

Lesson 7: Equivalent ratios have the same unit rates

Goals

- Apply reasoning about unit rates to complete a table of equivalent ratios, and explain (orally and in writing) the solution method.
- Explain (orally) that if two ratios are equivalent, they have the same unit rate.
- Generalise that the unit rate is the factor that takes you from one column to the other column in a table of equivalent ratios.

Learning Targets

- I can give an example of two equivalent ratios and show that they have the same unit rates.
- I can multiply or divide by the unit rate to calculate missing values in a table of equivalent ratios.

Lesson Narrative

The purpose of this lesson is to make it explicit to students that equivalent ratios have the same unit rates. For instance, students can see that the ratios $10 : 4$, $15 : 6$, and $20 : 8$ all have unit rates of $\frac{2}{5}$ and $\frac{5}{2}$. Interpreted in a context, this might mean, for example, that no matter how many ounces of raisins are purchased in bulk and how much is paid, the price per ounce will always match the £0.40 per ounce rate marked on the price label.

a		b
10		4
15	$\times \frac{2}{5}$	6
20	$\times \frac{2}{5}$	8

$\times \frac{2}{5}$

This understanding gives new insights as students reason with tables. Up to this point, students have often been reasoning about the relationship from row to row, understanding that the rows contain equivalent ratios and the values in any row can be found by multiplying both quantities in another row by a scale factor. Here students see that they can also reason across columns, because the unit rate is the factor that relates the values in one column to those in the other. Later in KS3, students will call the unit rate the *constant of proportionality* and write equations of the form $y = kx$ to characterise these relationships.

Later in the lesson, students practise using unit rates and tables of equivalent ratios to find unknown quantities and compare rates in context.

Addressing

- Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a : b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. Expectations for unit rates in this stage are limited to non-complex fractions.
- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Instructional Routines

- Stronger and Clearer Each Time
- Compare and Connect
- Discussion Supports
- Think Pair Share
- Which One Doesn't Belong?

Student Learning Goals

Let's revisit equivalent ratios.

7.1 Which One Doesn't Belong: Comparing Speeds

Warm Up: 10 minutes

This warm-up prompts students to compare four rates. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about rates in comparison to one another. To allow all students to access the activity, each rate has one obvious reason it does not belong. During the discussion, listen for the term "unit rate," speed, pace, and ways that students reason about whether two rates indicate the same speed.

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the rates for all to see. Ask students to indicate when they have noticed one that does not belong and can explain why. Give students 1

minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular question does not belong and together find at least one reason each question doesn't belong.

Student Task Statement

Which one doesn't belong? Be prepared to explain your reasoning.

5 miles in 15 minutes

3 minutes per mile

20 miles per hour

32 kilometres per hour

Student Response

- 5 miles in 15 minutes is the only rate not expressed as a unit rate.
- 3 minutes per mile is the only rate expressed as a pace instead of a speed.
- 20 miles per hour is the only one that sounds like we are used to talking about speeds or the only one that would be on a road sign.
- 32 kilometres per hour is the only one using metric units of length and is not *exactly* the same speed as the other 3 (32 km = 19.8839 miles).

Activity Synthesis

Ask each group to share one reason why a particular rate does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct.

During the discussion, ask students to explain the meaning of any terminology they use, such as unit rate, speed, or pace. Also, press students on unsubstantiated claims.

7.2 Price of Burritos

10 minutes

In a previous lesson about treadmill workouts, students calculated unit rates and identified ratios that have matching unit rates as equivalent (i.e., if $\frac{a}{b} = \frac{c}{d}$, then $a : b$ is equivalent to $c : d$). Here they make sense of this relationship from the other direction; they see that when two ratios are equivalent, they have the same unit rate (i.e., if $a : b$ is equivalent to $c : d$, then $\frac{a}{b} = \frac{c}{d}$).

Students explore the above by noticing structures in a table of equivalent ratios — that in addition to the values in the rows being equivalent ratios, the values in the columns have a

multiplicative relationship. They see, specifically, that dividing the values of the two quantities in the ratio result in the same quotient — the associated unit rate — and that it can be used to reason about one quantity of the ratio when the other is known.

Students also begin to transition from numerical examples to encapsulating a relationship with variables as they generalise their observations above.

As students work and discuss, identify those who observed structures in the table and can describe them well. Also look for students who could explain how they know the per-item cost is the same given two ratios expressed in variables.

Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

Launch

Arrange students in groups of 2. Give students a few minutes of quiet think time to complete the first two questions, and then 1–2 minutes to discuss with a partner their observations about the values in the table. Ask them to complete the last two questions together afterwards.

Representation: Internalise Comprehension. Use colour and annotations to illustrate student thinking. As students describe their calculations and the relationships they noticed in the tables, use colour and annotation to scribe their thinking on a display of each problem so that it is visible for all students.

Supports accessibility for: Visual-spatial processing; Conceptual processing Writing, Speaking: Stronger and Clearer Each Time. Use this routine to help students refine their justifications for the final question, “Explain why, if you can buy b burritos for 4 pounds, or buy $2 \times b$ burritos for $2 \times c$ pounds, the cost per item is the same in either case.” Listeners should press for details and clarity as appropriate based on what each speaker produces. Provide students with prompts for feedback that will help individuals strengthen their ideas and clarify their language (e.g., “Why do you think that?”, “How could you use values to show your thinking?”, “Would your explanation work if you bought 4 burritos?”, etc.). Students can borrow ideas and language from each partner to strengthen their final product.

Design Principle(s): Optimise output (for justification)

Anticipated Misconceptions

Students may not realise that the third column asks for pounds per 1 burrito and instead write 14 pounds per 2 burritos or 28 pounds per 8 burritos. If this happens, remind students that “per burrito” means “per 1 burrito.”

Student Task Statement

1. Two burritos cost £14. Complete the table to show the cost for 4, 5, and 10 burritos at that rate. Next, find the cost for a single burrito in each case.

number of burritos	cost in pounds	unit price (pounds per burrito)
2	14	
4		
5		
10		
b		

- What do you notice about the values in this table?
- Noah bought b burritos and paid c pounds. Lin bought twice as many burritos as Noah and paid twice the cost he did. How much did Lin pay per burrito?

	number of burritos	cost in pounds	unit price (pounds per burrito)
Noah	b	c	$\frac{c}{b}$
Lin	$2 \times b$	$2 \times c$	

- Explain why, if you can buy b burritos for c pounds, or buy $2 \times b$ burritos for $2 \times c$ pounds, the cost per item is the same in either case.

Student Response

1.

number of burritos	cost in pounds	unit price (pounds per burrito)
2	14	7
4	28	7
5	35	7
10	70	7
b	$b \times (7)$	7

- Answers vary. Sample responses: The ratios are equivalent. No matter how many burritos you buy, it costs £7 per burrito.

	number of burritos	cost in pounds	unit price (pounds per burrito)
Noah	b	c	$\frac{c}{b}$
Lin	$2 \times b$	$2 \times c$	$\frac{c}{b}$

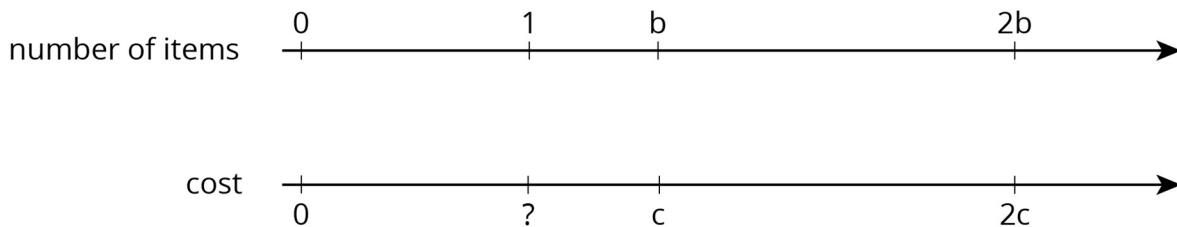
3. If you have b items for c pounds, then the unit rate is $\frac{c}{b}$ pounds per burrito. If you have $2b$ items for $2c$ pounds, then the unit rate is $\frac{2c}{2b}$ pounds per burrito. But $\frac{c}{b}$ and $\frac{2c}{2b}$ are equivalent fractions, so Noah and Lin each paid $\pounds\frac{c}{b}$ per burrito.

Activity Synthesis

After students have conferred with a partner, debrief as a class. Focus the discussion on students' calculations and the relationships they noticed in the tables. Display a completed version of the first table for all to see. Select previously identified students to share their observations. As they explain, illustrate their comments on the table. Students may bring up that:

- The ratios shown in the first two columns are equivalent across all rows.
- The ratios in all rows have the same pound-per-item value (unit rate).
- The value in the last column can be found by dividing the pound amount by the number of burritos. (If students use phrasing such as, "divide the second column by the first column," encourage them to use more precise terms.) Highlight the first two observations above, or bring them up if students do not. The last observation above is welcome but do not need to be emphasised as it will be the focus of the next activity.

Invite selected students to share their reasoning on the last two questions. Attend to the last question in particular, as it may be challenging to digest given its abstract nature. Consider using a double number line to help students visualise how the unit rate is the same for $b : c$ and $2b : 2c$, as shown.



Connect the location of $1 : ?$ with the unit rate, which students have by now recognised as having an unchanging value. Discuss how the unit rate is the same for any positive multiplier applied to $b : c$, since the multiplier would produce equivalent ratios.

7.3 Making Bracelets

10 minutes

At this point students understand that tables are a flexible tool for working with equivalent ratios. Up to this point, however, all actions performed on a table have started with both

values of a known ratio $a:b$ allowing students to move from row to row using multiplicative reasoning.

In this activity, however, students are asked to determine an unknown value of a ratio given only the other value of the ratio and a unit rate. Using their understanding of unit rate, equivalent ratios, and the relationship between the two, students learn how a unit rate is a factor that takes you from one column to another column in a table of equivalent ratios.

As students work, notice different approaches taken. For some students, the structure of the information in the table may not be apparent. Encourage them to refer to the tables in the preceding activity and think about the relationship between the ratios and unit rates there. Other students may be inclined to create a different table—such as the ones below—as an intermediate step for completing the given table. If so, consider asking them to share first during whole-class discussion.

time in hours	number of bracelets	speed (bracelets per hour)
1	6	6
2	12	6

This intermediate strategy makes good use of the “1” rows in the table, but is less efficient than directly dividing or multiplying by the unit rate to move from one column value to another since students have to work out both unit rates instead of using only the given unit rate.

Instructional Routines

- Compare and Connect
- Think Pair Share

Launch

Ask students to read the opening sentence and table. Explain that the first two columns show the ratio and the third column shows a unit rate associated with that ratio, similar to the structure of the tables in the previous activity.

Arrange students in groups of 2. Give students 3–4 minutes of quiet think time to complete the table and answer the questions, and then time to discuss their responses with a partner. Ask students to be mindful of how they go about completing the table and be prepared to explain their thinking.

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, review the relationship between the number of burritos purchased, cost in pounds, and the unit price of a burrito from the previous task.

Supports accessibility for: Social-emotional skills; Conceptual processing

Student Task Statement

Complete the table. Then, explain the strategy you used to do so.

time in hours	number of bracelets	speed (bracelets per hour)
2		6
5		6
7		6
	66	6
	100	6



Here is a partially filled table from an earlier activity. Use the same strategy you used for the bracelet problem to complete this table.

number of burritos	cost in pounds	unit price (pounds per burrito)
	14	7
	28	7
5		7
10		7

Next, compare your results with those in the first table in the previous activity. Do they match? Explain why or why not.

Student Response

time in hours	number of bracelets	speed (bracelets per hour)
2	12	6
5	30	6
7	42	6
11	66	6
$16\frac{2}{3}$	100	6

Since the number of bracelets divided by the time in hours is 6 bracelets per hour, then the time in hours multiplied by 6 should give the number of bracelets. Using similar thinking, the number of bracelets divided by 6 gives the time in hours.

number of burritos	cost in pounds	unit price (pounds per burrito)
2	14	7
4	28	7
5	35	7
10	70	7

Answers vary. Sample response: Yes, the strategy works and my table matches. To figure out cost in pounds from number of burritos and pounds per burrito, I can multiply 7 times the number of burritos. For example, 5 burritos cost £35. To find values in the first column using the second two columns, I can divide cost in pounds by pounds per burrito. For example, 14 divided by 7 is 2, so I can buy 2 burritos for £14.

Activity Synthesis

After they have a chance to discuss with a partner, select a few students to share with the class their strategies for completing the table. Start with students using less efficient strategies, such as those that worked out the 1 rows. Progress toward using the given unit rate to navigate from column to column in efficient ways, such as multiplying the time in hours by 6 to find number of bracelets, and multiplying the number of bracelets by $\frac{1}{6}$ to find time in hours.

Writing, Listening, Conversing: Compare and Connect. Assign students to prepare a visual display that shows how they completed the table for the first question, including a brief explanation of the strategy they used. Ask students to investigate each other’s work by taking a tour of the visual displays and facilitate discussion about comparisons and connections of the different approaches or representations in their work. Prompt students with questions such as, “Did anyone solve the problem in a similar way, but would explain it differently?,” “Why does this approach use ____, and this one does not? Is the outcome the same?.” During the discussion, amplify language students use to communicate about unit rate, equivalent ratios, and the relationship between the two. This helps students to reflect on, and linguistically respond to, the comparisons and connections about the mathematical features that are key to this lesson.

Design Principle(s): Cultivate conversation; Maximise meta-awareness

7.4 How Much Applesauce?

Optional: 10 minutes

This optional activity is a chance to apply more sophisticated, newly-learned techniques to a familiar-looking problem about equivalent ratios. None of the given numbers are

multiples of each other from row to row (for example, 7 isn't a multiple of 4), so this problem lends itself to reasoning about unit rates.

Instructional Routines

- Discussion Supports

Launch

If any student is familiar with making applesauce, ask them to explain how it is made. If not, explain: to make applesauce, you peel, core, and chop apples. Then, heat the apples gently in a saucepan for a while until they break down into a sauce. Finally, add flavours like lemon juice and cinnamon. If you know how many pounds of apples you start with, you can predict how many cups of applesauce after cooking.

Student Task Statement

It takes 4 pounds of apples to make 6 cups of applesauce.

1. At this rate, how much applesauce can you make with:
 - a. 7 pounds of apples?
 - b. 10 pounds of apples?
2. How many pounds of apples would you need to make:
 - a. 9 cups of applesauce?
 - b. 20 cups of applesauce?

pounds of apples	cups of applesauce
4	6
7	
10	
	9
	20

Student Response

To find the cups of applesauce, multiply the pounds of apples by 1.5.

To find the pounds of apples, divide the cups of applesauce by 1.5, or multiply by $\frac{2}{3}$.

pounds of apples	cups of applesauce
4	6
7	10.5
10	15
6	9
$13\frac{1}{3}$	20

Are You Ready for More?

1. Jada eats 2 scoops of ice cream in 5 minutes. Noah eats 3 scoops of ice cream in 5 minutes. How long does it take them to eat 1 scoop of ice cream working together (if they continue eating ice cream at the same rate they do individually)?
2. The garden hose at Andre's house can fill a 5-gallon bucket in 2 minutes. The hose at his next-door neighbour's house can fill a 10-gallon bucket in 8 minutes. If they use both their garden hoses at the same time, and the hoses continue working at the same rate they did when filling a bucket, how long will it take to fill a 750-gallon pool?

Student Response

1. 1 minute. Jada eats $\frac{2}{5}$ scoop in 1 minute and Noah eats $\frac{3}{5}$ scoop in 1 minute. So together, they eat 1 scoop in 1 minute.
2. 200 minutes or 3 hours and 20 minutes. The rate for Andre's hose is 2.5 gallons per minute, and the rate for his neighbour's hose is 1.25 gallons per minute. If they use the hoses at the same time, the pool will fill at a rate of 3.75 gallons per minute. $750 \div 3.75 = 200$, so it will take 200 minutes for the hoses to emit 750 gallons of water.

Activity Synthesis

Highlight approaches where students compute how many pounds of apples per cup of applesauce and how many cups of applesauce per pound of apples and use multiplicative reasoning to move from column to column.

Speaking, Listening, Conversing: Discussion Supports. As students are discussing how they make applesauce using the given information in the problem, press for details in students' explanations when finding how many pounds of apples are needed to make a recipe or how many cups of applesauce are made from some pounds of apples. Encourage think aloud by talking through their approaches when calculating the missing values in the table of equivalent ratios. This will help students make sense of the problem and the approaches used to complete the table.

Design Principle(s): Support sense-making; Cultivate conversation

Lesson Synthesis

In this lesson, students learned about two new ideas around ratios and rates:

- Equivalent ratios have the same unit rate.
- Unit rates are the factors that takes you from one column to the other column in a table of equivalent ratios.

Summarise for students the two new ideas and, if possible, highlight how students used them in the final activity of the lesson.

7.5 Cheetah Speed

Cool Down: 5 minutes

Student Task Statement

A cheetah can run at its top speed for about 25 seconds. Complete the table to represent a cheetah running at a constant speed. Explain or show your reasoning.

time (seconds)	distance (metres)	speed (metres per second)
4	120	
25		
	270	

Student Response

time (seconds)	distance (metres)	speed (metres per second)
4	120	30
25	750	30
9	270	30

Sample reasoning: Since the cheetah runs 120 metres in 4 seconds, this is 30 metres per second because $120 \div 4 = 30$. Since it says the cheetah runs at a constant speed, the speed in each row is 30 metres per second. To find the distance run in 25 seconds, multiply 25 by 30. To find the time it takes to run 270 metres, divide 270 by 30.

Student Lesson Summary

The table shows different amounts of apples selling at the same rate, which means all of the ratios in the table are equivalent. In each case, we can find the *unit price* in pounds per pound by dividing the price by the number of pounds.

apples (pounds)	price (pounds)	unit price (pounds per pound)
4	10	$10 \div 4 = 2.50$
8	20	$20 \div 8 = 2.50$
20	50	$50 \div 20 = 2.50$

The unit price is always the same. Whether we buy 4 pounds of apples for 10 pounds or 8 pounds of apples for 20 pounds, the apples cost 2.50 pounds per pound.

We can also find the number of pounds of apples we can buy per pound by dividing the number of pounds by the price.

apples (pounds)	price (pounds)	pounds per pound
4	10	$4 \div 10 = 0.4$
8	20	$8 \div 20 = 0.4$
20	50	$20 \div 50 = 0.4$

The number of pounds we can buy for a pound is the same as well! Whether we buy 4 pounds of apples for 10 pounds or 8 pounds of apples for 20 pounds, we are getting 0.4 pounds per pound.

This is true in all contexts: when two ratios are equivalent, their unit rates will be equal.

quantity x	quantity y	unit rate 1	unit rate 2
a	b	$\frac{a}{b}$	$\frac{b}{a}$
$s \times a$	$s \times b$	$\frac{s \times a}{s \times b} = \frac{a}{b}$	$\frac{s \times b}{s \times a} = \frac{b}{a}$

Lesson 7 Practice Problems

Problem 1 Statement

A car travels 55 miles per hour for 2 hours. Complete the table.

time (hours)	distance (miles)	miles per hour
1	55	55
$\frac{1}{2}$		
$1\frac{1}{2}$		
	110	

Solution

time (hours)	distance (miles)	miles per hour
1	55	55
$\frac{1}{2}$	27.5	55
$1\frac{1}{2}$	82.5	55
2	110	55

Problem 2 Statement

The table shows the amounts of onions and tomatoes in different-sized batches of a salsa recipe.

Elena notices that if she takes the number in the tomatoes column and divides it by the corresponding number in the onions column, she always gets the same result.

What is the meaning of the number that Elena has calculated?

onions (ounces)	tomatoes (ounces)
2	16
4	32
6	48

Solution

The recipe calls for 8 ounces of tomatoes per ounce of onions.

Problem 3 Statement

A restaurant is offering 2 specials: 10 burritos for £12, or 6 burritos for £7.50. Noah needs 60 burritos for his party. Should he buy 6 orders of the 10-burrito special or 10 orders of the 6-burrito special? Explain your reasoning.

Solution

Answers vary. Possible reasoning: Noah should get 6 orders of the 10-burrito special. The 10-burrito special sells burritos at a rate of £1.20 per burrito, because $12 \div 10 = 1.20$. The 6-burrito special sells at a rate of £1.25 per burrito, because $7.5 \div 6 = 1.25$. The 10-burrito special is a better deal.

Problem 4 Statement

Complete the table so that the cost per banana remains the same.

number of bananas	cost in pounds	unit price (pounds per banana)
4		0.50
6		0.50
7		0.50
10		0.50
	10.00	0.50

	16.50	0.50
--	-------	------

Solution

number of bananas	cost in pounds	pounds per banana
4	2.00	0.50
6	3.00	0.50
7	3.50	0.50
10	5.00	0.50
20	10.00	0.50
33	16.50	0.50

Problem 5 Statement

Two planes travel at a constant speed. Plane A travels 2 800 miles in 5 hours. Plane B travels 3 885 miles in 7 hours. Which plane is faster? Explain your reasoning.

Solution

Plane A is faster. Plane A travels $2\,800 \div 5 = 560$ or 560 miles per hour. Plane B travels $3\,885 \div 7 = 555$, or 555 miles per hour. Plane A travels a farther distance in one hour.

Problem 6 Statement

A car has 15 gallons of petrol in its tank. The car travels 35 miles per gallon of petrol. It uses $\frac{1}{35}$ of a gallon of petrol to go 1 mile.

- How far can the car travel with 15 gallons? Show your reasoning.
- How much petrol does the car use to go 100 miles? Show your reasoning.

Solution

a. 525 miles. Possible reasoning:

gallons of petrol	miles car can travel
1	35
5	175
15	525

b. $\frac{100}{35}$ (or $\frac{20}{7}$ or $2\frac{6}{7}$) gallons. Possible reasoning:

gallons of petrol	miles car can travel
$\frac{1}{35}$	1

$\frac{10}{35}$	10
$\frac{100}{35}$	100

Problem 7 Statement

A box of cereal weighs 600 grams. How much is this weight in pounds? Explain or show your reasoning. (Note: 1 kilogram = 2.2 pounds)

Solution

1.32 pounds. Explanations vary. Possible explanation:

grams	pounds
1 000	2.2
100	0.22
500	1.1
600	1.32

(Note that for the first line of the table, 1 kilogram is written as 1 000 grams.)



© These materials were derived and adapted from Illustrative Mathematics's IM 6–8 Math™. IM 6–8 Math was originally developed by Open Up Resources and authored by Illustrative Mathematics®, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0) <https://creativecommons.org/licenses/by/4.0/>. OUR's 6–8 Math Curriculum is available at <https://openupresources.org/math-curriculum/>. Adaptations and updates to IM 6–8 Math™ are copyright 2019 by Illustrative Mathematics®, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). Further adaptations have been made by MEI.