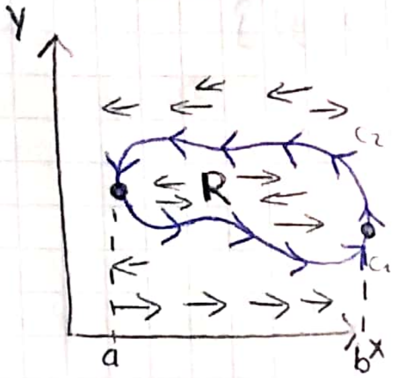
DEMOSTRACION DEL TEOREMA DE GREEN

$$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$



$$\vec{p}(x,y) = p(x,y)\,\hat{\imath}$$

$$\oint_C \vec{p} \cdot d\vec{r}$$

$$\oint_C p(x,y)\,dx + 0$$

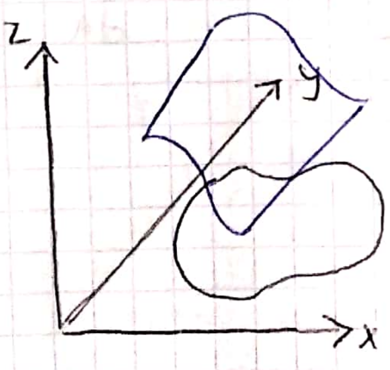
$$\oint_C p(x,y)\,dx = \underbrace{\int_a^b p(x,y_1(x))\,dx}_{C_1} + \underbrace{\int_b^a p(x,y_2(x))\,dx}_{C_2}$$

$$\oint_C p(x,y)\,dx = \int_a^b p(x,y_1(x))\,dx - \int_a^b p(x,y_2(x))\,dx$$

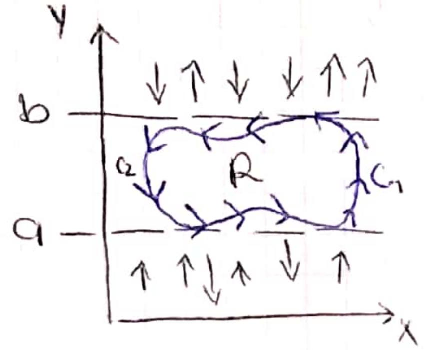
$$= \oint_C p(x,y)\,dx = \int_a^b \big(p(x,y_1(x)) - p(x,y_2(x))\big)\,dx$$

$$= -\int_a^b \big(p(x,y_2(x)) - p(x,y_1(x))\big)\,dx$$

$$= -\int_a^b p(x,y)\Big|_{y=y_1(x)}^{y=y_2(x)} dx = -\int_a^b \int_{y_1(x)}^{y_2(x)} \frac{dp}{dy}\,dy\,dx$$



$$\oint_C p(x,y)\,dx = -\iint_R \frac{dp}{dy}\,dy\,dx$$



$$\vec{q}\,(x,y) = Q\,(x,y)\,\hat{j}$$

$C_2 :\ x = x_2(y)$

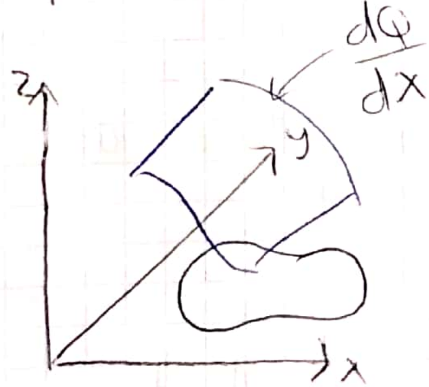
$C_1 :\ x = x_1(y)$

$$\oint_c \vec{q}\cdot d\vec{r} = \oint_c Q\,(x,y)\,dy = \int_b^a Q\,(x_2(y)\,, y\ dy + \int_a^b Q(x_1(y))\,, y\ dy$$

$$\oint_c Q\,(x,y)dy = \int_a^b Q\,(x_2(y))\,, y\ dy + \int_a^b Q\,(x_1(y))\,, y\ dy$$

$$= \int_a^b \left(Q\,(x_1(y), y) - Q(x_2\,(y), y)\right) dy = \underbrace{\int_a^b Q\,(x,y)\Big|_{x = x_2(y)}^{x = x_1(y)} dy}$$

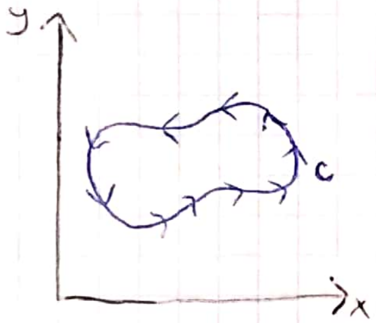
$$\underbrace{\int_a^b \int_{x_2(y)}^{x_1(y)}}_{R} \frac{dQ}{dx}\ \underbrace{dx\,dy}_{dA}$$



$$\oint_c \vec{q}\cdot di = \oint_c Q\,(x,y)\,dy = \iint_R \frac{dQ}{dx}\,dA$$

$$\oint_C P(x,y)\,dx = -\iint_R \frac{dP}{dy}\,dy\,dx \qquad \oint_C Q(x,y)\,dy = \iint_R \frac{dQ}{dx}\,dA$$

$$\vec{F}(x,y) = P(x,y)\hat{\imath} + Q(x,y)\hat{\jmath} \qquad d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$$



$$\oint_C \vec{F}\cdot d\vec{r} = \oint_C P(x,y)\,dx + Q(x,y)\,dy$$

$$= \oint_C P(x,y)\,dx + \oint_C Q(x,y)\,dy$$

$$= \iint_R -\frac{dP}{dy} = dA + \iint_R \frac{dQ}{dx}\,dA$$

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_R \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA$$

Si $\vec{F}$ es conservativo $\oint_C \vec{F}\cdot dr = 0$ $\quad \boxed{\frac{dQ}{dx} = \frac{dP}{dy}}$

$$\Rightarrow \frac{dQ}{dx} \cdot \frac{dP}{dy} = 0$$