Lesson 9: Solving rate problems

Goals

- Apply reasoning about ratios and rates to convert and compare (in writing) distances expressed in different units.
- Apply reasoning about ratios and rates to justify (orally) whether a given price is a good deal.
- Practise arithmetic with fractions and decimals.

Learning Targets

- I can choose how to use unit rates to solve problems.

Lesson Narrative

In previous lessons, students have used tables of equivalent ratios to reason about unit rates. In this lesson, students gain fluency working with unit rates without scaffolding. They choose what unit rate they want to use to solve a problem, divide to find the desired unit rate, and multiply or divide by the unit rate to answer questions. They may choose to create diagrams to represent the situations, but the problems do not prompt students to do so. The activity about which animal ran the farthest requires students to use multiple unit rates in a sequence to be able to convert all the measurements to the same unit.

Addressing

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, bar models, double number line diagrams, or equations.
- Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
- Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Building Towards

- Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Instructional Routines

- Collect and Display
- Compare and Connect
- Poll the Class

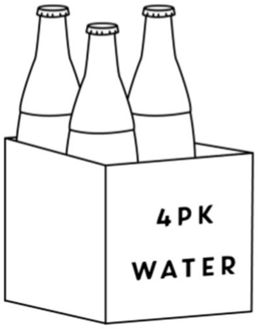
- Take Turns

Required Materials

Four-function calculators
Pre-printed slips, cut from copies of the blackline master

Card Sort: Is it a Deal?

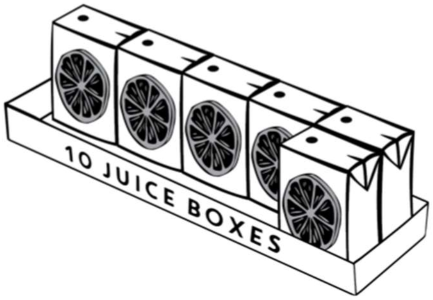
A. Drink pack



Original: 4 for £3.16
New deal: 3 for £2.25

Card Sort: Is it a Deal?

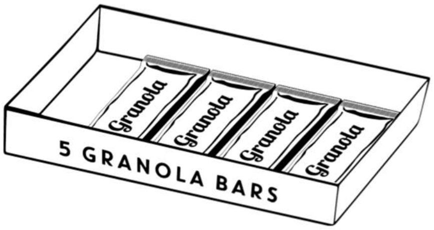
B. Juice boxes



Original: 10 for £3.50
New deal: 6 for £2.40

Card Sort: Is it a Deal?

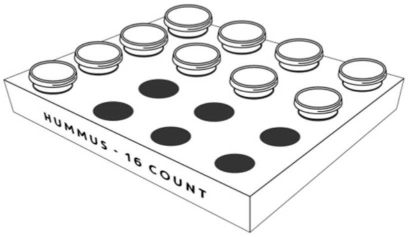
C. Granola bars



Original: 5 for £4.40
New deal: 4 for £3.12

Card Sort: Is it a Deal?

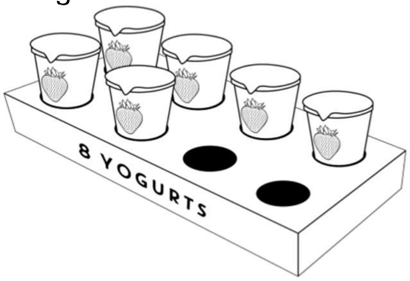
D. Hummus



Original: 16 for £14.40
New deal: 10 for £9.00

Card Sort: Is it a Deal?

E. Yoghurt



Original: 8 for £6.80
New deal: 6 for £5.22

Card Sort: Is it a Deal?

F. Fruit snacks

Original: 12 for £9.12
New deal: 9 for _______

Required Preparation

Print and cut the blackline master so that each group of two students gets five cards A–E (or six cards A–F if you expect students to tackle the extension problem).

Optionally, purchase a four-pack of drinks for demonstration purposes in the Deal or No Deal activity.

Providing access to calculators is optional. All of the calculations in this lesson can be done using KS2 techniques. If you would like students to practise arithmetic, don't offer calculators. If you think the calculations will present too much of a barrier make them available.

Student Learning Goals

Let's use unit rates like a pro.

9.1 Grid of 100

Warm Up: 5 minutes

In this warm-up, students are asked to name the shaded portion of a 10-by-10 grid, which is equal to 1. To discourage students from counting every square, flash the image for a few seconds and then hide it. Flash it once more for students to check their thinking. Ask, “How did you see the shaded portion?” instead of “How did you solve for the shaded portion?” so students can focus on the structure of the fractional pieces and tenths in the image. Encourage students to name the shaded portion in fractions or decimals. Some students may also bring up percentages.

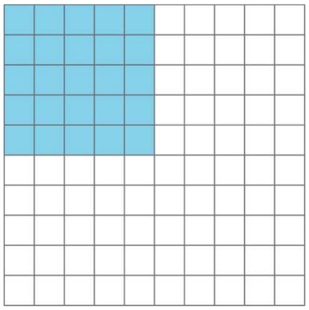
Launch

Tell students you will show them a 10×10 grid for 3 seconds and that the entire grid represents 1. Their job is to find how much is shaded in the image and explain how they saw it.

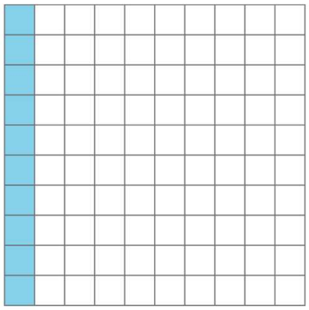
Display the image for 3 seconds and then hide it. Do this twice. Give students 15 seconds of quiet think time between each flash of the image. Encourage students who have one way of seeing the grid to consider another way to determine the size of the shaded portion while they wait.

Student Task Statement

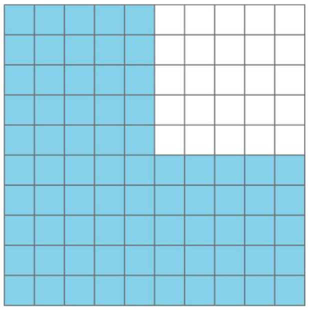
How much is shaded in each one?



A



B



C

Student Response

A: $\frac{25}{100}$, 0.25, $\frac{1}{4}$

B: $\frac{1}{10}$, $\frac{10}{100}$, 0.1, 0.10

C: $\frac{75}{100}$, 0.75, $\frac{3}{4}$

Activity Synthesis

Invite students to share how they visualised the shaded portion of each image. Record and display their explanations for all to see. Solicit from the class alternative ways of quantifying the shaded portion and alternative ways of naming the size of the shaded portion (to elicit names in fractions, decimals, and percentages). To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone solve the shaded portion the same way but would explain it differently?”
- “Did anyone solve the shaded portion in a different way?”
- “Does anyone want to add on to _____’s strategy?”
- “Do you agree or disagree? Why?”

9.2 Card Sort: Is it a Deal?

20 minutes

Students are given cards, each of which contains an original price and a new price, as shown.

B. Juice Boxes

Original: 10 for £3.50

New Deal: 6 for £2.40

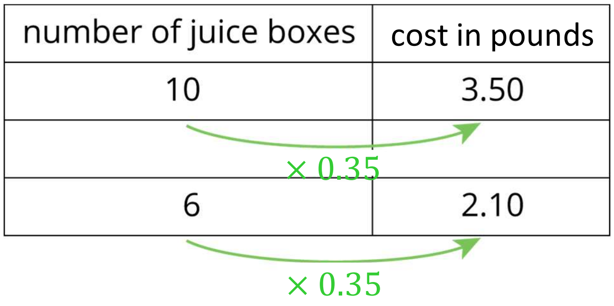
Their job is to sort the cards into two piles: one pile for deals they would take, and another for those they would reject. There are many paths students could use to reason about whether or not to accept a deal. For example, if the original deal was £3.50 for 10 juice boxes and the new deal is £2.40 for 6 juice boxes, they could:

- Find and compare the unit rates for both the original pack and the new pack. If the unit rate is the same, the deal is fair. If the unit rate is lower, the clerk is offering a discount. If the unit rate is higher, the clerk is not being fair.

number of juice boxes	cost in pounds	pounds per box
10	3.50	0.35
6	2.40	0.40

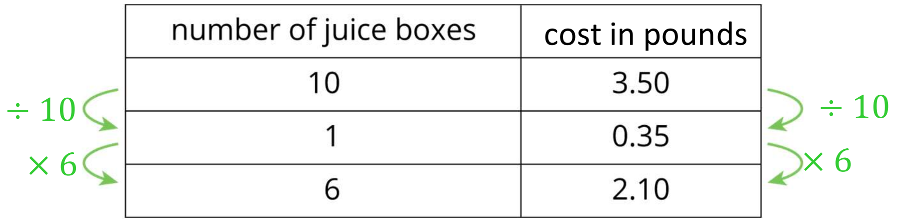
- Find the unit rate in the original pack, apply it to the number of items in the new pack, and compare the costs for the same number of items in the original and new pricing schemes. This can be done in two ways, one focused more on column reasoning and the other on row reasoning, as shown.

1.



number of juice boxes	cost in pounds
10	3.50
6	2.10

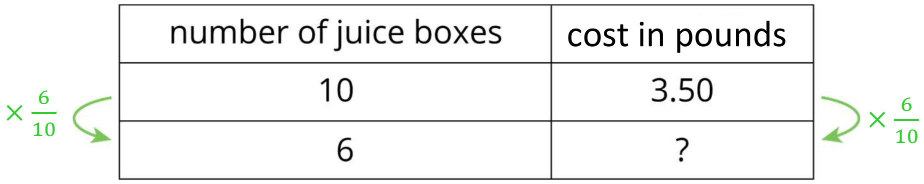
2.



number of juice boxes	cost in pounds
10	3.50
1	0.35
6	2.10

- Use an abbreviated table and bypass calculating the unit rate. Find the multiplier to get from the original to the new number of items, and use the same multiplier to find what the price would be if the deal has not changed. Compare the actual new price to this projected price.

3.



number of juice boxes	cost in pounds
10	3.50
6	?

As students work, attend to how they reason about the deals and make their decisions (deal or no deal).

Instructional Routines

- Collect and Display
- Poll the Class
- Take Turns

Launch

Show the picture on card A (or use an actual 4-pack of a beverage with a missing bottle.) Present the following situation:

"You've entered a local shop to buy a 4-pack of drinks. You find one last pack of the drink you want on the shelf and, unfortunately, only 3 bottles remain in that pack. You decide to buy it anyway. You take the 3-pack to the check-out counter and ask the clerk to consider a fair price for the incomplete pack. If the cost of a 4-pack was £3.16 and the clerk offers to sell the 3 pack for £2.25, will you take the deal?"

Poll the class for their response and display how many students would and would not take the deal. Then, ask "How could you figure out if the deal is good or not?" Give students a moment of quiet think time to come up with strategies for solving such a problem and then invite a few students to share.

Arrange students in groups of 2. Give each group a set of five cards A–E (or six cards A–F if including the extension problem). Tell students their job is to sort the cards into a 'Deal' pile and a 'No Deal' pile. Instruct partners to collaborate in finding the answer for card A and divide up the remaining cards between them. Ask students to first work on their cards individually, then share their reasoning with their partner, and lastly, sort the cards together.

Representation: Internalise Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have identified which initial cards were good deals.
Supports accessibility for: Conceptual processing; Organisation

Anticipated Misconceptions

Students who are not fluent in multiplication and division computation work from KS2 may need some review in order to be successful in this activity.

Student Task Statement

Your teacher will give you a set of cards showing different offers.

1. Find card A and work with your partner to decide whether the offer on card A is a good deal. Explain or show your reasoning.

2. Next, split cards B–E so you and your partner each have two.
    a. Decide individually if your two cards are good deals. Explain your reasoning.
    b. For each of your cards, explain to your partner if you think it is a good deal and why. Listen to your partner's explanations for their cards. If you disagree, explain your thinking.
    c. Revise any decisions about your cards based on the feedback from your partner.
3. When you and your partner are in agreement about cards B–E, place all the cards you think are a good deal in one stack and all the cards you think are a bad deal in another stack. Be prepared to explain your reasoning.

Student Response

Card A: Deal! The price per bottle is £0.79 so a 3 pack should be £2.37.

Card B: No Deal! The price per juice box is £0.35 so 6 juice boxes should be £2.10.

Card C: Deal! The price per bar is £0.88 so 4 bars should be £3.52.

Card D: Deal! The price per hummus container is £0.90 so the cost for 10 should be £9.00.

Card E: No Deal! The price per yogurt container is £0.85 so containers should cost £5.10.

Are You Ready for More?

Time to make your own deal! Read the information on card F and then decide what you would charge if you were the clerk. When your teacher signals, trade cards with another group and decide whether or not you would take the other group's offer.

Keep in mind that you may offer a fair deal or an unfair deal, but the goal is to set a price close enough to the value it should be that the other group cannot immediately tell if the deal you offer is a good one.

Student Response

Answers vary. A fair deal for F would be £6.84 for 9 packs.

Activity Synthesis

Select 2–3 students who used different but effective strategies to share their thinking with the class. Encourage students to listen to others' reasoning. Record the different strategies in one place and display them for all to see. Highlight any similarities and differences (e.g., whether a unit rate was used, whether students compare the original unit rate to the new quantity or the other way around, etc.)

Writing, Listening, Conversing: Collect and Display. Listen and observe how students reason about the deals and make their decisions (deal or no deal), and make note of the different

strategies students use to compare unit rates. Listen for language such as "the same," "equal," "unit rate," "cost for the same number of items," etc., and display these visually for the whole class to use as a reference. Continue to add to and refer to the display during the whole class discussion, making explicit connections between the language and the strategies used (i.e., whether students compare the original unit rate to the new quantity or the other way around). This will help students make sense of calculating and comparing unit rates while increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximise meta-awareness

9.3 The Fastest of All

15 minutes

In this activity, students convert between imperial and metric units in order to compare lengths. To make some measurements comparable to others, students need to perform multistep conversions and activate arithmetic skills from previous years. Support students with computations as needed and provide access to calculators as appropriate. Share the following information with students when requested.

- 1 mile = 1 760 yards = 5 280 feet
- 1 yard = 3 feet
- 1 foot = 12 inches
- 1 kilometre = 1 000 metres
- 1 metre = 100 centimetres

Expect students to choose different units of measurements to make comparisons. As students work, identify those who opt for the same unit so that they can partnered or grouped together for discussion.

Instructional Routines

- Compare and Connect

Launch

Give students 1–2 minutes to read the task, and then ask how they think they could compare these lengths. Students are likely to suggest converting all the lengths into the same unit of measurement. Ask students which units might be appropriate in this case and why. (Feet, yards, and metres are better choices than inches or miles.) After discussing some appropriate options, give students quiet think time to complete the activity, and then time to share their explanation with one or more students who have chosen to use the same unit of measurement.

Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2-3 minutes of work time. Look

for students who are converting all the distances to the same length.
Supports accessibility for: Memory; Organisation

Anticipated Misconceptions

Some students may need to be prompted about the intermediate steps needed to compare units that require several conversions before they can be compared.

Student Task Statement

Wild animals from around the world wanted to hold an athletic competition, but no one would let them on an airplane. They decided to just measure how far each animal could sprint in one minute and send the results to you to decide the winner.

You look up the following information about converting units of length:

1 inch = 2.54 centimetres

animal	sprint distance
cougar	1 408 yards
antelope	1 mile
hare	49 632 inches
kangaroo	1 073 metres
ostrich	1.15 kilometres
coyote	3 773 feet

1. Which animal sprinted the furthest?
2. What are the place rankings for all of the animals?

Student Response

1. Antelope wins first place.

2. Antelope, Cougar, Hare, Coyote, Ostrich, Kangaroo. Possible strategies:

- Converting all measurements to feet:

  Antelope sprinted 5 280 feet, because $1 \times 1\,760 \times 3 = 5\,280$.

  Cougar sprinted 4 224 feet, because $(1\,408) \times 3 = 4\,224$.

  Hare sprinted 4 136 feet, because $49\,632 \div 12 = 4\,136$.

  Coyote sprinted 3 773 feet.

  Ostrich sprinted almost 3 773 feet, because $(1.15) \times (1\,000) \times 100 \div 2.54 \div 12 = 3\,772.97$.

  Kangaroo sprinted 3 520 feet, because $(1\,073) \times 100 \div 2.54 \div 12 = 3\,520.34$.

- Converting all measurements to yards:

  Antelope sprinted 1 760 yards, because 1 mile = 1 760 yards.

  Cougar sprinted 1 408 yards.

  Hare sprinted 1 378$\frac{2}{3}$ yards, because $49\,632 \div 12 \div 3 = 1\,378\frac{2}{3}$.

  Coyote sprinted 1 257$\frac{2}{3}$ yards, because $3\,773 \div 3 = 1\,257\frac{2}{3}$.

  Ostrich sprinted almost 1 257$\frac{2}{3}$ yards, because $(1.15) \times (1\,000) \times 100 \div 2.54 \div 12 \div 3 = 1\,257.66$.

  Kangaroo sprinted 1 173$\frac{1}{3}$ yards, because $(1\,073) \times 100 \div 2.54 \div 12 \div 3 \approx 1\,173\frac{1}{3}$.

- Converting all measurements to metres:

  Antelope sprinted 1 609.34 metres, because $1 \times (1\,760) \times 3 \times 12 \times (2.54) \div 100 = 1\,609.34$.

  Cougar sprinted 1 287.48 metres, because $(1\,408) \times 3 \times 12 \times 2.54 \div 100 = 1\,287.48$.

  Hare sprinted 1 260.65 metres, because $(49\,632) \times (2.54) \div 100 = 1\,260.65$.

  Coyote sprinted just over 1 150 metres, because $3\,773 \times 12 \times 2.54 \div 100 = 1\,150.01$.

  Ostrich sprinted 1 150 metres, because $(1.15 \times 1\,000 = 1\,150$.

  Kangaroo sprinted 1 073 metres.

Activity Synthesis

Poll the class to see if they agree on who took first, second, third, and last place. If there is widespread agreement, invite two students to share: one student who converted all measurements to feet or yards, and another who converted everything to metres. If there are discrepancies, list the distances run by each animal in each unit of measurement and display them for all to analyse and double check. While the numerical values of the measurements in feet will all be greater than those in metres, the rank order will come out the same.

Speaking, Representing: Compare and Connect. Use this routine when students present their strategy and representation for determining the place rankings for all of the animals. Ask students to consider what is the same and what is different about each approach. Draw students' attention to the different units of measurements used to make comparisons, while making connections to the strategies used to make conversions. These exchanges can strengthen students' mathematical language use and reasoning to make sense of strategies used to convert units to be able to make comparisons.
Design Principle(s): Maximise meta-awareness

Lesson Synthesis

Emphasise that when we want to compare rates, a straightforward way is to compare unit rates. For example, when we were comparing the best deal, an example was 10 juice boxes for £3.50 or 6 juice boxes for £2.40. It may be helpful to draw two tables or two double number lines to facilitate discussion. Questions to discuss:

- "What are two associated unit rates that we could compare?" (0.35 and 0.4)
- "How were they computed?" (Divide 3.5 by 10 and divide 2.4 by 6)
- "What do these numbers mean in this context?" (They are each the price per bottle for the different offers. For example, £0.35 for 1 bottle.)

9.4 Tacos by the Pack

Cool Down: 5 minutes

Student Task Statement

A restaurant sells 10 tacos for £8.49, or 6 of the same kind of taco for £5.40.

Which is the better deal? Explain how you know.

Student Response

The 10-taco offer is a better deal.

Based on the price per taco: The 10-taco offer is about £0.85 per taco because $8.49 \div 10 = 0.849$. The 6-taco offer is £0.90 per taco because $5.40 \div 6 = 0.90$.

Or, look at the cost of a common multiple number of tacos. The least common multiple is 30. Using the 10-taco offer, 30 tacos cost £25.47. Using the 6-taco offer, 30 tacos cost £27.00. The 10-taco offer is better.

Based on how much you get for a pound (the calculations are more difficult with this approach): With the 10-taco offer, you get around 1.2 tacos per pound because $10 \div 8.49 \approx 1.2$. With the 6-taco offer, you get around 1.1 tacos per pound because $6 \div 5.40 \approx 1.1$.

Student Lesson Summary

Sometimes we can find and use more than one unit rate to solve a problem.

Suppose a supermarket is having a sale on shredded cheese. A small bag that holds 8 ounces is sold for £2. A large bag that holds 2 kilograms is sold for £16. How do you know which is a better deal?

Here are two different ways to solve this problem:

Compare pounds per kilogram.

- The large bag costs £8 per kilogram, because $16 \div 2 = 8$.
- The small bag holds $\frac{1}{2}$ pound of cheese, because there are 16 ounces in 1 pound, and $8 \div 16 = \frac{1}{2}$.

The small bag costs £4 per pound, because $2 \div \frac{1}{2} = 4$. This is about £8.80 per kilogram, because there are about 2.2 pounds in 1 kilogram, and $4.00 \times 2.2 = 8.80$.

The large bag is a better deal, because it costs less money for the same amount of cheese.

Compare ounces per pound.

- With the small bag, we get 4 ounces per pound, because $8 \div 2 = 4$.
- The large bag holds 2,000 grams of cheese. There are 1 000 grams in 1 kilogram, and $2 \times 1\,000 = 2\,000$. This means 125 grams per pound, because $2\,000 \div 16 = 125$.

There are about 28.35 grams in 1 ounce, and $125 \div 28.35 \approx 4.4$, so this is about 4.4 ounces per pound.

The large bag is a better deal, because you get more cheese for the same amount of money.

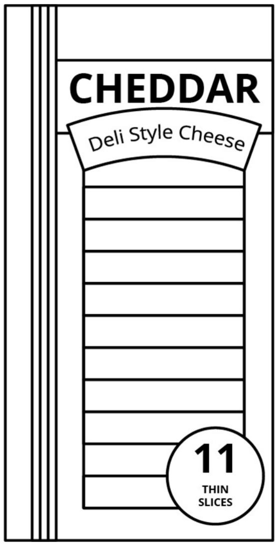
Another way to solve the problem would be to compare the unit prices of each bag in pounds per ounce. Try it!

Lesson 9 Practice Problems

Problem 1 Statement

This package of sliced cheese costs £2.97.

How much would a package with 18 slices cost at the same price per slice? Explain or show your reasoning.



Solution

£4.86. Sample reasoning: The package of 11 slices costs £2.97, so this is 27p per slice. A package of 18 slices at 27p per slice would cost £4.86.

Problem 2 Statement

A copy machine can print 480 copies every 4 minutes. For each question, explain or show your reasoning.

a. How many copies can it print in 10 minutes?

b. A teacher printed 720 copies. How long did it take to print?

Solution

a. 1 200 copies, because the rate is 120 copies per minute, and $120 \times 10 = 1\,200$.

b. 6 minutes, because $720 \div 120 = 6$

Problem 3 Statement

Order these objects from heaviest to lightest.

(Note: 1 pound = 16 ounces, 1 kilogram ≈ 2.2 pounds, and 1 ton = 2 000 pounds)

item	weight
school bus	9 tons
horse	1 100 pounds
elephant	5 500 kilograms
grand piano	15 840 ounces

Solution

school bus, elephant, horse, grand piano

item	weight	weight in pounds
school bus	9 tons	18 000
horse	1 100 pounds	1 100
elephant	5 500 kilograms	12 100
grand piano	15 840 ounces	990

Problem 4 Statement

Andre sometimes mows lawns on the weekend to make extra money. Two weeks ago, he mowed a neighbour's lawn for $\frac{1}{2}$ hour and earned £10. Last week, he mowed his uncle's lawn for $\frac{3}{2}$ hours and earned £30. This week, he mowed the lawn of a community centre for 2 hours and earned £30.

Which jobs paid better than others? Explain your reasoning.

Solution

The first two jobs paid better. His neighbour and his uncle both paid £20 per hour. For his neighbour, an hour of lawn mowing pays 10×2 or £20. His uncle paid £30 per $\frac{3}{2}$ hours, which means £10 every $\frac{1}{2}$ hour and £20 every hour. The third job at the community centre paid £15 per hour, since $30 \div 2 = 15$.

Problem 5 Statement

Calculate and express your answer in decimal form.

a. $\frac{1}{2} \times 17$

b. $\frac{3}{4} \times 200$

c. $(0.2) \times 40$

d. $(0.25) \times 60$

Solution

a. 8.5

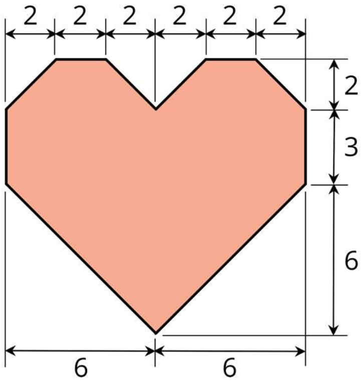
b. 150

c. 8

d. 15

Problem 6 Statement

Here is a polygon.

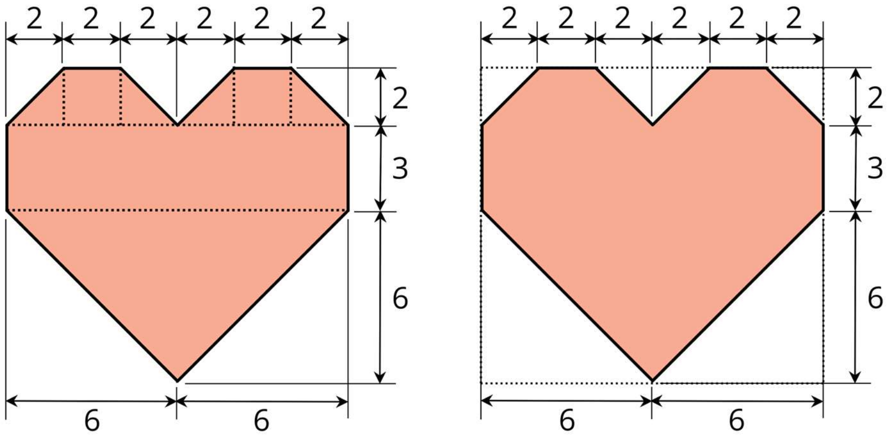


a. Decompose this polygon so that its area can be calculated. All measurements are in centimetres.

b. Calculate its area. Organise your work so that it can be followed by others.

Solution

a. Answers vary. One strategy is to decompose the polygon into triangles and rectangles and adding up their areas. Another is to enclose it with a rectangle, find its area, and subtract the unshaded right triangles from it.



b. 88 square centimetres. Reasonings vary.