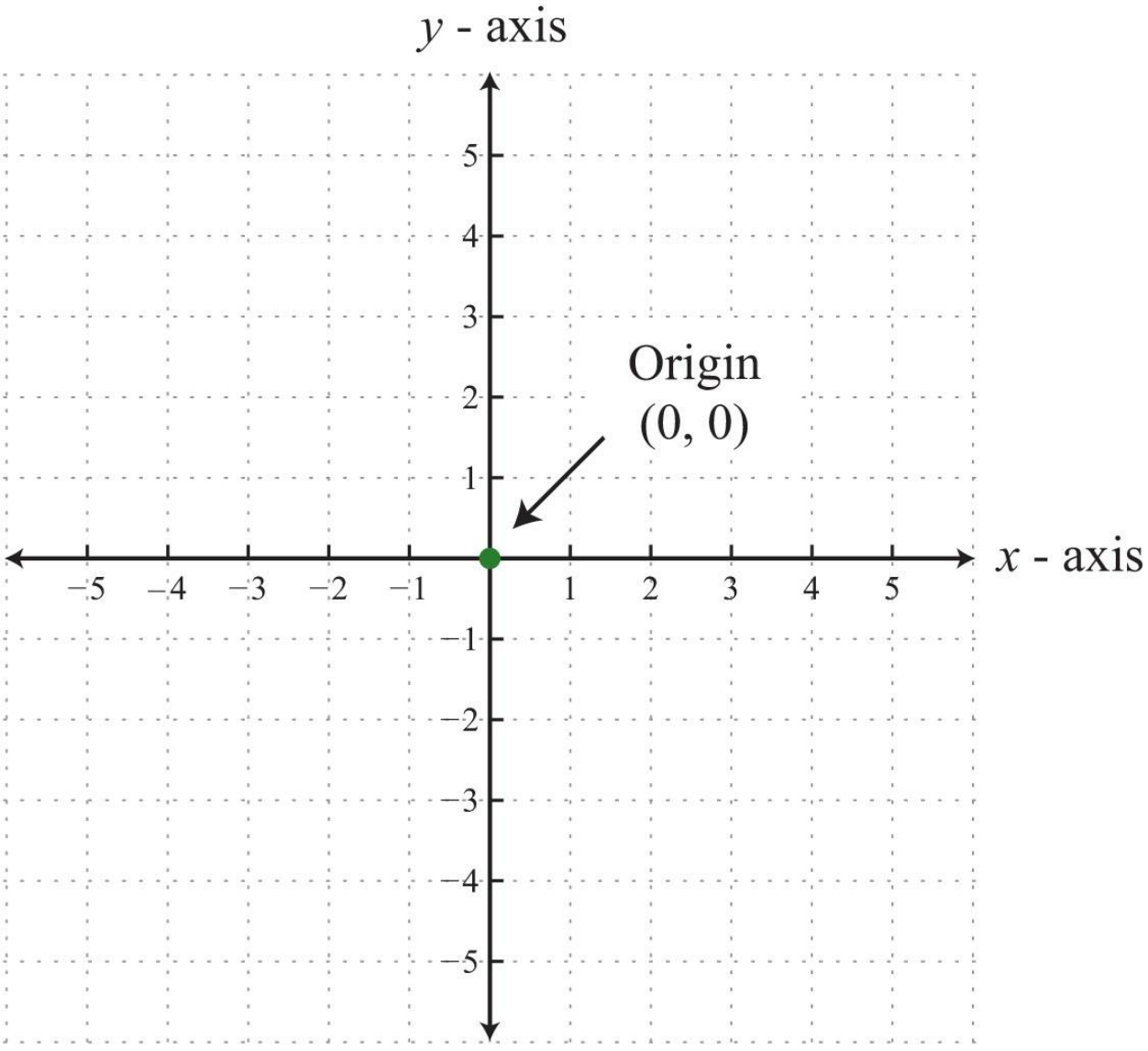
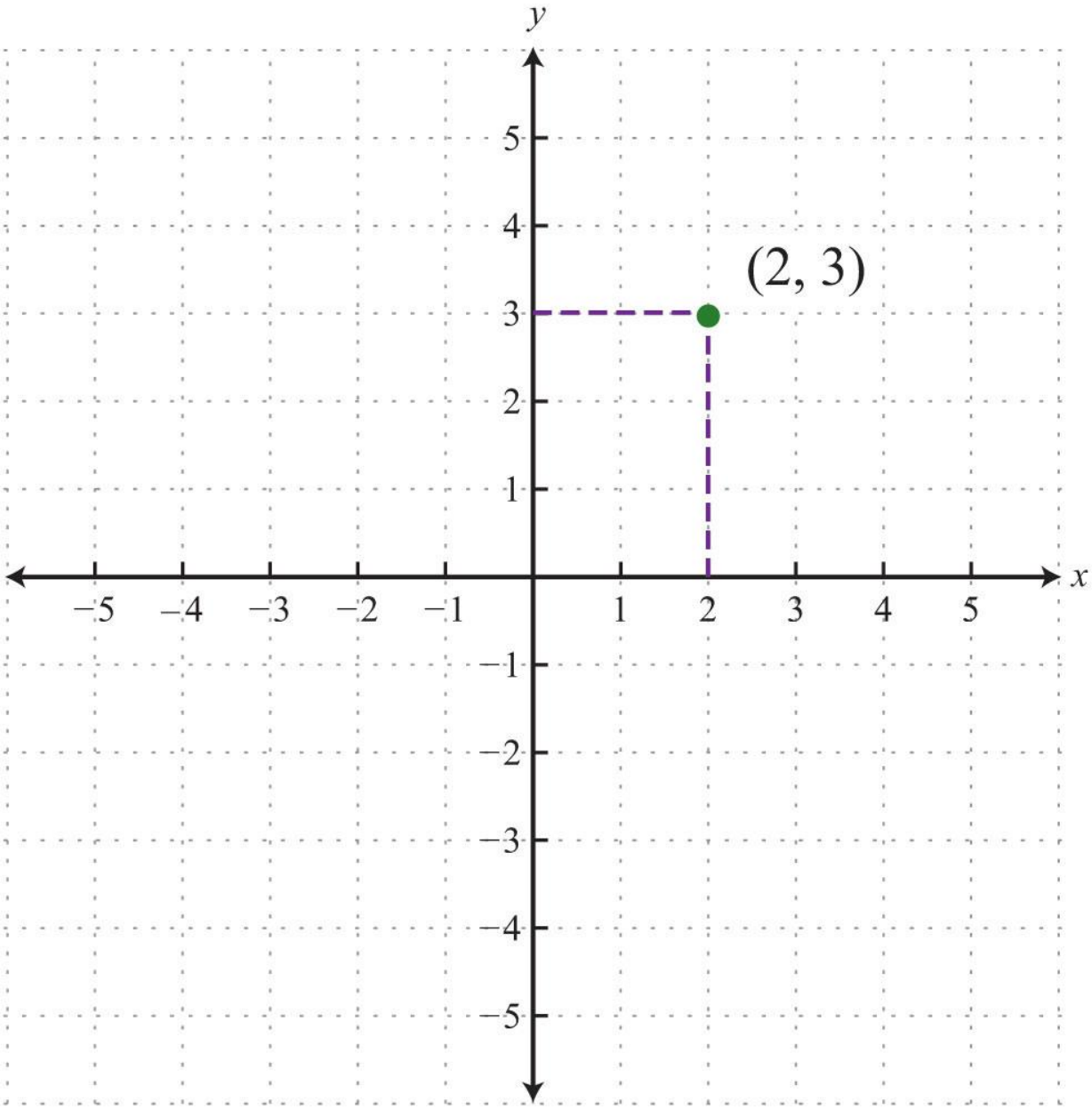
Rectangular Coordinate System

The rectangular coordinate system consists of two real number lines that intersect at a right angle. The horizontal number line is called the x-axis, and the vertical number line is called the y-axis. These two number lines define a flat surface called a plane, and each point on this plane is associated with an ordered pair of real numbers (x, y). The first number is called the ***x*-coordinate**, and the second number is called the ***y*-coordinate**. The intersection of the two axes is known as the origin, which corresponds to the point (0, 0).



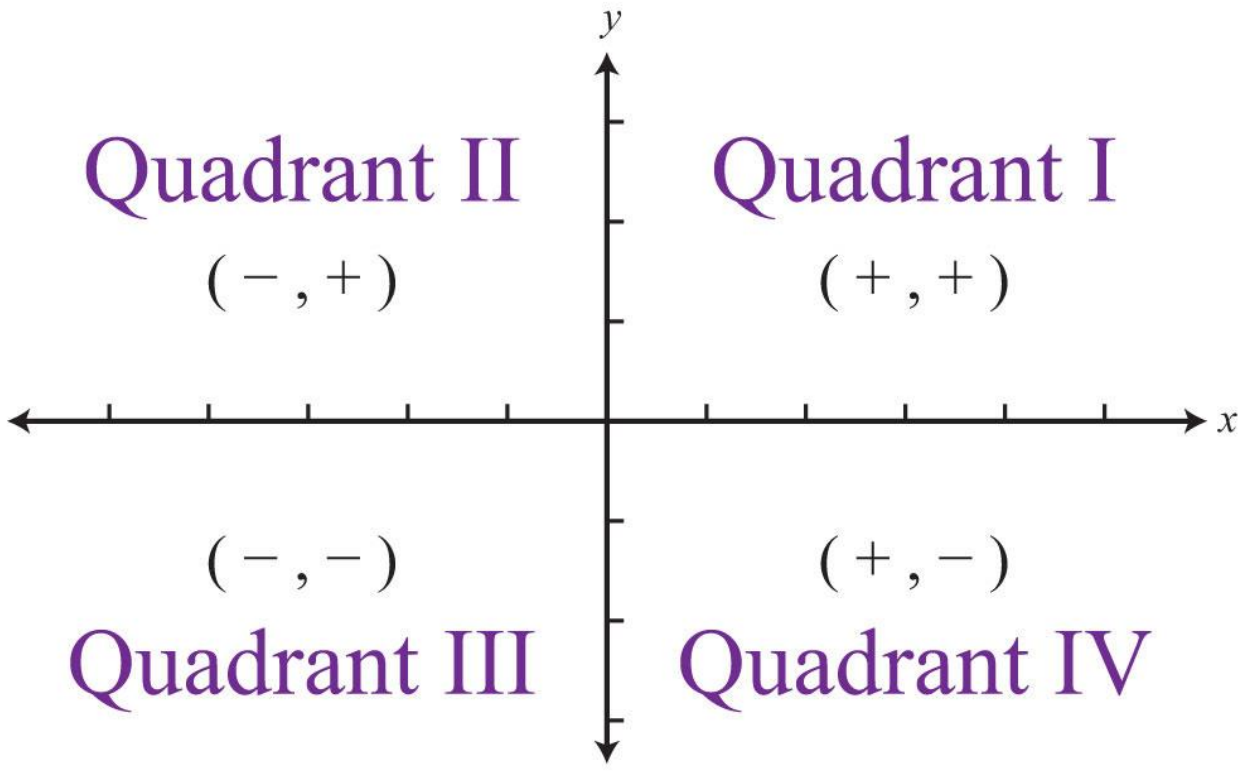
An ordered pair (x, y) represents the position of a point relative to the origin. The x-coordinate represents a position to the right of the origin if it is positive and to the left of the origin if it is negative. The y-coordinate represents a position above the origin if it is positive and below the origin if it is negative. Using this system, every position (point) in the plane is uniquely identified. For example, the pair (2, 3) denotes the position relative to the origin as shown:



Portrait of René Descartes (1596–1650) after Frans Hals, from http://commons.wikimedia.org/wiki/File:Frans_Hals_-_Portret_van_Ren%C3%A9_Descartes.jpg.

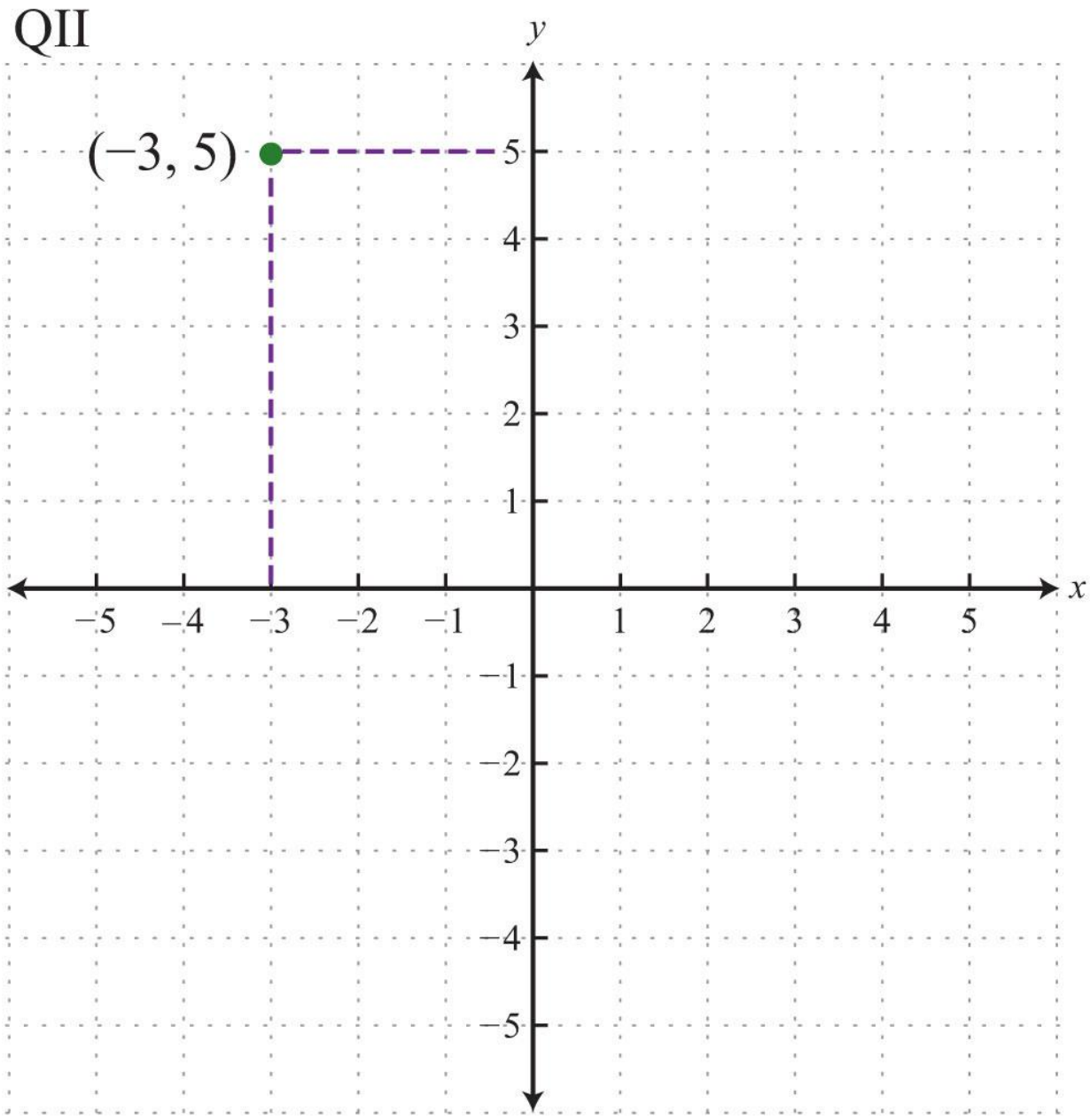
This system is often called the Cartesian coordinate system, named after the French mathematician René Descartes (1596–1650).

The x- and y-axes break the plane into four regions called quadrants, named using roman numerals I, II, III, and IV, as pictured. In quadrant I, both coordinates are positive. In quadrant II, the x-coordinate is negative and the y-coordinate is positive. In quadrant III, both coordinates are negative. In quadrant IV, the x-coordinate is positive and the y-coordinate is negative.



Example 1: Plot the ordered pair (–3, 5) and determine the quadrant in which it lies.

Solution: The coordinates x=−3 and y=5 indicate a point 3 units to the left of and 5 units above the origin.

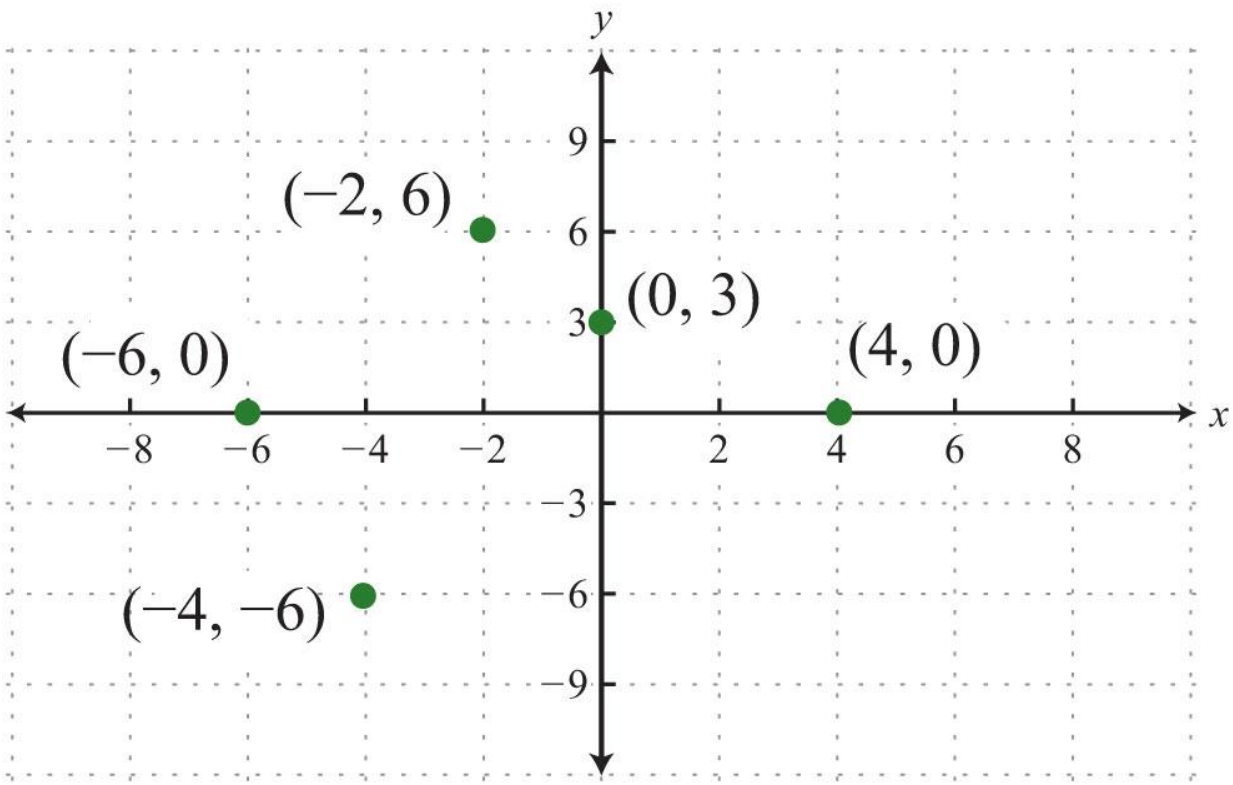


Answer: The point is plotted in quadrant II (QII) because the x-coordinate is negative and the y-coordinate is positive.

Ordered pairs with 0 as one of the coordinates do not lie in a quadrant; these points are on one axis or the other (or the point is the origin if both coordinates are 0). Also, the scale indicated on the *x*-axis may be different from the scale indicated on the *y*-axis. Choose a scale that is convenient for the given situation.

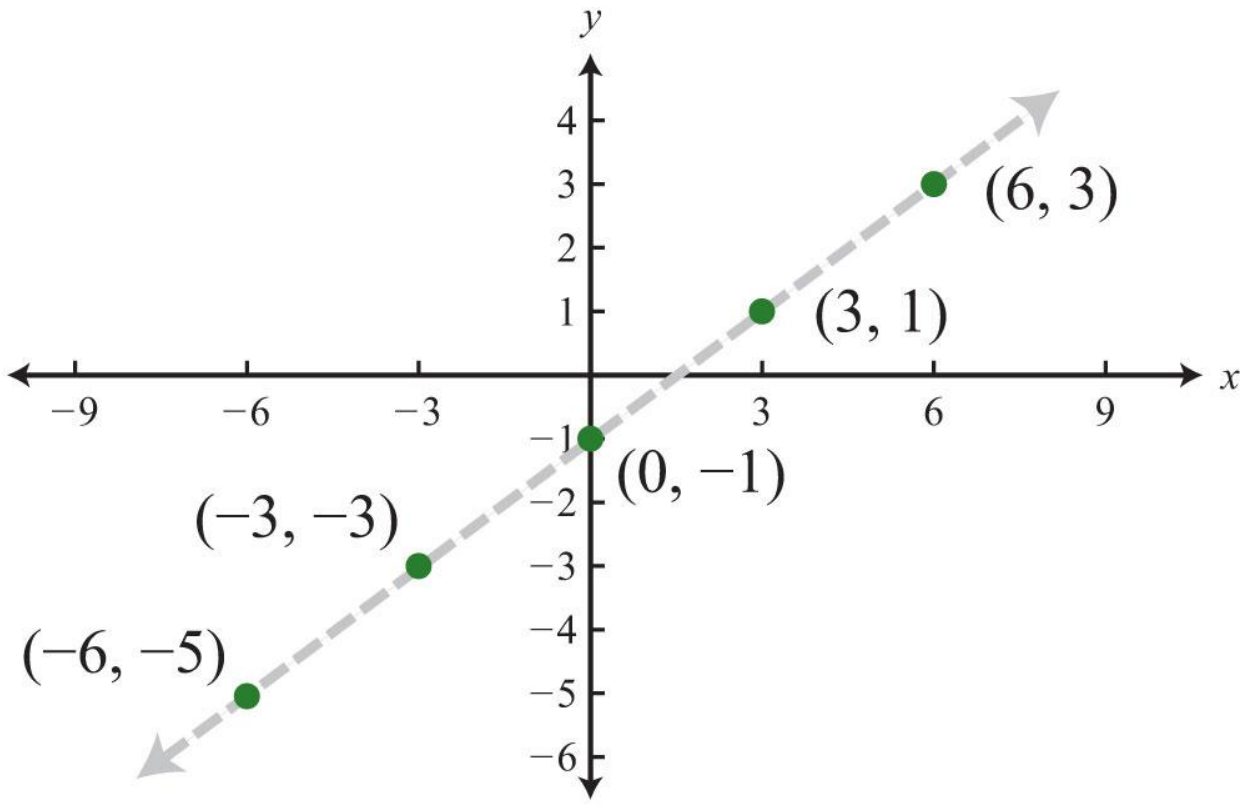
Example 2: Plot this set of ordered pairs: {(4, 0), (−6, 0), (0, 3), (−2, 6), (−4, −6)}.

Solution: Each tick mark on the *x*-axis represents 2 units and each tick mark on the *y*-axis represents 3 units.



Example 3: Plot this set of ordered pairs: {(−6, −5), (−3, −3), (0, −1), (3, 1), (6, 3)}.

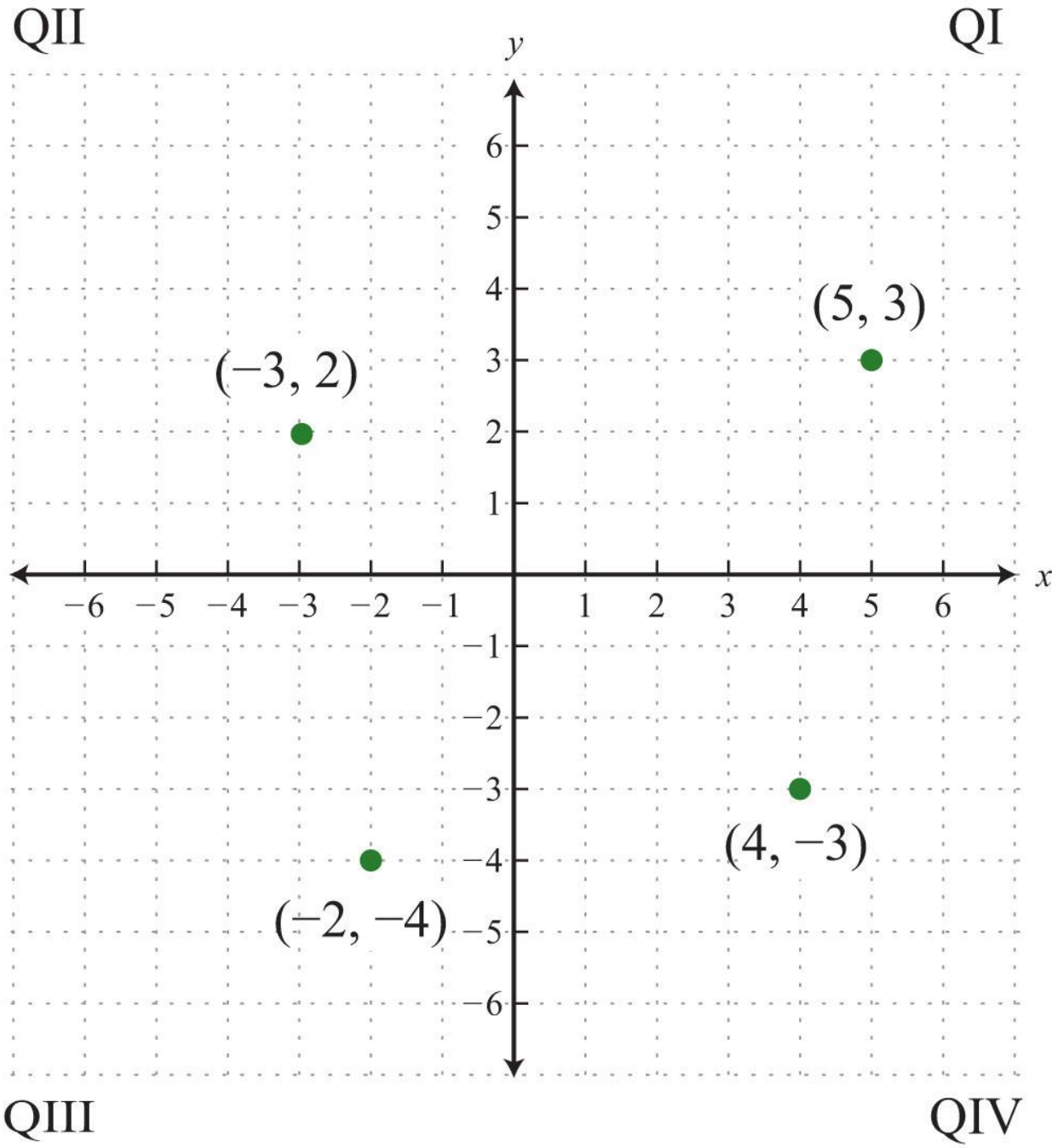
Solution:



In this example, the points appear to be collinear, or to lie on the same line. The entire chapter focuses on finding and expressing points with this property.

Try this! Plot the set of points {(5, 3), (−3, 2), (−2, −4), (4, −3)} and indicate in which quadrant they lie. ([Link: Click here for printable graph paper in PDF.])

Answer:


QII
QI
y
6
5
4
3
2
1
(5, 3)
(−3, 2)
−6 −5 −4 −3 −2 −1
1 2 3 4 5 6
x
−1
−2
−3
−4
−5
−6
(4, −3)
(−2, −4)
QIII
QIV