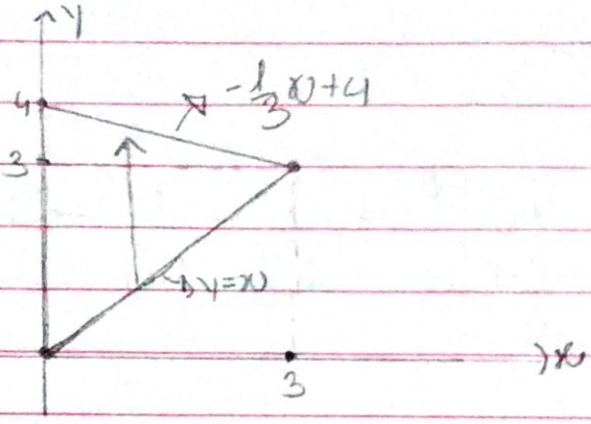
credeal

b) Região do tipo II



$$\int_0^4\int_0^{-\frac{3}{4}y+3} f(x,y)\,dx\,dy$$

3) $\overset{x\,y}{(0,0)}$, $\overset{x\,y}{(3,3)}$ e $\overset{x\,y}{(0,4)}$ a área de R é $A(R)=$



$$\int_0^3\int_x^{-\frac{1}{3}x+4} f(x,y)\,dy\,dx$$

$\overset{x_1\,y_1}{0,4}$ $\overset{x_2\,y_2}{3,3}$

$m=\dfrac{y_2-y_1}{x_2-x_1}$

$m=\dfrac{3-4}{3-0}=\boxed{-\dfrac{1}{3}}$

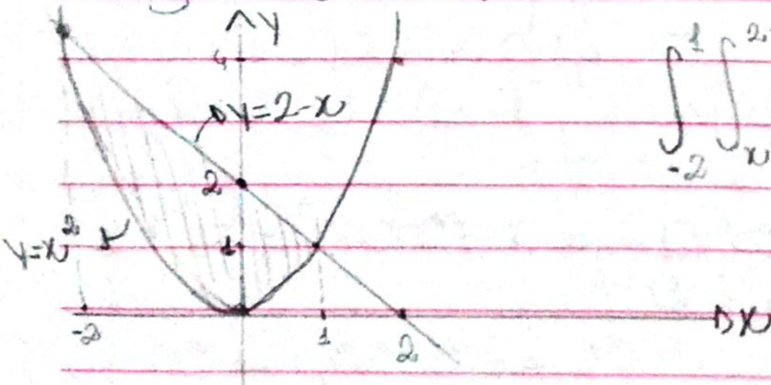
$y=mx+n$

$4=-\dfrac{1}{3}0+n$

$n=4$

$y=-\dfrac{1}{3}x+4$

4) A reta $y=2-x$ e a parábola $y=x^2$ intersectam nos pontos $\overset{x\,y}{(-2,4)}$ e $(1,1)$. Se R denotar a região englobada então:



$$\int_{-2}^{1}\int_{x^2}^{2-x}(1+2y)\,dy\,dx$$

1) Calcule as integrais iteradas

4) $\int_0^1 \int_{x^2}^{x} xy^2 \, dy \, dx$

$\int_{x^2}^{x} xy^2 \, dy = \frac{xy^3}{3} \Big|_{x^2}^{x} = \frac{x \cdot x^3}{3} - \frac{x(x^2)^3}{3}$

$\Rightarrow \frac{x^4}{3} - \frac{x^7}{3}$

$\int_0^1 \frac{x^4}{3} - \frac{x^7}{3} \, dx = \frac{x^5}{15} - \frac{x^8}{24} \Big|_0^1 = \frac{1}{15} - \frac{1}{24} - 0 = \frac{1}{40}$

5) $\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_0^{x^3} \operatorname{sen} \frac{y}{x} \, dy \, dx$

$\int_0^{x^3} \operatorname{sen} \frac{y}{x} \, dy = \int_0^{x^3} \operatorname{sen}(u) x \, du = x \int_0^{x^3} \operatorname{sen}(u) du$

$u = \frac{y}{x} dy \qquad x - \cos(u) \Big|_0^{x^3}$

$du = \frac{1}{x} dy \qquad \Rightarrow x - \cos \frac{y}{x} \Big|_0^{x^3} =$

$x \, du = dy$

$\Rightarrow x - \cos \frac{x^3}{x} - x - \cos \frac{0}{x} = x - \cos x^2 - x + 1$

$= -\cos x^2 + 1$

4) $\int_{1/4}^{1}\int_{x^2}^{x}\sqrt{\dfrac{x}{y}}\,dy\,dx$

$$\int_{x^2}^{x}\sqrt{\frac{x}{y}}\,dy = \int_{x^2}^{x}\frac{\sqrt{x}}{\sqrt{y}}\,dy = \int_{x^2}^{x} x^{1/2}\cdot y^{-1/2}\,dy$$

$$\sqrt{x}\cdot\frac{y^{1/2}}{1/2} = \sqrt{x}\,2\sqrt{y}\,\Big|_{x^2}^{x} = \sqrt{x}\cdot 2\sqrt{x} - \sqrt{x}\cdot 2\sqrt{x^2}$$

$$\Rightarrow 2x - \sqrt{x}\cdot 2x = 2x - 2x\sqrt{x}$$ //

$$\int_{1/4}^{1} 2x - 2x\sqrt{x}\,dx = \frac{\cancel{2}x^2}{\cancel{2}} - \frac{2x^{5/2}}{5/2} = x^2 - \frac{4}{5}x^{5/2}\Big|_{1/4}^{1}$$

$2x\cdot x^{1/2}$

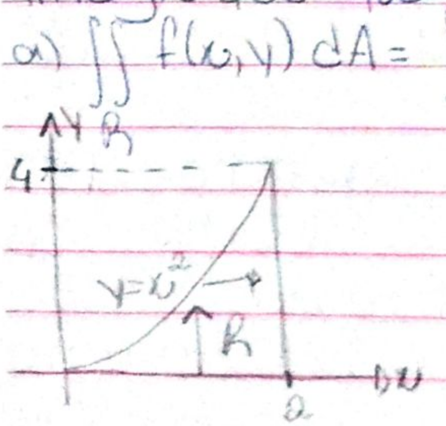
$2x^{3/2}$

$$\Rightarrow 1^2 - \frac{4}{5}1^{5/2} - \left(\left(\frac{1}{4}\right)^2 - \frac{4}{5}\left(\frac{1}{4}\right)^{5/2}\right)$$

$$\frac{1}{5} - \frac{3}{80} = \frac{13}{80}$$ //

9) Preecha as lacunas com os extremos de integração que faltam.

a) $\iint f(x,y)\,dA = \int_{\square}^{\square}\int_{\square}^{\square} f(x,y)\,dy\,dx$ $dx\,dy$



$$\int_0^2\int_0^{x^2} f(x,y)\,dy\,dx$$ //

$$\int_0^4\int_{\sqrt{y}}^{2} f(x,y)\,dx\,dy$$ //

11) a) $\iint_R f(x,y)\,dA = \int_1^2 \int_\square^\square f(x,y)\,dy\,dx$ $\int_1^2 \int_{-2x+5}^{3} f(x,y)\,dy\,dx$

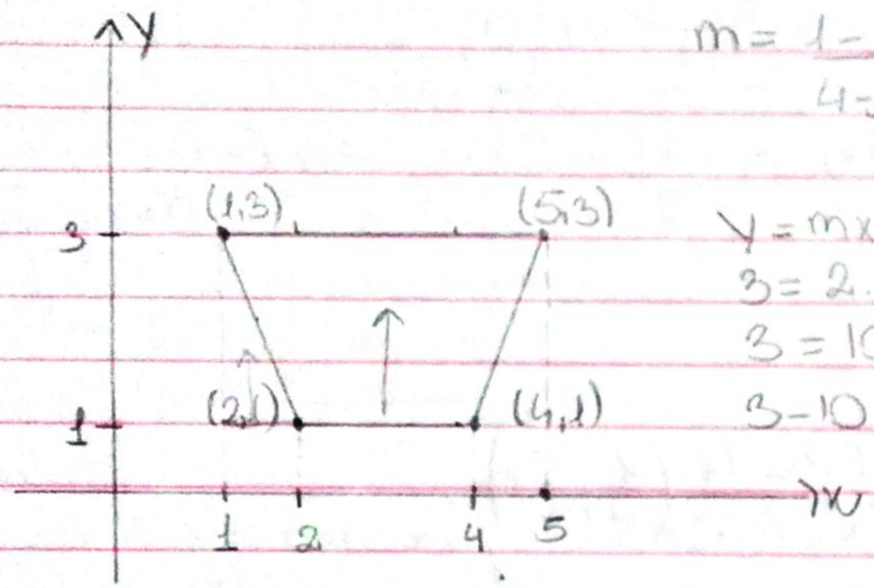
$+ \int_2^4 \int_\square^\square f(x,y)\,dy\,dx$ $\int_2^4 \int_1^3 f(x,y)\,dy\,dx$

$+ \int_4^5 \int_\square^\square f(x,y)\,dy\,dx$ $\int_4^5 \int_{2x-7}^{3} f(x,y)\,dy\,dx$

b) $\iint_R f(x,y)\,dA = \int_1^3 \int_{-y+5/2}^{y+7/2} f(x,y)\,dx\,dy$

$(5,3)\times(4,1)$

$m = \frac{1-3}{4-5} = \frac{-2}{-1} = 2$



$y = mx + n$

$3 = 2 \cdot 5 + n$

$3 = 10 + n$

$3 - 10 = n$

$n = -7$

$y = 2x - 7$

$x = \frac{y+7}{2}$

$\overset{x_1\,y_1}{(2,1)} \times \overset{x_2\,y_2}{(1,3)}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{1-2} = \frac{2}{-1} = \boxed{-2}$

$y = mx + n$

$3 = -2 \cdot 1 + n$

$3 = -2 + n$

$5 = n$

$\boxed{n = 5}$

$y = -2x + 5$

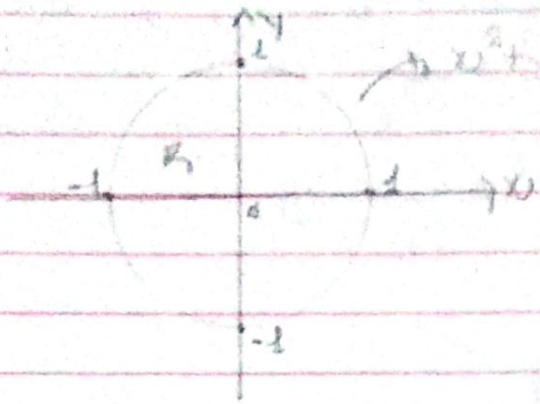
$y - 5 = -2x$

$\frac{-y+5}{2} = x$

12) [illegible]

a) $\iint_R f(x,y)\,dA$ $\int_{-1}^{1}\int_{0}^{1} f(x,y)\,dy\,dx$

" " " $dx\,dy$



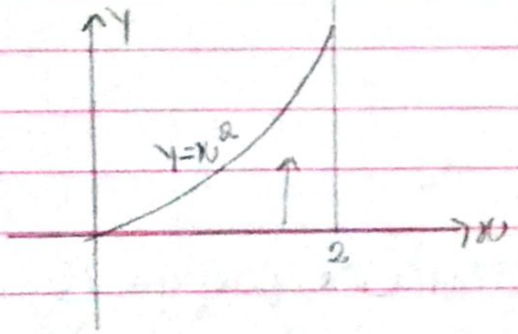
$A = \pi r^2$

$A = \pi \cdot 1^2$

$A = \pi$

13) Calcule $\iint_R xy\,dA$ onde R é a região no

a) Exercício 9.



$$\int_0^2\int_0^{x^2} xy\,dy\,dx$$

$$\int_0^{x^2} xy\,dy = \frac{xy^2}{2}\Big|_0^{x^2}$$

$$\Rightarrow \frac{x\cdot(x^2)^2}{2} - 0 = \frac{x^5}{2}$$

$$\int_0^2 \frac{x^5}{2}\,dx = \frac{x^6}{12}\Big|_0^2 = \frac{2^6}{12} - 0 = \boxed{\frac{16}{3}}$$

b) Exercício 11.

$$\int_1^3\int_{\frac{-y+5}{2}}^{\frac{y+7}{2}} xy\,dx\,dy$$

$$\int_1^3 \int_{(5-y)/2}^{(y+7)/2} xy\,dx\,dy = \frac{x^2 y}{2}\Big|_{(5-y)/2}^{(y+7)/2} = \frac{\left(\frac{y+7}{2}\right)^2 y}{2} - \frac{\left(\frac{5-y}{2}\right)^2 y}{2}$$

$$\frac{\boxed{\frac{(y+7)(y+7).y}{4}} - \left[\frac{(5-y)(5-y)y}{4}\right]}{2}$$

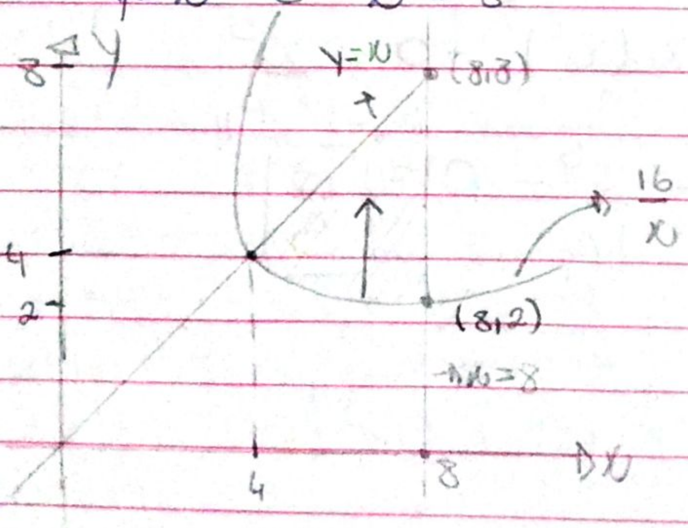
$$\frac{(y^3+14y^2+49y)y - (y^3-10y^2+25y)y}{8}$$

$$\frac{24y^2+24y}{8} = 3y^2+3y$$

$$\int_1^3 3y^2+3y\,dy = \frac{\cancel{3}y^3}{\cancel{3}} + \frac{3y^2}{2}\Big|_1^3 = \left(3^3 + \frac{3 \cdot 3^2}{2}\right) - \left(1^3 + \frac{3 \cdot 1^2}{2}\right)$$

$$\Rightarrow \frac{81}{2} - \frac{5}{2} = \boxed{38}$$

15) $\iint_R x^2\,dA$; R é a região delimitada por $y = \frac{16}{x}$, $y = x$ e $x = 8$



$$\int_4^8 \int_{16/x}^{x} x^2\,dy\,dx$$

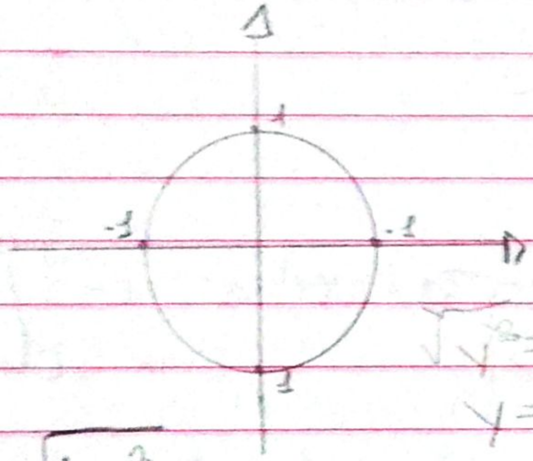
$\int_{16/x}^{x} x^2 dy = x^2 y \Big|_{16/x}^{x} = (x^2 \cdot x) - \left(x^2 \cdot \frac{16}{x}\right)$

$x^3 - 16x$

$\int_4^8 x^3 - 16x \, dx = \frac{x^4}{4} - \frac{16x^2}{2} \Big|_4^8 = \left(\frac{8^4}{4} - 8 \cdot 8^2\right) -$

$\left(\frac{4^4}{4} - 8 \cdot 4^2\right) = 512 + 64 = 576$

17) $\iint_R (3x - 2y) dA$; R é a região compreendida pelo circulo $x^2 + y^2 = 1$.



$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (3x - 2y) \, dy \, dx$

$\sqrt{y^2} = \sqrt{1 - x^2}$

$y = \pm\sqrt{1 - x^2}$

$\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (3x - 2y) \, dy = 3xy - \frac{2y^2}{2} \Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}$

$\Rightarrow \left[3x(\sqrt{1-x^2}) - (\sqrt{1-x^2})^2\right] - \left[3x(-\sqrt{1-x^2}) - (-\sqrt{1-x^2})^2\right]$

$-3x\sqrt{1-x^2}$