Lesson 14: Finding cylinder dimensions

Goals

- Calculate the value of one dimension of a cylinder, and explain (orally and in writing) the reasoning.
- Create a table of dimensions of cylinders, and describe (orally) patterns that arise.

Learning Targets

- I can find missing information about a cylinder if I know its volume and some other information.

Lesson Narrative

In this lesson, students use the formula $V = \pi r^2 h$ for the volume of a cylinder to solve a variety of problems. They calculate volumes given radius and height, and find radius or height given a cylinder's volume and the other dimension by reasoning about the structure of the volume formula.

Addressing

- Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Instructional Routines

- Stronger and Clearer Each Time
- Discussion Supports
- Think Pair Share

Student Learning Goals

Let's figure out the dimensions of cylinders.

14.1 A Cylinder of Unknown Height

Warm Up: 5 minutes

The purpose of this warm-up is to assess students' understanding of the volume of a cylinder. Students learned that the volume of either a cylinder or prism is found by multiplying the area of the base by its height. In this warm-up, students are given information to find the area of a cylinder's base, but they are not given the height. Students propose a volume for the cylinder and explain why it works. Since the diameter of the base is 8, the area of the base is 16π.

If students have trouble getting started, ask them:

- "Do you have enough information to calculate the area of the base?"

- "What is the radius?"

Identify students who use these strategies:

- find the area of the base first then set up the equation $V = 16\pi h$.
- choose a specific value for h then solve for the volume.

Instructional Routines

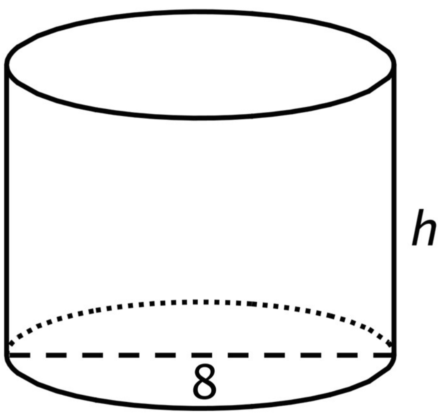
- Think Pair Share

Launch

Arrange students in groups of 2. Tell students that in a previous lesson, they learned how to find the volume of a cylinder if they know the cylinder's radius and height. Draw their attention to where volume formulas are displayed in the classroom as the unit progresses. Give students 1–2 minutes of quiet work time followed by time to explain their reasoning to their partner. Follow this with a whole-class discussion.

Student Task Statement

What is a possible volume for this cylinder if the diameter is 8 cm? Explain your reasoning.



Student Response

Answers vary. Sample response: The radius of the cylinder's base is 4 cm, which means the area of the base is 16π cm^2 since $4^2 \times \pi = 16\pi$. If the height is 1 cm, then the volume would be 16π cm^3 since $16\pi \times 1 = 16\pi$.

Activity Synthesis

The goal of this discussion is for students to communicate how the height of a cylinder is related to its volume. Invite students to share their solutions and their reasoning. Record and display the dimensions and volumes of cylinders that correspond to solutions given by students.

14.2 What's the Dimension?

15 minutes

In this activity, students find the missing dimensions of cylinders when given the volume and the other dimension. A volume equation representing the cylinder is given for each problem.

Identify students who use these strategies: guess and check, divide each side of the equation by the same value to solve for missing variable, or use the structure of the volume equation to reason about the missing variable

Instructional Routines

- Stronger and Clearer Each Time

Launch

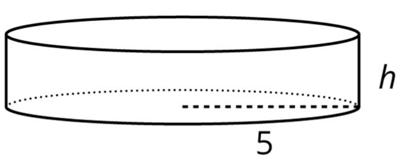
Arrange students in groups of 2. Give students 2–3 minutes of quiet work time followed by time to share their explanation for the first problem with their partners.

Representation: Internalise Comprehension. Activate or supply background knowledge of strategies students can use to solve for unknown variables in equations. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing Writing, Conversing: Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise a response to one or both of the questions. Ask each student to meet with 2–3 other partners in a row for feedback. Provide student with prompts for feedback that will help students strengthen their ideas and clarify their language (e.g., "What did you do first?", "How do you know h is...?", and "How did you use the volume formula?", etc.). Students can borrow ideas and language from each partner to strengthen their final explanation.
Design Principle(s): Optimise output (for explanation)

Student Task Statement

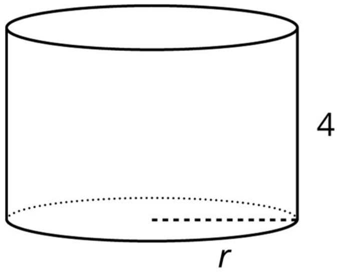
The volume V of a cylinder with radius r is given by the formula $V = \pi r^2 h$.

1. The volume of this cylinder with radius 5 units is 50π cubic units. This statement is true: $50\pi = 5^2\pi h$



What does the height of this cylinder have to be? Explain how you know.

2. The volume of this cylinder with height 4 units is 36π cubic units. This statement is true: $36\pi = r^2\pi 4$



What does the radius of this cylinder have to be? Explain how you know.

Student Response

1. The statement $50\pi = 5^2\pi h$ is equivalent to $50 = 25h$. Since 50 is 25 times 2, $h = 2$ units.

2. The statement $36\pi = r^2\pi 4$ is equivalent to $36 = r^2 \times 4$. Since 36 is 4 times 9, $r^2 = 9$. This implies $r = 3$ units.

Are You Ready for More?

Suppose a cylinder has a volume of 36π cubic inches, but it is not the same cylinder as the one you found earlier in this activity.

1. What are some possibilities for the dimensions of the cylinder?

2. How many different cylinders can you find that have a volume of 36π cubic inches?

Student Response

1. The volume for the cylinder is $36\pi = \pi \times r^2 \times h$, which implies $36 = h \times r^2$. Answers vary. Sample responses: the cylinder could have $r = 3$ and $h = 4$ or $r = 9$ and $h = \frac{4}{9}$.

2. There are an infinite number of cylinders with a volume of 36π cubic inches. No matter what value for r is chosen, a value for h can be calculated using the formula $36\pi = h \times r^2 \times \pi$.

Activity Synthesis

Select previously identified students to explain the strategies they used to find the missing dimension in each problem. If not brought up in students' explanations. Discuss the following strategies and explanations:

- Guess and check: plug in numbers for h, a value that make the statements true. Since the solutions for these problems are small whole numbers, this strategy works well. In other situations, this strategy may be less efficient.

- Divide each side of the equation by the same value to solve for the missing variable: for example, divide each side of $36\pi = r^2\pi 4$ by the common factor, 4π. It's important to remember π is a number that can be multiplied and divided like any other factor.

- Use the structure of the equation to reason about the missing variable: for example, 50π is double 25π, so the missing value must be 2.

14.3 Cylinders with Unknown Dimensions

15 minutes

The purpose of this activity is for students to use the structure of the volume formula for cylinders to find missing dimensions of a cylinder given other dimensions. Students are given the image of a generic cylinder with marked dimensions for the radius, diameter, and height to help their reasoning about the different rows in the table..

While completing the table, students work with approximations and exact values of π as well as statements that require reasoning about squared values. The final row of the table asks students to find missing dimensions given an expression representing volume that uses letters to represent the height and the radius. This requires students to manipulate expressions consisting only of variables representing dimensions.

Encourage students to make use of work done in some rows to help find missing information in other rows. Identify students who use this strategy and ask them to share during the discussion.

Instructional Routines

- Discussion Supports

Launch

Give students 6–8 minutes of work time followed by a whole-class discussion.

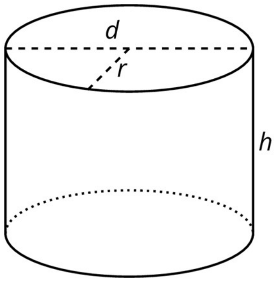
If short on time, consider assigning students only some of the rows to complete.

Engagement: Internalise Self-Regulation. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to choose and complete five out of eight rows in the table. Encourage students to select rows that contain varied information.
Supports accessibility for: Organisation; Attention

Anticipated Misconceptions

Students might try to quickly fill in the missing dimensions without the proper calculations. Encourage students to use the volume of a cylinder equation and the given dimensions to figure out the unknown dimensions.

Student Task Statement



Each row of the table has information about a particular cylinder. Complete the table with the missing dimensions.

diameter (units)	radius (units)	area of the base (square units)	height (units)	volume (cubic units)
	3		5	
12				108π
			11	99π
8				16π
			100	16π
	10			20π
20				314
			b	$\pi \times b \times a^2$

Student Response

diameter (units)	radius (units)	area of the base (square units)	height (units)	volume (cubic units)
6	3	9π	5	45π
12	6	36π	3	108π
6	3	9π	11	99π
8	4	16π	1	16π
0.8	0.4	0.16π	100	16π
20	10	100π	0.2 (or $\frac{1}{5}$)	20π
20	10	100π	1	314
$2a$	a	$a^2\pi$	b	$\pi \times b \times a^2$

Activity Synthesis

Select previously identified students to share their strategies. Ask students:

- "What patterns did you see as you filled out the table?" (Sample reasoning: Rows that had the same base area were easier to compare because their volume was the base area times height.)
- "Look at rows 1 and 3 in the table. How did having one row filled out help you fill out the other more efficiently?" (If the base areas were the same, then the radius and diameter must be the same also.)
- "How did you reason about the last row?"

Speaking: Discussion Supports. Provide sentence frames to support students as they share their strategies for completing the table. For example, "I noticed _____ in the (rows/columns)." or "I noticed _____, and it tells me that _____ ."
Design Principle(s): Support sense-making; Optimise output (for justification)

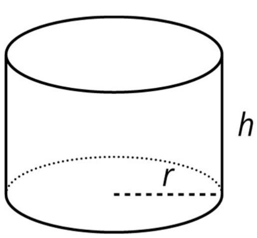
Lesson Synthesis

Working in groups of 2, tell students to choose one partner to name a value for the radius and one partner to name a value for the volume of a cylinder. Together, partners determine the height and make a sketch or their cylinder, including labels on the dimensions of their sketch. Display sketches and invite students to share their strategies for determining height.

14.4 Find the Height

Cool Down: 5 minutes

Student Task Statement



This cylinder has a volume of 12π cubic inches and a diameter of 4 inches. Find the cylinder's radius and height.

Student Response

The radius is 2 inches, and the height is 3 inches. Since the diameter is 4 inches, the radius is half of 4 inches. The volume is $12\pi = 2^2\pi h$, which means $12\pi = 4\pi h$ and $h = 3$.

Student Lesson Summary

In an earlier lesson we learned that the volume, V, of a cylinder with radius r and height h is $V = \pi r^2 h$.

We say that the volume depends on the radius and height, and if we know the radius and height, we can find the volume. It is also true that if we know the volume and one dimension (either radius or height), we can find the other dimension.

For example, imagine a cylinder that has a volume of 500π cm³ and a radius of 5 cm, but the height is unknown. From the volume formula we know that

$$500\pi = \pi \times 25 \times h$$

must be true. Looking at the structure of the equation, we can see that $500 = 25h$. That means that the height has to be 20 cm, since $500 \div 25 = 20$.

Now imagine another cylinder that also has a volume of 500π cm³ with an unknown radius and a height of 5 cm. Then we know that

$$500\pi = \pi \times r^2 \times 5$$

must be true. Looking at the structure of this equation, we can see that $r^2 = 100$. So the radius must be 10 cm.

Lesson 14 Practice Problems

1. **Problem 1 Statement**

   Complete the table with all of the missing information about three different cylinders.

diameter of base (units)	area of base (square units)	height (units)	volume (cubic units)
4		10	
6			63π
	25π	6	

Solution

Answers vary. Sample response:

diameter of base (units)	area of base (square units)	height (units)	volume (cubic units)
4	4π	10	40π
6	9π	7	63π
10	25π	6	150π

2. **Problem 2 Statement**

A cylinder has volume 45π and radius 3. What is its height?

Solution

5 units (Solve $45\pi = \pi \times 3^2 \times h$.)

3. **Problem 3 Statement**

Three cylinders have a volume of 2 826 cm^3. Cylinder A has a height of 900 cm. Cylinder B has a height of 225 cm. Cylinder C has a height of 100 cm. Find the radius of each cylinder. Use 3.14 as an approximation for π.

Solution

- Cylinder A has a radius of 1 cm.
- Cylinder B has a radius of 2 cm.
- Cylinder C has a radius of 3 cm.

4. **Problem 4 Statement**

A petrol company's delivery truck has a cylindrical tank that is 14 feet in diameter and 40 feet long.

a. Sketch the tank, and mark the radius and the height.

b. How much petrol can fit in the tank?

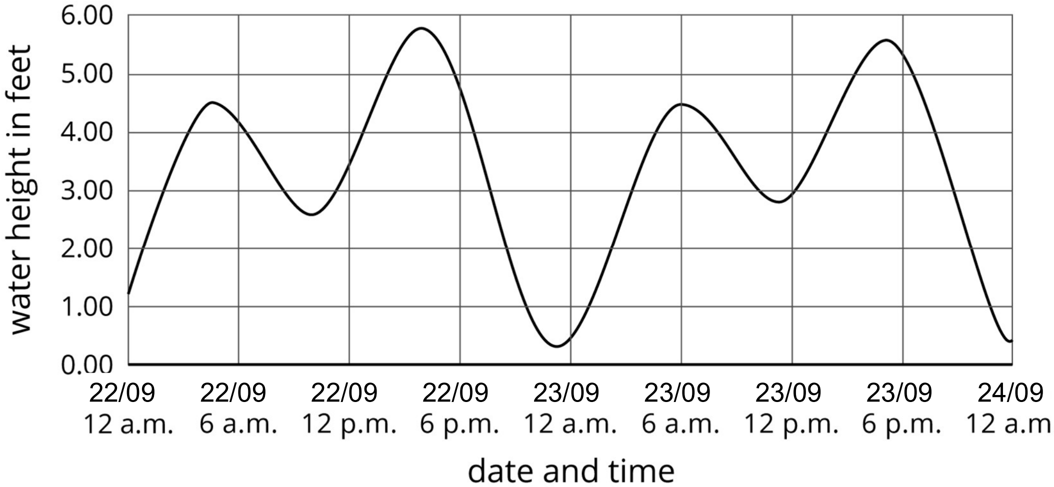
Solution

a. Answers vary. The radius is 7 feet, and the height is 40 feet.

b. About 6 158 cubic feet (The volume of the cylinder is given by $V = \pi \times r^2 \times h$ where $r = 7$ and $h = 40$. Using a close approximation of π gives an approximate volume of 6 158 cubic feet, but different answers may be found if a different approximation of π is used.)

5. **Problem 5 Statement**

Here is a graph that shows the water height of the ocean between September 22 and September 24, 2016 in Bodega Bay, CA.



a. Estimate the water height at 12 p.m. on September 22.

b. How many times was the water height 5 feet? Find two times when this happens.

c. What was the lowest the water got during this time period? When does this occur?

d. Does the water ever reach a height of 6 feet?

Solution

a. 3.5 feet

b. 4 times: approximately 2 p.m. and 6 p.m. on 22/9, 3 p.m. and 7 p.m. on 23/9

c. The lowest height was at about 11 p.m. on 22/9. The water height then was about 0.3 feet.

d. No, the highest it ever got was about 5.8 feet.