Lesson 9: When are they the same?

Goals

- Create an equation in one variable to represent a situation in which two conditions are equal.
- Interpret the solution of an equation in one variable in context.

Learning Targets

- I can use an expression to find when two things, like height, are the same in a real-world situation.

Lesson Narrative

In this lesson students apply their knowledge of solving equations by considering two real world situations: two tanks where one is filling and the other is emptying and two elevators traveling above and below ground level. Using the given expressions for each situation, students are asked to determine when the amount of water in the tanks or the travel time of elevators will be the same. It is the work of the student to recognise that they can set the two expressions equal and solve the equation for the unknown and this work sets up the concept of substitution for the coming section on systems of linear equations.

Addressing

- Analyse and solve linear equations and pairs of simultaneous linear equations.
- Solve linear equations in one variable.
- Analyse and solve pairs of simultaneous linear equations.

Building Towards

- Analyse and solve pairs of simultaneous linear equations.

Instructional Routines

- Collect and Display
- Three Reads
- Think Pair Share

Student Learning Goals

Let’s use equations to think about situations.

9.1 Which Would You Choose?

Warm Up: 5 minutes

The purpose of this warm-up is for students to reason about two situations that can be represented with linear equations. Since the number of babysitting hours determines which situation would be most profitable, there is no one correct answer to the question. Students are asked to explain their reasoning.

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

If you were babysitting, would you rather

- Charge £5 for the first hour and £8 for each additional hour?

Or

- Charge £15 for the first hour and £6 for each additional hour?

Explain your reasoning.

Student Response

Answers vary. Students may choose to charge £15 for the first hour and £6 for each additional hour if they are only babysitting for up to 5 hours. Students may say it doesn't matter which one they choose if they babysit for 6 hours (the first hour plus 5 additional hours) because the amount they will earn is the same. Students may choose £5 for the first hour and £8 for each additional hour if they are babysitting more than 6 hours.

Activity Synthesis

Poll the class on which situation they would choose. Invite students from each side to explain their reasoning. Record and display these ideas for all to see. If no one reasoned about babysitting for less than 5 hours and therefore chose the second option, mention this idea to students.

Students may not use linear equations or graphs to decide which situation they would choose. If there is time, ask students for the equation and graph we could use to model each scenario.

9.2 Water Tanks

10 minutes

The goal of this activity, and the two that follow, is for students to solve an equation in a real-world context while previewing some future work solving systems of equations. Here, students first make sense of the situation using a table of values describing the water heights of two tanks and then use the table to estimate when the water heights are equal. A key point in this activity is the next step: taking two expressions representing the water heights in two different tanks for a given time and recognising that the equation created by setting the two expressions equal to one another has a solution that is the value for time, t, when the water heights are equal.

Instructional Routines

- Collect and Display

Launch

Give students 2–3 minutes to read the context and answer the first problem.
Select students to share their answer with the class, choosing students with different representations of the situation if possible. Give 3–4 minutes for the remaining problems followed by a whole-class discussion.

Representation: Internalise Comprehension. Represent the same information through different modalities by using diagrams. If students are unsure where to begin, suggest that they draw a diagram to help illustrate the information provided.
Supports accessibility for: Conceptual processing; Visual-spatial processing Representing, Conversing: Collect and Display. To begin the whole-class discussion, give students the opportunity to discuss their solutions to the first question in groups of 3–4. Circulate through the groups and record language students use to describe what is happening in each tank. Listen for language related to rate of change, differences between rates, initial amounts of water, etc. If groups are stuck, consider asking, “What happens to the amount of water in Tank 1 (or 2) as time goes on?” and “Can you draw a picture of Tank 1 (or 2) at 5 minutes and then another picture at 10 minutes? What do you notice?” Post the collected language in the front of the room so that students can refer to it throughout the rest of the activity and lesson. This will help students talk about the relationship between the two tanks prior to being asked to find the time when they are equal.
Design Principle(s); Maximise meta-awareness

Student Task Statement

The amount of water in two tanks every 5 minutes is shown in the table.

time (minutes)	tank 1 (litres)	tank 2 (litres)
0	25	1 000
5	175	900
10	325	800
15	475	700
20	625	600
25	775	500
30	925	400

35	1 075	300
40	1 225	200
45	1 375	100
50	1 525	0

1. Describe what is happening in each tank. Either draw a picture, say it verbally, or write a few sentences.

2. Use the table to estimate when the tanks will have the same amount of water.

3. The amount of water (in litres) in tank 1 after t minutes is $30t + 25$. The amount of water (in litres) in tank 2 after t minutes is $-20t + 1\,000$. Find the time when the amount of water will be equal.

Student Response

1. Answers vary. Sample response: the water in Tank 1 is increasing while the water in Tank 2 is decreasing.

2. The tanks will have the same amount of water between 15 and 20 minutes, but closer to 20 minutes.

3. When the amounts of water in the two tanks are equal, $30t + 25 = -20t + 1\,000$. This happens when $t = 19.5$, so 19 and a half minutes after the start.

Activity Synthesis

The purpose of this discussion is to elicit student thinking about why setting the two expressions in the task statement equal to one another is both possible and a way to solve the final problem.

Consider asking the following questions:

- “What does t represent in the first expression? The second?” (In each expression, t is the time in minutes since the tank’s water level started being recorded.)

- “After we substitute a time in for t and simplify one of the expressions to be a single number, what does that number represent? What units does it have?” (The number represents the amount of litres in the water tank.)

- “How accurate was your estimate about the water heights using the table?” (My estimate was within a few minutes of the actual answer.)

- “If you didn’t know which expression in the last problem belonged to which tank, how could you figure it out?” (One of the expressions is increasing as t increased, which means it must be Tank 1. The other is decreasing as t increases, so it must be Tank 2.)

- “How did you find the time the two water heights were equal using the expressions?” (Since each expression gives the height for a specific time, t, and we want to know when the heights are equal, I set the two expressions equal to each other and then solved for the t-value that made the new equation true.)

9.3 Elevators

15 minutes

In this activity, students work with two expressions that represent the travel time of an elevator to a specific height. As with the previous activity, the goal is for students to work within a real-world context to understand taking two separate expressions and setting them equal to one another as a way to determine more information about the context.

Instructional Routines

- Three Reads
- Think Pair Share

Launch

Give students 1 minute to read the context and the problems. You may wish to share with the class that programming elevators in buildings to best meet the demands of the people in the building can be a complicated task depending on the number of floors in a building, the number of people, and the number of elevators. For example, many large buildings in cities have elevators programmed to stay near the ground floor in the morning when employees are arriving and then stay on higher floors in the afternoon when employees leave work.

Arrange students in groups of 2. Give 2–3 minutes of quiet work time for the first two questions and then ask students to pause and discuss their solutions with their partner. Give 3–4 minutes for partners to work on the remaining questions followed by a whole-class discussion.

Reading, Representing: Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. Display only the diagram and the description of the situation, without revealing the questions. In the first read, students read the problem with the goal of comprehending the situation (e.g., A building has two elevators that go above and below ground.). Use the second read to identify important quantities without yet focusing on specific values. Listen for, and amplify, the important quantities that vary in relation to each other in this situation: height of elevator A, in metres; height of elevator B, in metres; elapsed time, in seconds. For the third read, reveal the questions and ask students to brainstorm possible ways to use the given expressions to determine when the elevators will reach ground level at the same time. This will help students connect the language in the word problem and the reasoning needed to solve the problem.
Design Principle(s): Support sense-making

Anticipated Misconceptions

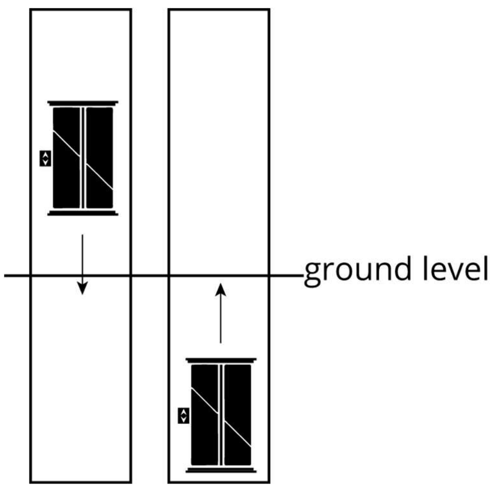
Students may mix up height and time while working with these expressions. For example, they may think that at $h = 0$, the height of the elevators is 16 metres and 12 metres, respectively, instead of the correct interpretation that the elevators reach a height of 0 metres at 16 seconds and 12 seconds, respectively.

Student Task Statement

A building has two elevators that both go above and below ground.

At a certain time of day, the travel time it takes elevator A to reach height h in metres is $0.8h + 16$ seconds.

The travel time it takes elevator B to reach height h in metres is $-0.8h + 12$ seconds.



1. What is the initial height of each elevator?

2. How long would it take each elevator to reach ground level?

3. If the two elevators travel toward one another, at what height do they pass each other? How long would it take?

4. If you are on an underground parking level 14 metres below ground, which elevator would reach you first?

Student Response

1. Elevator A is 20 metres below ground. At $t = 0$, the initial height of Elevator A can be found by solving the equation $0 = 0.8h + 16$. Elevator B is 15 metres above ground. At $t = 0$, the initial height of Elevator B can be found by solving the equation $0 = -0.8h + 12$.

2. The elevator reaches the ground when $h = 0$. Elevator A reaches ground level after 16 seconds because $0.8(0) + 16 = 16$. Elevator B reaches ground level after 12 seconds because $-0.8(0) + 12 = 12$.

3. At 14 seconds both elevators are 2.5 metres below ground. The solution can be found by solving the equation $0.8h + 16 = -0.8h + 12$.

4. Elevator A will reach you first, in 4.8 seconds, because $0.8(-14) + 16 = 4.8$. Elevator B will reach your level in 23.2 seconds because $-0.8(-14) + 12 = 23.2$.

Are You Ready for More?

1. In a two-digit number, the ones digit is twice the tens digit. If the digits are reversed, the new number is 36 more than the original number. Find the number.

2. The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number is 45 less than the original number. Find the number.

3. The sum of the digits in a two-digit number is 8. The value of the number is 4 less than 5 times the ones digit. Find the number.

Student Response

1. $10x + 2x + 36 = 10(2x) + x$ or equivalent, where x represents the tens digit. $x = 4$ so the number is 48.

2. $10x + (11 - x) - 45 = 10(11 - x) + x$ or equivalent, where x represents the tens digit. $x = 8$ so the number is 83.

3. $10x + (8 - x) = 5(8 - x) - 4$ or equivalent, where x represents the tens digit. $x = 2$ so the number is 26.

Activity Synthesis

This discussion should focus on the act of setting the two expressions equal and what that means in the context of the situation.

Consider asking the following questions:

- "If someone thought that the height of Elevator A before we started timing was 16 metres because they substituted 0 for the variable of the expression $0.8h + 16$ and got 16, how would you help them correct their answer?" (I would remind them that h is height and $0.8h + 16$ is the time, so when we start timing at 0 that means $0.8h + 16 = 0$, not that $h = 0$.)

- "Which of the elevators in the image is A and which is B? How do you know?" (A is the elevator on the right since at time 0 the height is negative, while B is the elevator on the left since at time 0 the height is positive.)

- "How did you find the height when the travel times are equal?" (Since each expression gives the time to travel to a height h, I solved the equation $0.8h + 16 = -0.8h + 12$, which gives the value of h when the two expressions are equal.)

Representation: Internalise Comprehension. Use colour and annotations to illustrate student thinking. As students describe their strategies, use colour and annotations to scribe their

thinking on a display of the problem so that it is visible for all students.
Supports accessibility for: Visual-spatial processing; Conceptual processing

Lesson Synthesis

The work in this lesson is a prelude to a simple form of a system of equations, where each equation can be written in the form $y =$ some expression (though students do not need to know the term "system of equations" at this point).

Arrange students in groups of 2. Ask partners to think of other situations where two quantities are changing and they want to know when the quantities are equal. Give groups time to discuss and write down a few sentences explaining their situation. Invite groups to share their situation with the class. (For example, in a race where participants walk at steady rates but the slower person has a head start, when will they meet?) Consider allowing groups to share their situation by making a picture, a graph, in words, or by acting it out.

9.4 Printers and Ink

Cool Down: 5 minutes

Student Task Statement

To own and operate a home printer, it costs £100 for the printer and an additional £0.05 per page for ink. To print out pages at an office store, it costs £0.25 per page. Let p represent number of pages.

1. What does the equation $100 + 0.05p = 0.25p$ represent?

2. The solution to that equation is $p = 500$. What does the solution mean?

Student Response

1. The equation represents when the cost for owning and operating a home printer is equal to the cost for printing at an office store.

2. The solution of $p = 500$ means that the costs are equal for printing 500 pages.

Cost to own and operate a home printer: $100 + 0.05(500) = 125$ pounds.

Cost to print out pages at an office store: $0.25(500) = 125$ pounds.

Student Lesson Summary

Imagine a full 1 500 litre water tank that springs a leak, losing 2 litres per minute. We could represent the number of litres left in the tank with the expression $-2x + 1\,500$, where x represents the number of minutes the tank has been leaking.

Now imagine at the same time, a second tank has 300 litres and is being filled at a rate of 6 litres per minute. We could represent the amount of water in litres in this second tank with the expression $6x + 300$, where x represents the number of minutes that have passed.

Since one tank is losing water and the other is gaining water, at some point they will have the same amount of water—but when? Asking when the two tanks have the same number of litres is the same as asking when $-2x + 1\,500$ (the number of litres in the first tank after x minutes) is equal to $6x + 300$ (the number of litres in the second tank after x minutes),

$-2x + 1\,500 = 6x + 300$.

Solving for x gives us $x = 150$ minutes. So after 150 minutes, the number of litres of the first tank is equal to the number of litres of the second tank. But how much water is actually in each tank at that time? Since both tanks have the same number of litres after 150 minutes, we could substitute $x = 150$ minutes into either expression.

Using the expression for the first tank, we get $-2(150) + 1\,500$ which is equal to $-300 + 1\,500$, or 1 200 litres.

If we use the expression for the second tank, we get $6(150) + 300$, or just $900 + 300$, which is also 1 200 litres. That means that after 150 minutes, each tank has 1 200 litres.

Lesson 9 Practice Problems

1. **Problem 1 Statement**

   Cell phone Plan A costs £70 per month and comes with a free £500 phone. Cell phone Plan B costs £50 per month but does not come with a phone. If you buy the £500 phone and choose Plan B, how many months is it until your cost is the same as Plan A's?

   Solution

   25 months

2. **Problem 2 Statement**

   Priya and Han are biking in the same direction on the same path.

   a. Han is riding at a constant speed of 16 miles per hour. Write an expression that shows how many miles Han has gone after t hours.

   b. Priya started riding a half hour before Han. If Han has been riding for t hours, how long has Priya been riding?

   c. Priya is riding at a constant speed of 12 miles per hour. Write an expression that shows how many miles Priya has gone after Han has been riding for t hours.

   d. Use your expressions to find when Han and Priya meet.

Solution

a. $16t$ miles

b. $t + \frac{1}{2}$ hours

c. $12(t + \frac{1}{2})$

d. $t = \frac{3}{2}$. To find when Han and Priya meet, set the two expressions equal to one another: $16t = 12(t + \frac{1}{2})$. They meet after Han rides for one and a half hours and Priya rides for two hours.

3. **Problem 3 Statement**

Which story matches the equation $-6 + 3x = 2 + 4x$?

a. At 5 p.m., the temperatures recorded at two weather stations in Antarctica are -6 degrees and 2 degrees. The temperature changes at the same constant rate, x degrees per hour, throughout the night at both locations. The temperature at the first station 3 hours after this recording is the same as the temperature at the second station 4 hours after this recording.

b. Elena and Kiran play a card game. Every time they collect a pair of matching cards, they earn x points. At one point in the game, Kiran has -6 points and Elena has 2 points. After Elena collects 3 pairs and Kiran collects 4 pairs, they have the same number of points.

Solution A

4. **Problem 4 Statement**

For what value of x do the expressions $\frac{2}{3}x + 2$ and $\frac{4}{3}x - 6$ have the same value?

Solution

$x = 12$

5. **Problem 5 Statement**

Decide whether each equation is true for all, one, or no values of x.

a. $2x + 8 = -3.5x + 19$

b. $9(x - 2) = 7x + 5$

c. $3(3x + 2) - 2x = 7x + 6$

Solution

a. True for one value of x.

b. True for one value of x.

c. True for all values of x.

6. **Problem 6 Statement**

Solve each equation. Explain your reasoning.

$3d + 16 = -2(5 - 3d)$

$2k - 3(4 - k) = 3k + 4$

$\frac{3y-6}{9} = \frac{4-2y}{-3}$

Solution

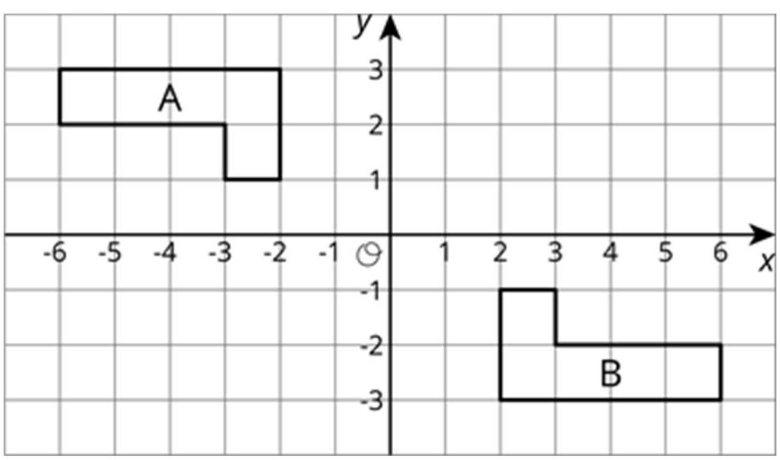
a. $d = \frac{26}{3}$. Explanations vary. Sample response: Expand the right side of the equation, add 10 to each side, subtract $3d$ from each side, then divide each side by 3.

b. $k = 8$. Explanations vary. Sample response: Expand and combine like terms on the left side, subtract $3k$ on each side, add 12 to each side, and then divide each side by 2.

c. $y = 2$. Explanations vary. Sample response: Multiply each side by 9, multiply out by -3 on the right side, subtract $3y$ on each side, add 12 to each side, and then divide each side by 3.

7. **Problem 7 Statement**

Describe a transformation that takes Polygon A to Polygon B.



Solution

Answers vary. Sample response: Rotate Polygon A 180 degrees around (0,0).