Lesson 15: Estimating population measures of centre

Goals

- Calculate and interpret (orally and in writing) the range of a sample.
- Generalise that an estimate for the centre of a population distribution is more likely to be accurate when it is based on a random sample with less variability.
- Use the mean of a random sample to make inferences about the population, and explain (orally and in writing) the reasoning.

Learning Targets

- I can consider the variability of a sample to get an idea for how accurate my estimate is.
- I can estimate the mean or median of a population based on a sample of the population.

Lesson Narrative

In this lesson students calculate measures of centre and variation for samples from different populations and consider the meaning of these quantities in terms of the situation. Students see that when there is less variability in the data from different samples from a population, then there is reason to believe that the measure of centre from a sample is a better estimate for the measure of centre from a population than when a sample has greater variability.

Building On

- Relating the choice of measures of centre and variability to the shape of the data distribution and the context in which the data were gathered.

Addressing

- Understand that statistics can be used to gain information about a population by examining a sample of the population; generalisations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.
- Use measures of centre and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example,

decide whether the words in a chapter of a year 8 science book are generally longer than the words in a chapter of a year 5 science book.

Building Towards

- Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.
- Use measures of centre and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a year 8 science book are generally longer than the words in a chapter of a year 5 science book.

Instructional Routines

- Co-Craft Questions
- Discussion Supports
- Think Pair Share

Student Learning Goals

Let's use samples to estimate measures of centre for the population.

15.1 Describing the Centre

Warm Up: 5 minutes

This warm-up asks students to decide whether to use the mean or median based on the distribution of the data. As students compare groups in this section, the choice of measure of centre will be important.

Instructional Routines

- Think Pair Share

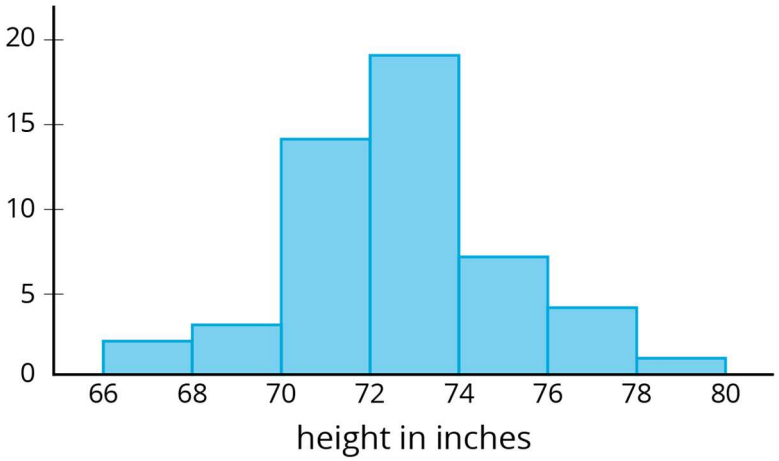
Launch

Arrange students in groups of 2. Give students 1 minute quiet work time, followed by 2 minutes to discuss their work with a partner, followed by a whole-class discussion.

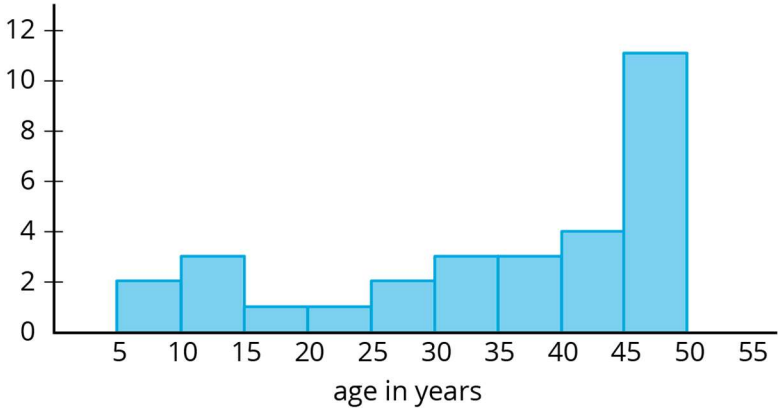
Student Task Statement

Would you use the median or mean to describe the centre of each data set? Explain your reasoning.

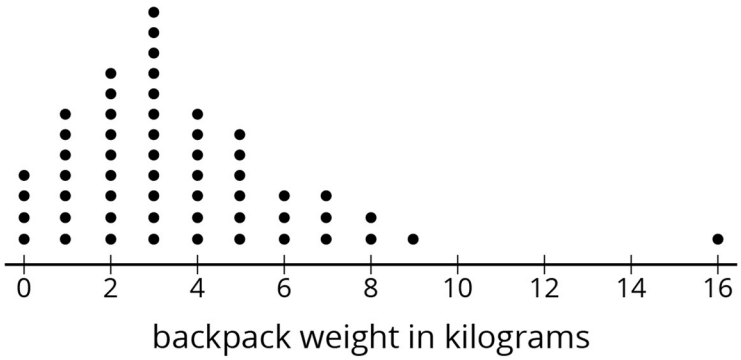
Heights of 50 basketball players



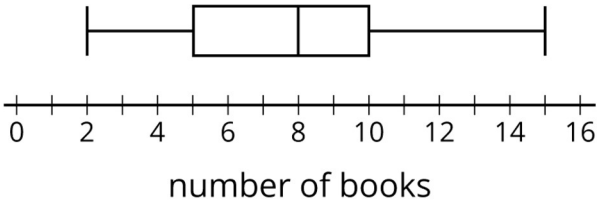
Ages of 30 people at a family dinner party



Backpack weights of year 7 students



How many books students read over the summer holiday



Student Response

- Basketball players: Mean. The distribution is symmetric.
- Ages at a party: Median. The distribution is not symmetric.
- Backpacks: Median. The point at 16 would affect the mean much more than the median.
- Books: You could use the mean if you know what it is because the data set is approximately symmetric, but if you only know what's given on the box and whisker plot, then you should use the median.

Activity Synthesis

Select students to share their chosen measure of centre and reasoning for their choice. Ask students what measures of variability should be used with each measure of centre.

15.2 Three Different TV Shows

5 minutes

In this activity, students analyse data from samples of viewers for different TV shows. The data in this activity is used to begin the analysis as well as to get students thinking about the different shows the sample could represent. The purpose of the activity is to get students thinking about how measures of centre from a sample might be used to make decisions about the population of a group.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 3. Tell students that each person in the group should work on a different sample then share their results with their group. Give students 1 minute quiet work time and then 1 minute to share their work with the group followed by a whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge of calculating measures of centre. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing Speaking, Representing: Discussion

Supports. Use this routine to support small-group discussion. Display the following sentence frames for students to respond to their group members' explanations: "I agree because . . ." or "I disagree because . . . ." If necessary, revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.
Design Principle(s): Support sense-making

Student Task Statement

Here are the ages (in years) of a random sample of 10 viewers for 3 different television shows. The shows are titled, "Science Experiments YOU Can Do," "Learning to Read," and "Trivia the Game Show."

sample 1

6	6	5	4	8	5	7	8	6	6

sample 2

15	14	12	13	12	10	12	11	10	8

sample 3

43	60	50	36	58	50	73	59	69	51

1. Calculate the mean for *one* of the samples. Make sure each person in your group works with a different sample. Record the answers for all three samples.
2. Which show do you think each sample represents? Explain your reasoning.

Student Response

- Sample 1 mean: 6.1 years
- Sample 2 mean: 11.7 years
- Sample 3 mean: 54.9 years
- Sample 1 is probably "Learning to Read" since it is the youngest group.
- Sample 2 is probably "Science Experiments YOU Can Do" since that sounds like a show young people would like, but should be a little older than 6 years old.
- Sample 3 is probably "Trivia the Game Show" since that is the oldest group.

Activity Synthesis

Select students to share how they determined which shows matched with which data set. The purpose of the discussion is for students to notice that the shows are meant to appeal to different age groups.

15.3 Who's Watching What?

15 minutes

This activity continues the work begun in the previous activity for this lesson. Students calculate the means for sample ages to determine what shows might be associated with each sample. They also consider the variability to assess the accuracy of population estimates. A sample from a population with less variability should provide a more accurate estimate than a sample that came from a population with more spread in the data. In the discussion, students think about why a sample is used and why an estimate of the mean is helpful, but it may miss some important aspects of the data. The discussion following the activity also asks students to think again about why different samples from the same population may produce different results.

Launch

Keep students in groups of 3.

Tell students that advertisers are interested in age groups for certain television shows so that they can try to sell appropriate items to the audience. For example, it does not make sense to advertise tricycles during a night-time crime drama show nor to show an ad for an expensive sports car during a children's cartoon.

The samples given in this activity are related to the shows mentioned in the previous activity for this lesson.

Tell students to divide samples 4 through 6 among the group members so that each person only needs to find one mean and share their answer with the group so that the group has access to all 3 answers.

Give students 3 minutes quiet work time for the first 3 problems, then pause the class after the third problem for a quick discussion and to assign items to groups before they continue.

After students have completed the first 3 problems, ask students to indicate which of the shows seem to go with each of the 6 samples (samples 1 through 3 from the previous activity as well as 4 through 6 in this activity). Discuss any disagreements until the class can agree on which samples correspond to which shows. Tell half of the groups that they will use samples 1 through 3 from the previous activity for the last 3 problems and the other half of the groups that they will use samples 4 through 6 for the last 3 problems.

Give students another 3 minutes of quiet work time to finish the activity followed by a whole-class discussion.

Representation: Internalise Comprehension. Use colour and annotations to illustrate student strategies. As students describe how they calculated the measures of centres, use colour and annotations to scribe their thinking on a display. Ask students how they matched samples and television shows, and label each representation accordingly.
Supports accessibility for: Visual-spatial processing; Conceptual processing

Student Task Statement

Here are three more samples of viewer ages collected for these same 3 television shows.

sample 4

57 71 5 54 52 13 59 65 10 71

sample 5

15 5 4 5 4 3 25 2 8 3

sample 6

6 11 9 56 1 3 11 10 11 2

1. Calculate the mean for *one* of these samples. Record all three answers.
2. Which show do you think each of these samples represents? Explain your reasoning.
3. For each show, estimate the mean age for all the show's viewers.
4. Calculate the range for *one* of the shows' samples. Make sure each person in your group works with a different sample. Record all three answers.

	Learning to Read	Science Experiments YOU Can Do	Trivia the Game Show
Which sample?			
Range			

5. What do the different values for the range tell you about each group?
6. An advertiser has a commercial that appeals to 15- to 16-year-olds. Based on these samples, are any of these shows a good fit for this commercial? Explain or show your reasoning.

Student Response

1. Sample 4 mean: 45.7 years. Sample 5 mean: 7.4 years. Sample 6 mean: 12 years.

2. Sample 4: "Trivia the Game Show." Sample 5: "Learning to Read." Sample 6: "Science Experiments YOU Can Do."

3. Mean age of "Learning to Read:" about 7 years. Mean age of "Science Experiments YOU Can Do:" about 12. Mean age of "Trivia the Game Show:" about 45

   - ranges for "Learning to Read" Sample 1: 4 years. Sample 5: 23, but 13 years after extreme value removed.
   - ranges for "Science Experiments YOU Can Do" Sample 2: 7 years. Sample 6: 5 years, but 10 after extreme value removed.
   - ranges for "Trivia the Game Show" Sample 3: 37 years. Sample 4: 66 years.

4. The different values tell you how spread out the values are for each sample. For example, most of the ages for "Learning to Read" are close to the mean, but for "Trivia the Game Show," the ages are spread out more.

5. It might work best with "Science Experiments YOU Can Do," but since the average age is 11.7 years old (or 12 years old) and the range is 7 years (or 10 years), there may or may not be very many 15 and 16 year olds watching this show.

Activity Synthesis

The purpose of the discussion is to understand why it might be helpful to estimate the mean of a population based on a sample.

Some questions for discussion:

- "Why do you think a sample was used in this situation rather than data from the population?" (There are probably millions of people who watch these shows and it would be difficult to collect data about their ages from all of them.)
- "How could we improve the estimate of the mean for the populations?" (Include more viewers in the sample.)
- "If a sample has a large range, what does that imply about the population?" (That the data in the population is very spread out.)
- "If a sample has a small range, what is the relationship between the data and the mean?" (Most of the data is close to the mean.)
- "Which estimate of the mean for the population do you expect to be more accurate: the mean from a sample with a large range or the mean from a sample with a small range? Explain or show your reasoning." (The mean from a sample with a small range. If the data in the sample is close to the mean, then most of the data from the population is

also probably close to the mean. Therefore the data in the sample is probably close to the mean of the population and will provide a good estimate.)

- "What do you notice about the different answers for the same show, but the different samples?" (The means are close, but the ranges can be different by a lot.)
- "Why were the answers different for the same show, but different samples?" (Different people were included in the samples, so the numbers may change some, but if they are representative they should be close. In these examples, even though the ranges may seem very different, the relative size compared to the other shows is similar.)

Some students may wonder why they need to calculate the mean when it might be obvious how to match the titles by just looking at the data. This example included 10 ages in each sample so that the important information could be calculated quickly. In a more realistic scenario, the sample may include hundreds of ages. A computer could still calculate the mean quickly, but scanning through all of the data may not make the connection to the correct show as obvious.

- "Notice that there is a 56 year old in sample 6. What are some reasons you think they might be watching this show?" (Maybe a grandmother is watching with her grandchild. Maybe an older person is interested in how science is shown on TV.)
- "The questions asked you to consider means, but are there any data sets for which median might be a better measure of centre? Explain your reasoning." (Yes, sample 6 has a wide range of data ranging from a 1 year old to a 56 year old, but most of the data are around 10, so median might be better to use for that sample.)
- "A lot of families might be watching 'Learning to Read' with their children or older people may be using the show to learn English. How might this affect the mean? How could you recognise that there are two main age groups that watch this show?" (It would bring the mean up from just the kids who watch the show. The mean would not make the two age groups obvious, so looking at a dot plot or histogram might be more helpful with this group.)

15.4 Film Reviews

10 minutes

In this activity, students use data from a sample of film reviews to estimate information about all the reviews for the film. Based on the distribution of the data, students are asked to choose an appropriate measure of centre and measure of variation then apply their calculations to the entire population. Finally, students gauge their trust in the measure of centre they have chosen based on the associate measure of variation.

Instructional Routines

- Co-Craft Questions

Launch

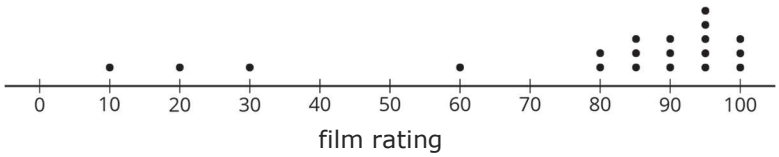
Keep students in groups of 3. Allow students 5 minutes work time in their groups followed by a whole-class discussion.

It may be helpful to use the warm-up for this lesson to review how to choose mean and median based on the distribution of data.

Writing, Conversing: Co-Craft Questions. Display the scenario and the film rating dot plot without revealing the questions that follow. Ask students to write down possible mathematical questions that can be asked about the situation. Invite students to share their questions with a partner before selecting 1–2 pairs to share their questions with the class. Listen for and amplify questions about how different measures of centre and variability give more or less accurate information about the population. Then reveal and ask students to work on the actual questions of the task. This will help students justify their choices of using a particular measure of centre to represent the population.
Design Principle(s): Optimise output (for justification); Maximise meta-awareness

Student Task Statement

A film rating website has many people rate a new film on a scale of 0 to 100. Here is a dot plot showing a random sample of 20 of these reviews.



1. Would the mean or median be a better measure for the centre of this data? Explain your reasoning.

2. Use the sample to estimate the measure of centre that you chose for *all* the reviews.

3. For this sample, the range is 90, and the interquartile range is 15. Which of these values is associated with the measure of centre that you chose?

4. Films must have an average rating of 75 or more from all the reviews on the website to be considered for an award. Do you think this film will be considered for the award? Use the measure of centre and measure of variability that you chose to justify your answer.

Student Response

1. Median since most of the data is above 80, but there are a few points that are much lower, so the data is not symmetric.

2. 90 is the median.

3. IQR is better to use with median.

4. Answers vary. Sample response: Yes. Since the first quartile for this data set is 80, at least 75% of the ratings in the sample were 80 or higher. Therefore, it is likely that the overall rating for the entire population is 75 or higher.

Are You Ready for More?

Estimate typical temperatures in the United Kingdom today by looking up current temperatures in several places across the country. Use the data you collect to decide on the appropriate measure of centre for the country, and calculate the related measure of variation for your sample.

Student Response

Answers vary.

Activity Synthesis

The purpose of the discussion is for students to review how to choose a measure of centre and its associated measure of variation. Additionally, students use the measure of variation to help them think about how much to trust their population characteristic estimate.

Consider asking these discussion questions:

- "Which measure of centre did you choose and why?" (Median, since the distribution is not symmetric.)
- "Based on the context, do you think other film reviews would have non-symmetrical distributions as well?" (Yes, usually a lot of people will agree whether a film is good or bad, but a few people will have strong opinions on the other end of the scale.)
- "A random sample of 20 reviews for another film has a median of 90 as well, but its IQR is 30. Do you think this film is more or less likely to be considered for the award?" (Less likely since there is more variability in the sample, so it is harder to estimate the median for all of the reviews.)
- "A random sample of 20 reviews for a third film has a median of 50 and an IQR of 20. Is it possible this third film will be considered for an award?" (It seems unlikely, but it is possible. The random sample may have randomly selected the 20 worst reviews and all the other reviews gave it a 100 rating.)

Action and Expression: Internalise Executive Functions. Provide students with a graphic organiser to support their participation during the synthesis. Invite students to describe what to look for to determine how to choose a measure of centre and its associated measure of variation, and to include examples for each.
Supports accessibility for: Conceptual processing; Organisation

Lesson Synthesis

Consider asking these discussion questions to review the main ideas from the lesson:

- "How do you determine which measure of centre will best describe the data in a sample?" (Base it on the distribution of the data.)
- "When you have the data from a sample, how can you estimate the value of a measure of centre for the population?" (If the sample is random, calculate the appropriate measure of centre for the sample and use that to estimate the same characteristic for the population.)
- "What does the variability of the sample tell you about your estimate for the measure of centre of the population?" (The greater the variability, the less certain I am of the estimate. If the data is spread widely in the sample, it might be spread even more widely in the population and this sample may not capture everything going on in the population, so the estimate may not be very accurate.)

15.5 More Accurate Estimate

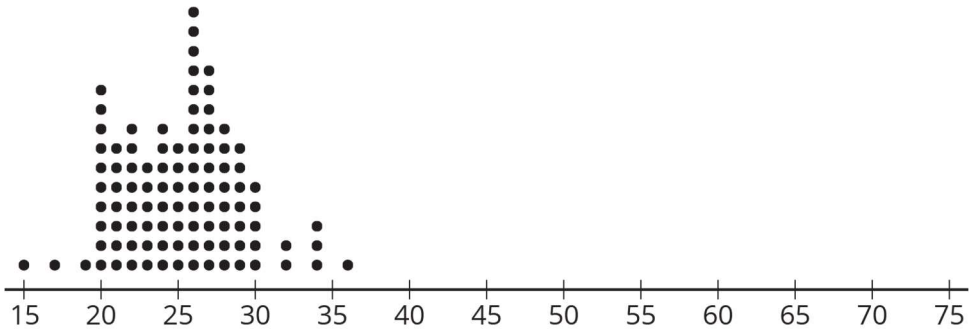
Cool Down: 5 minutes

This cool-down assesses student understanding of how variability within the data can help gauge how far off an estimate of a population characteristic might be from its actual value.

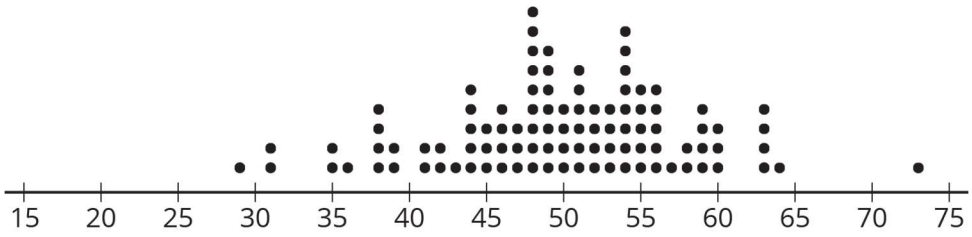
Student Task Statement

Here are dot plots that represent samples from two different populations.

Sample 1:



Sample 2:



1. Estimate the mean of each population using these samples.

2. Based on the dot plots, which estimate is more likely to be accurate? Explain your reasoning.

Student Response

1. Answers vary. Correct responses should be close to 25 and 50 respectively.
2. The estimate for sample 1 is probably more accurate since there is much less variability in the data.

Student Lesson Summary

Some populations have greater variability than others. For example, we would expect greater variability in the weights of dogs at a dog park than at a beagle meetup.

Dog park:

Mean weight: 12.8 kg Range: 6.9 kg

Beagle meetup:

Mean weight: 10.1 kg Range: 2.6 kg

The lower range indicates there is less variability in the weights of the beagles. We would expect that the mean weight from a sample that is randomly selected from a group of beagles will provide a more accurate estimate of the mean weight of all the beagles than a sample of the same size from the dogs at the dog park.

In general, a sample of a similar size from a population with *less* variability is *more likely* to have a mean that is close to the population mean.

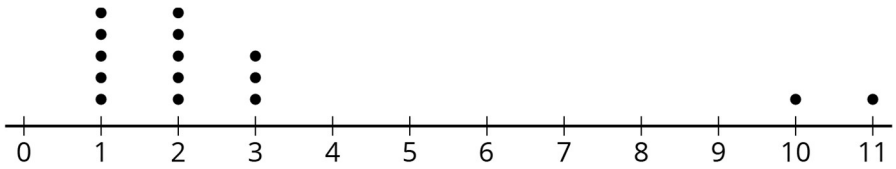
Glossary

- interquartile range (IQR)

Lesson 15 Practice Problems

Problem 1 Statement

A random sample of 15 items were selected.



For this data set, is the mean or median a better measure of centre? Explain your reasoning.

Solution

Median, since the data is not symmetrical with a couple of values far away from most of the other numbers.

Problem 2 Statement

A video game developer wants to know how long it takes people to finish playing their new game. They surveyed a random sample of 13 players and asked how long it took them (in minutes).

1 235	952	457	1 486	1 759	1 148	548	1 037
1 864	1 245	976	866	1 431			

a. Estimate the median time it will take *all* players to finish this game.

b. Find the interquartile range for this sample.

Solution

Median: 1 148 minutes; IQR: 549.5 minutes

Problem 3 Statement

Han and Priya want to know the mean height of the 30 students in their dance class. They each select a random sample of 5 students.

- The mean height for Han's sample is 59 inches.
- The mean height for Priya's sample is 61 inches.

Does it surprise you that the two sample means are different? Are the population means different? Explain your reasoning.

Solution

No. Explanations vary. Sample explanation: Even though they both selected a random sample, their samples probably included different people from the population, so the two sample means would not necessarily be the same, even though there is only one population mean.

Problem 4 Statement

Clare and Priya each took a random sample of 25 students at their school.

- Clare asked each student in her sample how much time they spend doing homework each night. The sample mean was 1.2 hours and the range was 2 hours.
- Priya asked each student in her sample how much time they spend watching TV each night. The sample mean was 2 hours and the range was 4.2 hours.

a. At their school, do you think there is more variability in how much time students spend doing homework or watching TV? Explain your reasoning.

b. Clare estimates the students at her school spend an average of 1.2 hours each night doing homework. Priya estimates the students at her school spend an average of 2 hours each night watching TV. Which of these two estimates is likely to be closer to the actual mean value for all the students at their school? Explain your reasoning.

Solution

a. There is more variability in the times spent watching TV or playing games. The range is a measure of variability, and the range for time spent watching TV is greater than the range for the time spent doing homework.

b. Clare's estimate is more likely to be closer, because the times spent doing homework don't vary as much. It is harder to get an accurate estimate of the population mean when there is a lot of variability in the population values.