Lesson 9: The distributive property (Part 1)

Goals

- Generate equivalent numerical expressions that are related by the distributive property, and explain (orally or using other representations) the reasoning.
- Use an area diagram to make sense of equivalent numerical expressions that are related by the distributive property.

Learning Targets

- I can use a diagram of a rectangle split into two smaller rectangles to write different expressions representing its area.
- I can use the distributive property to help do computations in my head.

Lesson Narrative

This is the first of three lessons about the distributive property. In this lesson students recall the use of rectangle diagrams to represent the distributive property, and work with equations involving the distributive property with both addition and subtraction.

Alignments

Building On

- Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
- Apply properties of operations as strategies to multiply and divide. Students need not use formal terms for these properties. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

Building Towards

- Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.
- Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

Instructional Routines

- Collect and Display
- Discussion Supports
- Number Talk

Student Learning Goals

Let's use the distributive property to make calculating easier.

9.1 Number Talk: Ways to Multiply

Warm Up: 5 minutes

Students perform mental calculations by applying strategies involving the distributive property.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Find each product mentally.

5×102

5×98

5×999

Student Response

- 510, because $5 \times 102 = 5(100 + 2) = 500 + 10$
- 490, because $5 \times 98 = 5(100 - 2) = 500 - 10$
- 4 995, because $5 \times 999 = 5(1\,000 - 1) = 5\,000 - 5$

Activity Synthesis

Once students have had a chance to share a few different ways of reasoning about this product, focus on explanations using the distributive property like $5 \times 98 = 5 \times (90 + 8)$ or $5 \times (100 - 2)$, then writing out the two products from distributing. Remind students of the distributive property, and let them know they will spend the next few lessons working with it.

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

9.2 Ways to Represent Area of a Rectangle

15 minutes

The purpose of this activity is to remind students of the rectangle diagrams they worked with in a previous unit to represent multiplication. It is also to introduce the convention that for example the expression $6 \times 3 + 2$ equals 20. If we want the sum to be carried out before the product, we would need to use brackets like $6 \times (3 + 2)$.

Instructional Routines

- Discussion Supports

Launch

Allow students 10 minutes of quiet work time, followed by a whole-class discussion.

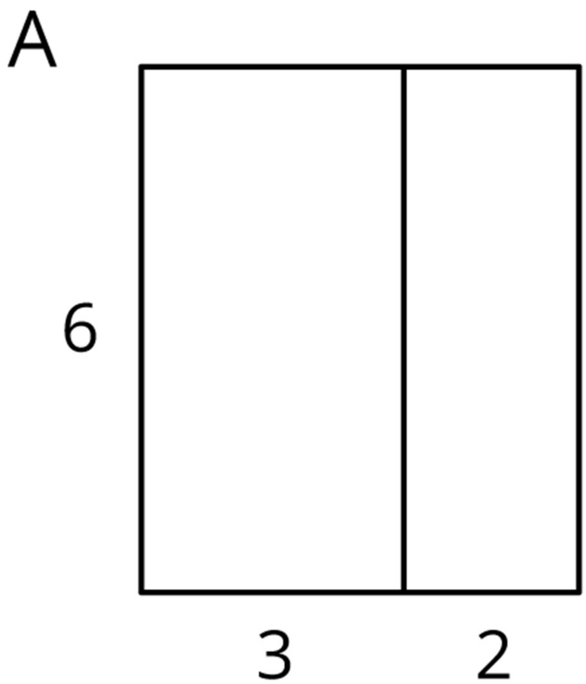
Representation: Internalise Comprehension. Activate or supply background knowledge about finding area. Some students may benefit from a review of the rectangle diagrams they used in a previous unit to represent multiplication.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

1. Select **all** the expressions that represent the area of the large, outer rectangle in figure A. Explain your reasoning.

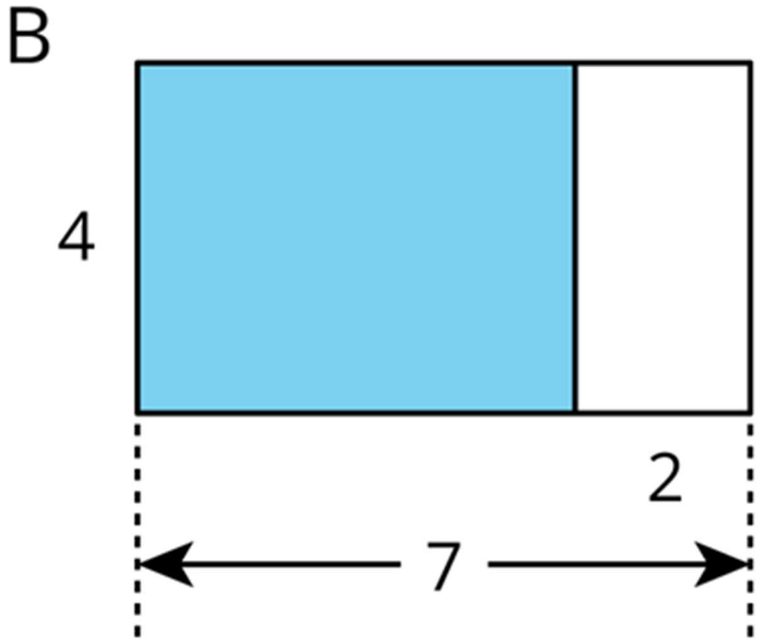
    - $6 + 3 + 2$
    - $6 \times 3 + 6 \times 2$
    - $6 \times 3 + 2$
    - 6×5
    - $6(3 + 2)$

- $6 \times 3 \times 2$



2. Select **all** the expressions that represent the area of the shaded rectangle on the left side of figure B. Explain your reasoning.

   - $4 \times 7 + 4 \times 2$
   - $4 \times 7 \times 2$
   - 4×5
   - $4 \times 7 - 4 \times 2$
   - $4(7 - 2)$
   - $4(7 + 2)$
   - $4 \times 2 - 4 \times 7$



Student Response

1. $6 \times 3 + 6 \times 2$, 6×5, and $6(3 + 2)$. Explanations vary. Sample responses:
   - These are all equal to 30.
   - $6 \times 3 + 6 \times 2$ is the sum of the areas of the two pieces, and the other two expressions are just the area of the whole rectangle.
2. 4×5, $4 \times 7 - 4 \times 2$, and $4(7 - 2)$. Explanations vary. Sample responses:
   - These are all equal to 20.
   - $4 \times 7 - 4 \times 2$ is the area of the whole rectangle minus the unshaded part, and the other two expressions are just the area of the shaded part.

Activity Synthesis

Students may conclude that $6 \times 3 + 2$ represents the area of the rectangle. This is a good opportunity to introduce a convention. When we have multiplication and addition in the same expression, it is the convention that the multiplication is done first. So $6 \times 3 + 2$ equals $18 + 2$, or 20, so it doesn't represent the area of the rectangle, which we know to be 30 square units. If you want the addition to be done first, you need to use brackets. Therefore, $6(3 + 2)$ does represent the area of the large rectangle. Remind students that "next to" implies multiplication.

By using the rectangle, we can tell that $6(3 + 2) = 6 \times 3 + 6 \times 2$. This is another example of two expressions that are equivalent because of the distributive property.

Speaking: Discussion Supports. Display sentence frames to support whole-class discussion. For example, "Expression _____ matches figure A because _____." or "Figure ___ cannot be represented by expression _____ because ________." Invite students to share their responses with a partner, and prompt them to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to clarify their thinking.
Design Principle(s): Optimise output (for explanation)

9.3 Distributive Practice

15 minutes

This is for practice going back and forth with the distributive property using numbers, but also to make the point that invoking the distributive property can help you do computations in your head. Some expressions that would be difficult to brute force become simpler after using the distributive property to write an equivalent expression.

Note that there is more than one way to rewrite the last row. For example, $24 - 16$ could also be written as $2(12 - 8)$ where $(12 - 8)$ is the difference of two terms. A *term* is a single number or variable, or variables and numbers multiplied together. This is fine, since there is no reason to insist that students use the greatest common factor at this time. Students should recognise that there is more than one factor that would work, and that the resulting expressions are equivalent. In a subsequent unit students will explicitly study the idea of a greatest common factor.

Instructional Routines

- Collect and Display

Launch

Give students the following setup: Suppose your business makes 15 items for £17 each and sells them for £20 each. How would you find your profit? One way is to write $15 \times 20 - 15 \times 17$. But that's a lot of calculations! An easier way to get the answer is to write $15(20 - 17)$, which is easier to figure out in your head.

Allow students 10 minutes of quiet work time, followed by a whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts. For example, after students have completed the first two rows of the table, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.
Supports accessibility for: Organisation; Attention

Anticipated Misconceptions

Students might understand how to expand an expression with brackets but struggle with how to approach a sum. Encourage students to think about the rectangle diagrams they have seen and draw a diagram of a partitioned rectangle. Ask students what the sum represents and help them to see that it can represent the sum of the areas of the two smaller rectangles. Remind students that the rectangles have the same width, and ask what that width might have been to produce the two areas, what factor the two areas have in common. Then have them consider the other factors (the lengths) that would produce those products for the areas.

Student Task Statement

Complete the table. If you get stuck, skip an entry and come back to it, or consider drawing a diagram of two rectangles that share a side.

column 1	column 2	column 3	column 4	value
5×98	$5(100 - 2)$	$5 \times 100 - 5 \times 2$	$500 - 10$	490
33×12	$33(10 + 2)$			
		$3 \times 10 - 3 \times 4$	$30 - 12$	
	$100(0.04 + 0.06)$			
		$8 \times \frac{1}{2} + 8 \times \frac{1}{4}$		
			$9 + 12$	
			$24 - 16$	

Student Response

column 1	column 2	column 3	column 4	value
5×98	$5(100 - 2)$	$5 \times 100 - 5 \times 2$	$500 - 10$	490
33×12	$33(10 + 2)$	$33 \times 10 + 33 \times 2$	$330 + 66$	396
3×6	$3(10 - 4)$	$3 \times 10 - 3 \times 4$	$30 - 12$	18
100×0.1	$100(0.04 + 0.06)$	$100 \times 0.04 + 100 \times 0.06$	$4 + 6$	10
$8 \times \frac{3}{4}$	$8\left(\frac{1}{2} + \frac{1}{4}\right)$	$8 \times \frac{1}{2} + 8 \times \frac{1}{4}$	$4 + 2$	6
3×7	$3(3 + 4)$	$3 \times 3 + 3 \times 4$	$9 + 12$	21
8×1	$8(3 - 2)$	$8 \times 3 - 8 \times 2$	$24 - 16$	8

Note that there is more than one correct response for the last row. For example, $24 - 16$ could also be rewritten as $2(12 - 8)$ or $4(6 - 4)$.

Are You Ready for More?

1. Use the distributive property to write two expressions that equal 360. (There are many correct ways to do this.)
2. Is it possible to write an expression like $a(b + c)$ that equals 360 where a is a fraction? Either write such an expression, or explain why it is impossible.
3. Is it possible to write an expression like $a(b - c)$ that equals 360? Either write such an expression, or explain why it is impossible.
4. How many ways do you think there are to make 360 using the distributive property?

Student Response

1. Answers vary. Possible expressions: $36(7 + 3)$, $10(20 + 16)$
2. Yes. For example, $\frac{1}{2}(700 + 20)$.
3. Yes. For example, $12(50 - 20)$.
4. There are infinite such expressions if you allow fractions or decimals, and quite a large number indeed even if you don't.

Activity Synthesis

Invite students to share their strategies and reasoning. Include students who used diagrams of partitioned rectangles. Ask if they noticed any interesting patterns, or if they want to share some examples of their own of calculations that can be made simpler by using the distributive property to write an equivalent expression.

Representing, Speaking, Listening: Collect and Display. During whole-class discussion, create a visual display to record a list of the strategies students describe, such as diagrams of partitioned rectangles. Amplify use of mathematical words and phrases that students use, such as patterns, equivalent, sum, etc. that describe their process. Ask students which strategies worked best for them, and ask them to say what they have in common. Remind students that they can borrow language and strategies from the display as needed.
Design Principle(s): Support sense-making; Maximise meta-awareness

Lesson Synthesis

Arrange students in groups of 2. One partner writes a product of the form $a(b + c)$ or $a(b - c)$. The other partner writes an equivalent expression using the distributive property, then each student evaluates their expression. The partners compare which computation was simpler, took less time, etc. Then have one partner write a sum and the other see if they can write an equivalent expression. Evaluate, compare, repeat as time allows. (It may be necessary to give students three numbers to use for the first round, instead of asking them to think of 3 numbers to use.)

9.4 Complete the Equation

Cool Down: 5 minutes

Student Task Statement

Write a number or expression in each empty box to create true equations.

1. $7(3+5) = \boxed{} + \boxed{}$

2. $15 - 10 = \boxed{}(3-2)$

Student Response

1. $7(3+5) = \boxed{21} + \boxed{35}$ or $7(3+5) = \boxed{7 \times 3} + \boxed{7 \times 5}$ or equivalent

2. $15 - 10 = \boxed{5}(3-2)$

Student Lesson Summary

A **term** is a single number or variable, or variables and numbers multiplied together. Some examples of terms are 10, $8x$, ab, and $7yz$.

When we need to do mental calculations, we often come up with ways to make the calculation easier to do mentally.

Suppose we are grocery shopping and need to know how much it will cost to buy 5 cans of beans at 79 pence a can. We may calculate mentally in this way:

5×79
$5 \times 70 + 5 \times 9$
$350 + 45$
395

In general, when we multiply two terms (or factors), we can break up one of the factors into parts, multiply each part by the other factor, and then add the products. The result will be the same as the product of the two original factors. When we break up one of the factors and multiply the parts we are using the distributive property.

The distributive property also works with subtraction. Here is another way to find 5×79:

5×79
$5 \times (80 - 1)$
$400 - 5$
395

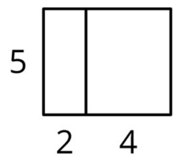
Glossary

- term

Lesson 9 Practice Problems

1. **Problem 1 Statement**

   Select **all** the expressions that represent the area of the large, outer rectangle.

   a. $5(2+4)$

   b. $5 \times 2 + 4$

   c. $5 \times 2 + 5 \times 4$

   d. $5 \times 2 \times 4$

   e. $5 + 2 + 4$

   f. 5×6

   Solution ["A", "C", "F"]

2. **Problem 2 Statement**

   Draw and label diagrams that show these two methods for calculating 19×50.

   – First find 10×50 and then add 9×50.

   – First find 20×50 and then take away 50.

   Solution

   a. A 19-by-50 rectangle partitioned into two rectangles with dimensions 10 by 50 and 9 by 50.

   b. A 20-by-50 rectangle partitioned into a 1 by 50 and a 19 by 50. Shading or arrows indicate that the 19-by-50 rectangle is the one we want.

3. **Problem 3 Statement**

   Complete each calculation using the distributive property.

   98×24 $(100 - 2) \times 24$ ...

21×15 $(20 + 1) \times 15$ …

0.51×40 $(0.5 + 0.01) \times 40$ …

Solution

a. $(100 - 2) \times 24 = 2400 - 48 = 2352$

b. $(20 + 1) \times 15 = 300 + 15 = 315$

c. $(0.5 + 0.01) \times 40 = 20 + 0.4 = 20.4$

4. **Problem 4 Statement**

A group of 8 friends go to the movies. A bag of popcorn costs £2.99. How much will it cost to get one bag of popcorn for each friend? Explain how you can calculate this amount mentally.

Solution

£23.92. Reasoning varies. Sample reasoning: If the bags of popcorn were £3 each, then this would be £24 (8×3). But $2.99 = 3 - 0.01$. So one pence has to be subtracted for each of the 8 bags of popcorn. That leaves £23.92.

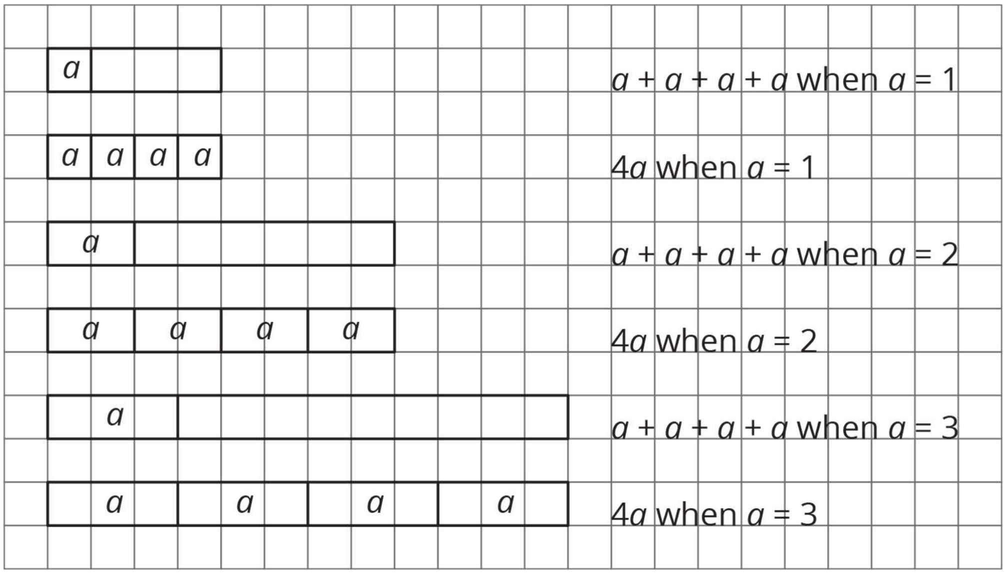
5. **Problem 5 Statement**

a. On graph paper, draw diagrams of $a + a + a + a$ and $4a$ when a is 1, 2, and 3. What do you notice?

b. Do $a + a + a + a$ and $4a$ have the same value for any value of a? Explain how you know.

Solution

a. See diagram



b. Yes, $4a$ can be rewritten as $a + a + a + a$, and this is true for any value of a. This can also be shown with a bar model.

6. **Problem 6 Statement**

120% of x is equal to 78.

a. Write an equation that shows the relationship of 120%, x, and 78.

b. Use your equation to find x. Show your reasoning.

Solution

a. $1.2x = 78$

b. $x = 65$ $(78 \div 1.2 = 65)$

7. **Problem 7 Statement**

Kiran’s aunt is 17 years older than Kiran.

a. How old will Kiran’s aunt be when Kiran is:

15 years old?

30 years old?

x years old?

b. How old will Kiran be when his aunt is 60 years old?

Solution

a. 32 years old ($156 + 17 = 32$), 47 years old ($30 + 17 = 47$), $x + 17$ years old.

b. 43 years old ($60 - 17 = 43$).