Grades 5-8 (AS), 9-12 (S)

Duration: 15-20 min

Tools: one Logifaces Set / class

Individual work

Keywords: Pythagorean theorem, Percent, Area of the triangle

411 - Area of Triangles

MATHS / 2D GEOMETRY

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

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DESCRIPTION

LEVEL 1 Students calculate the altitude of the top and the base triangles of the Logifaces blocks.

LEVEL 2 Students calculate the area of the top and the base triangles of the Logifaces blocks.

LEVEL 3 Students consider the percentage difference between the top and the base triangle.

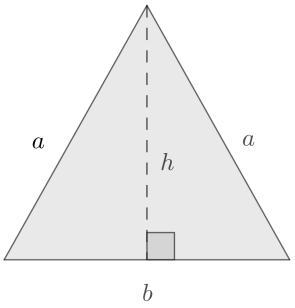
SOLUTIONS / EXAMPLES

LEVEL 1 There are 4 different types of triangles. The base faces are all equilateral triangles, the top faces are either equilateral triangles or isosceles triangles (3 types). The Pythagorean theorem is used to calculate the altitude of the triangles.

Pythagorean theorem in an isosceles triangle:

$$a^2 = \left(\frac{b}{2}\right)^2 + h^2$$

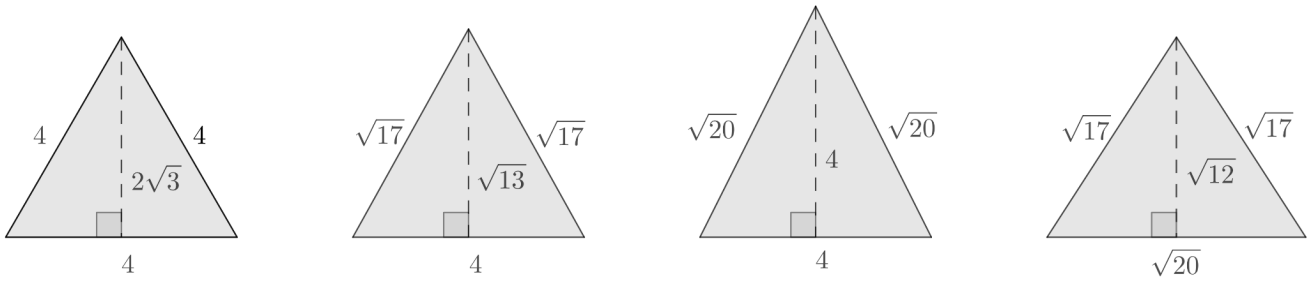
$$h = \sqrt{a^2 - \left(\frac{b}{2}\right)^2}$$



When the face is an equilateral triangle with edge length 4, this gives $h = \sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$

When the face is an isosceles triangle, the altitudes can be found in the table below:

lengths of the edges	altitude
$4, \sqrt{17}, \sqrt{17}$	$h = \sqrt{\sqrt{17}^2 - 2^2} = \sqrt{13}$
$4, \sqrt{20}, \sqrt{20}$	$h = \sqrt{\sqrt{20}^2 - 2^2} = \sqrt{16} = 4$
$\sqrt{20}, \sqrt{17}, \sqrt{17}$	$h = \sqrt{\sqrt{17}^2 - \left(\frac{\sqrt{20}}{2}\right)^2} = \sqrt{17 - \sqrt{5}^2} = \sqrt{12}$



LEVEL 2 The following formula is used to calculate the area of the triangles:

$A = \frac{1}{2} \times base \times altitude$

When the face is an equilateral triangle with edge length 4, this gives $A = \frac{1}{2} \times 4 \times 2\sqrt{3} = 4\sqrt{3}$

When the face is an isosceles triangle, the areas can be found in the table below:

lengths of the edges	area
$4, \sqrt{17}, \sqrt{17}$	$A = \frac{1}{2} \times 4 \times \sqrt{13} = 2\sqrt{13}$
$4, \sqrt{20}, \sqrt{20}$	$A = \frac{1}{2} \times 4 \times 4 = 8$
$\sqrt{20}, \sqrt{17}, \sqrt{17}$	$A = \frac{1}{2} \times \sqrt{20} \times \sqrt{12} = 2\sqrt{15}$

LEVEL 3 All the bases are equilateral triangles, their area is $4\sqrt{3}$.

- blocks 111, 222 and 333 have equilateral top faces: top triangle is 0% larger.

- blocks 112, 221, 223 and 332 have isosceles top faces with edges $4, \sqrt{17}, \sqrt{17}$: $\frac{2\sqrt{13}}{4\sqrt{3}} = \frac{\sqrt{39}}{6} \approx 1.04$. Top triangle is 4% larger.

- blocks 113 and 331 have isosceles top faces with edges $4, \sqrt{20}, \sqrt{20}$: $\frac{8}{4\sqrt{3}} = \frac{2}{\sqrt{3}} \approx 1.15$. Top triangle is 15% larger.

- blocks 123 and 132 have isosceles top faces with edges $\sqrt{20}, \sqrt{17}, \sqrt{17}$: $\frac{2\sqrt{15}}{4\sqrt{3}} = \frac{\sqrt{5}}{2} \approx 1.12$. Top triangle is 12% larger.

PRIOR KNOWLEDGE

Pythagorean theorem, Percentage, Area

RECOMMENDATIONS / COMMENTS

Exercises 406 - Different Triangles and 409 - Area of Rectangular Faces are recommended before this exercise. You find the dimensions of the blocks Measurements of the Logifaces blocks.

The calculations can be verified using GeoGebra, see exercise 528 - Read the Results in GeoGebra.