Grades 9-12 (A)

Duration: 20 min
Tools: one Logifaces Set / class
Individual work

Keywords: Coordinates, 3D
coordinate system, Spatial vision

529 - Coordinates of a Rotated Block


MATHS / COORDINATE GEOMETRY


LOGIFACES Erasmus+ TEACHER Logifaces

DESCRIPTION
Students rotate the Logifaces blocks to position them into the 3-dimensional coordinate system in particular ways. The terms "vertical edge", "base" and "vertical face" will be used for the normally placed block (see the introduction).

LEVEL 1 Students position a Logifaces block into the 3-dimensional coordinate system so that one of the vertical faces fits in the plane of the $x$ and $y$ axes, one base edge fits on the $x$-axis and one vertical edge fits on the $y$-axis. They calculate the coordinates of the other vertices of the block.

LEVEL 2 Students position a Logifaces block into the 3-dimensional coordinate system so that one of the vertical faces fits in the plane of the $x$ and $y$ axes, one base edge fits on the $y$-axis and one vertical edge fits on the x-axis. They calculate the coordinates of the other vertices of the block.

## POSSIBLE EXTENSION

Students discuss within small groups or with the whole class the relationship of the solutions of the Level 1 and Level 2 exercises. They also compare the results with exercise 526 - Calculate the Coordinates.

SOLUTIONS / EXAMPLES

## LEVEL 1

Blocks 111, 222 and 333 are the easiest:

- $111(0,0,0),(4,0,0),(4,1,0),(0,1,0),(2,0,2 \sqrt{3}),(2,1,2 \sqrt{3})$
- $222(0,0,0),(4,0,0),(4,2,0),(0,2,0),(2,0,2 \sqrt{3}),(2,2,2 \sqrt{3})$
- $333(0,0,0),(4,0,0),(4,3,0),(0,3,0),(2,0,2 \sqrt{3}),(2,3,2 \sqrt{3})$

In the notations of the figure the variable $a$ is 1 in the case of block 111, 2 in the case of block 222 and 3 in the case of block 333.


The other blocks' vertical edges have different heights (denoted by $a, b$ and $c$ ), so we use the notations of the figure below. These blocks can be placed into the coordinate system in three different ways by rotating the block, so that in each case a different vertical face is in the plane of the $x$ and $y$ axes.




The coordinates are listed in the table below:

| block | case 1 | case 2 | case 3 |
| :---: | :--- | :--- | :--- |
| 112 | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,2,0),(4,1,0),(2,1,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,1,0),(4,2,0),(2,1,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,1,0),(4,1,0),(2,2,2 \sqrt{3})$ |
| 122 | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,2,0),(4,2,0),(2,1,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,1,0),(4,2,0),(2,2,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,2,0),(4,1,0),(2,2,2 \sqrt{3})$ |
| 223 | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,3,0),(4,2,0),(2,2,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,2,0),(4,3,0),(2,2,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,2,0),(4,2,0),(2,3,2 \sqrt{3})$ |
| 233 | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,3,0),(4,3,0),(2,2,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,2,0),(4,3,0),(2,3,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,3,0),(4,2,0),(2,3,2 \sqrt{3})$ |
| 113 | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,3,0),(4,1,0),(2,1,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,1,0),(4,3,0),(2,1,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,1,0),(4,1,0),(2,3,2 \sqrt{3})$ |
| 133 | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,3,0),(4,3,0),(2,1,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,1,0),(4,3,0),(2,3,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,3,0),(4,1,0),(2,3,2 \sqrt{3})$ |
| 123 | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,3,0),(4,2,0),(2,1,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,1,0),(4,3,0),(2,2,2 \sqrt{3})$ | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,2,0),(4,1,0),(2,3,2 \sqrt{3})$ |
| 132 | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, | $(0,0,0),(4,0,0),(2,0,2 \sqrt{3})$, <br> $(0,2,0),(4,3,0),(2,1,2 \sqrt{3})$$(0,1,0),(4,2,0),(2,3,2 \sqrt{3})$ |

## LEVEL 2

- $\quad 111:(0,0,0),(1,0,0),(1,4,0),(0,4,0),(0,2,2 \sqrt{3}),(1,2,2 \sqrt{3})$
- $\quad 222:(0,0,0),(2,0,0),(2,4,0),(0,4,0),(0,2,2 \sqrt{3}),(2,2,2 \sqrt{3})$
- $\quad 333:(0,0,0),(3,0,0),(3,4,0),(0,4,0),(0,2,2 \sqrt{3}),(3,2,2 \sqrt{3})$

In the notations of the figure the variable $a$ is 1 in the case of block 111, 2 in the case of block 222 and 3 in the case of block 333.


The other blocks' vertical edges have different heights (denoted by $a, b$ and $c$ ), so we use the notations of the figure below. These blocks can be placed into the coordinate system in three different ways by rotating the block, so that in each case a different vertical face is in the plane of the $x$ and $y$ axes.




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| 122 | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(2,0,0),(2,4,0),(1,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(2,0,0),(1,4,0),(2,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(1,0,0),(2,4,0),(2,2,2 \sqrt{3})$ |
| 223 | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(2,0,0),(3,4,0),(2,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(3,0,0),(2,4,0),(2,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(2,0,0),(2,4,0),(3,2,2 \sqrt{3})$ |
| 233 | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(3,0,0),(3,4,0),(2,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(3,0,0),(2,4,0),(3,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(2,0,0),(3,4,0),(3,2,2 \sqrt{3})$ |
| 113 | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(1,0,0),(3,4,0),(1,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(1,0,0),(1,4,0),(3,2,2 \sqrt{3})$ |
| 133 | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(3,0,0),(3,4,0),(1,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(3,0,0),(1,4,0),(3,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(1,0,0),(3,4,0),(3,2,2 \sqrt{3})$ |
| 123 | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(2,0,0),(3,4,0),(1,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(3,0,0),(1,4,0),(2,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(1,0,0),(2,4,0),(3,2,2 \sqrt{3})$ |
| 132 | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(2,0,0),(1,4,0),(3,2,2 \sqrt{3})$ | $(0,0,0),(0,4,0),(0,2,2 \sqrt{3})$, <br> $(1,0,0),(3,4,0),(2,2,2 \sqrt{3})$ |

## POSSIBLE EXTENSION

Given any block and any fixed vertex of the block, the values of the coordinates are the same in the three solutions (Level 1, Level 2 and exercise 526 - Calculate the Coordinates), only the order of the coordinates differs. Comparing any two solutions, one can be obtained from another by performing the same rearrangement of the coordinates for each vertex.

The reason for this observation is that only the axes were rearranged.
PRIOR KNOWLEDGE
Coordinates of points in the 3 dimensional coordinate system
RECOMMENDATIONS / COMMENTS
Exercise 526 - Calculate the Coordinates is recommended before this exercise.

