Mathematics Education Innovation

Lesson 11: Dividing numbers that result in decimals

Goals

- Interpret different methods for calculating a quotient that is not a whole number, and express it (orally and in writing) in terms of ungrouping.
- Use long division to divide whole numbers that result in a quotient with a decimal, and explain (orally) the solution method.

Learning Targets

- I can use long division to find the quotient of two whole numbers when the quotient is not a whole number.

Lesson Narrative

So far, students have divided whole numbers that result in whole-number quotients. In the next three lessons, they work toward performing division in which the divisor, dividend, and quotient are decimals. In this lesson, they perform division of two whole numbers that result in a terminating decimal. Students divide using all three techniques introduced in this unit: base-ten diagrams, partial quotients, and long division. They apply this skill to calculate the (terminating) decimal expansion of some fractions.

Students analyse, explain, and critique various ways of reasoning about division.

Building On

- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Addressing

- Fluently divide multi-digit numbers using the standard algorithm.

Building Towards

- Apply and extend previous understandings of arithmetic to algebraic expressions.

Instructional Routines

- Stronger and Clearer Each Time
- Compare and Connect
- Discussion Supports
- Number Talk
- Think Pair Share

Required Preparation

Students may choose to draw base-ten diagrams in this lesson. If drawing them is a challenge, consider giving students access to:

- Commercially produced base-ten blocks, if available.
- Paper copies of squares and rectangles (to represent base-ten units), cut up from copies of the blackline master of the second lesson in the unit.
- Digital applet of base-ten representations https://www.geogebra.org/m/FXEZD466

Some students might find it helpful to use graph paper to help them align the digits as they divide using long division and the partial quotients method. Consider having graph paper accessible throughout the lesson.

Student Learning Goals

Let's find quotients that are not whole numbers.

11.1 Number Talk: Evaluating Quotients

Warm Up: 5 minutes

This number talk encourages students to think about the numbers in a calculation problem and rely on what they know about structure, patterns, and division to mentally solve a problem. Four expressions are given. The first three expressions are partial quotients that could help students evaluate the last expression of $496 \div 8$. Be sure to allocate more time to discuss the final expression and to draw out its connection to the other expressions.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one expression at a time, or ask students to work on one expression at a time and begin when cued. Give students 30 seconds of quiet think time per question and ask them to give a signal when they have an answer and can explain their strategy. Select 1–2 students to briefly discuss how they reasoned about the quotient.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Anticipated Misconceptions

Some students may start from scratch when evaluating the final question. Ask them how they could use what they have already done to help them with the final question.

Student Task Statement

Find the quotients mentally.

$400 \div 8$

$80 \div 8$

$16 \div 8$

$496 \div 8$

Student Response

- 50. Strategies vary. Possible strategy: $(40 \div 8) \times 10$.
- 10. Strategies vary.
- 2. Strategies vary.
- 62. Strategies vary. Possible strategy: $496 \div 8 = (400 \div 8) + (80 \div 8) + (16 \div 8)$.

Activity Synthesis

Ask students to share their reasoning for each quotient. Record and display their explanations for all to see; this will help students see the connections between the first three expressions and the last one. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone find the quotient the same way but would explain it differently?”
- “Did anyone find the quotient in a different way?”
- “Does anyone want to add on to _____’s reasoning?”
- “Do you agree or disagree? Why?”

If not mentioned by students, highlight how the quotients of $(400 \div 8) + (80 \div 8) + (16 \div 8)$ can be used to find $496 \div 8$.

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

11.2 Keep Dividing

Optional: 20 minutes

This activity extends division techniques used to find whole-number quotients to divide whole numbers resulting in (terminating) decimal quotients. In these problems, students see remainders in the ones place. In order to continue the division process, the ones are broken into tenths. Conceptually, this is the same ungrouping idea that is used when hundreds are broken into tens or when tens are broken into ones.

If students have trouble drawing the diagrams to represent ungrouping, consider providing actual base-ten blocks or paper cut-outs of base-ten units (from the blackline master used earlier in the unit) so that they can physically trade the pieces (e.g., 2 ones for 20 tenths).

Instructional Routines

- Compare and Connect
- Think Pair Share

Launch

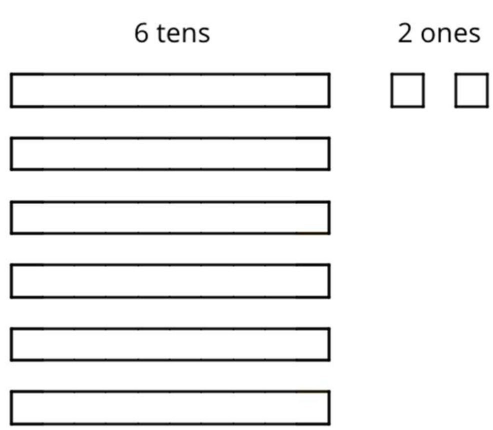
Arrange students in groups of 2. Provide access to graph paper. For the first question, give students 1 minute of quiet think time to analyse Mai's work and 2–3 minutes to discuss their observations with their partner. Pause for a whole-class discussion, making sure that all students understand how Mai dealt with the remainder.

Give students 5–7 minutes to complete the final two questions. Follow with a whole-class discussion.

Action and Expression: Develop Expression and Communication. Encourage students to begin with physical representations before drawing a diagram. Provide access to base-ten blocks or paper cut-outs of base-ten units to support drawing diagrams.
Supports accessibility for: Conceptual processing

Student Task Statement

Mai used base-ten diagrams to calculate $62 \div 5$. She started by representing 62.



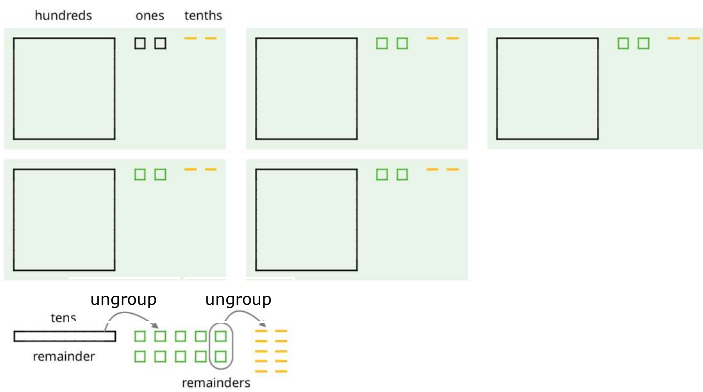
She then made 5 groups, each with 1 ten. There was 1 ten left. She ungrouped it into 10 ones and distributed the ones across the 5 groups.

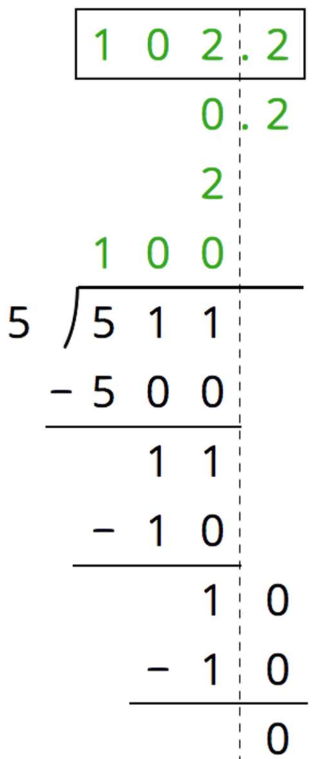
Here is Mai's diagram for 62 ÷ 5.

1. Discuss these questions with a partner and write down your answers:
    a. Mai should have a total of 12 ones, but her diagram shows only 10. Why?
    b. She did not originally have tenths, but in her diagram each group has 4 tenths. Why?
    c. What value has Mai found for 62 ÷ 5? Explain your reasoning.
2. Find the quotient of 511 ÷ 5 by drawing base-ten diagrams or by using the partial quotients method. Show your reasoning. If you get stuck, work with your partner to find a solution.
3. Four students share a £271 prize from a science competition. How much does each student get if the prize is shared equally? Show your reasoning.

Student Response

1.
    a. Mai ungrouped two ones blocks to make 20 tenths. So instead of 12 ones, her diagram has 10 ones and 20 tenths.
    b. In order to complete the division into 5 equal groups, Mai needed to ungroup 2 of her ones to make 20 tenths. She then placed 4 tenths in each of the equal groups.
    c. Mai has divided 62 into 5 equal groups of 1 ten, 2 ones, and 4 tenths. So 62 ÷ 5 = 12.4.
2. 102.2. Sample reasoning:





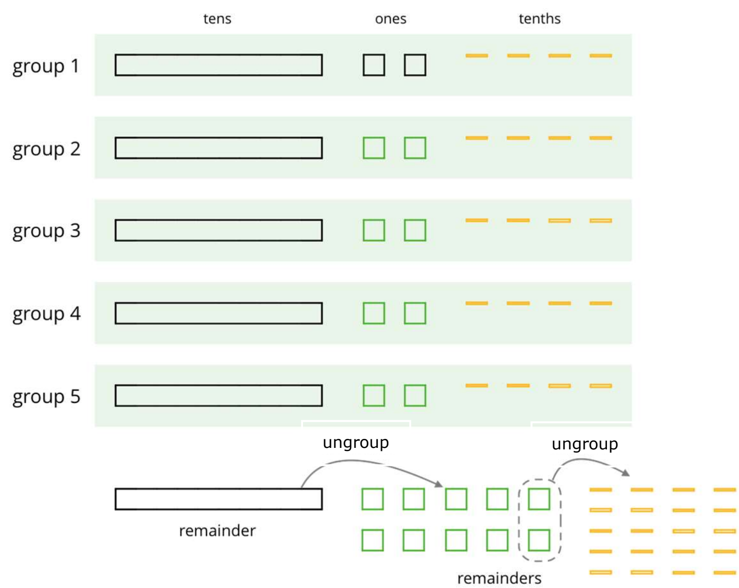
3. £67.75. Sample reasoning: There are 4 groups of £67 in £268, so each student gets £67 pounds, and the group must split the remaining £3 evenly. Since 300 pennies can be divided into 4 groups of 75 pennies, each student received £0.75. This means that each student gets £67.75.

Activity Synthesis

The goal of this discussion is for students to relate their earlier work on division, which resulted in whole-number quotients, to division involving decimal quotients. Below is an example of how the discussion may go, along with questions to ask students and some possible responses. Begin the discussion by reminding students of the work they have previously done to evaluate 657 ÷ 3.

How is calculation for 62 ÷ 5 similar to that for 657 ÷ 3?

- The method of division is the same: we divide a given number (62 or 657) into equal groups until everything is distributed.
- We divide by using place value, ungrouping one unit into ten of a smaller unit as needed. For 62 ÷ 5, the 2 ones can be broken into 20 tenths, while in 657 ÷ 3, the 2 tens were ungrouped into 20 ones.



How is calculation for 62 ÷ 5 different from that for 657 ÷ 3?

- There is no remainder for 657 ÷ 3, while there is a remainder of 2 for 62 ÷ 5.
- We need to write a decimal point and work with tenths in 62 ÷ 5.

It is important to stress that the methods and steps are the same in both calculations. The big new idea here is that sometimes a division problem of whole numbers does not end when we get to the ones place. In these cases, we have to add a decimal because the number being divided involves tenths, hundredths, or smaller base-ten units.

Representing, Speaking: Compare and Connect. Use this routine to give students an opportunity to compare approaches for finding the quotient of 511 ÷ 5. Ask students to share their approach with a partner. Invite groups to discuss what is the same and what it different about finding a quotient either by drawing base-ten diagrams or by using the partial quotients method. Listen for opportunities to highlight language such as "remainder," and "tenths." This will help students connect division techniques and extend them to find decimal quotients.
Design Principle(s): Optimise output; Cultivate conversation

11.3 Using Long Division to Calculate Quotients

25 minutes

In this activity, students use long division to divide whole numbers whose quotient is not a whole number. Previously, students found the quotient of $62 \div 5$ using base-ten diagrams and the partial quotients method. Because the long division is a particular version of the partial quotients method, and because students have been introduced to long division, they have the tools to divide whole numbers that result in a (terminating) decimal. In this activity, students evaluate and critique the reasoning of others.

Instructional Routines

- Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Provide access to graph paper. Give students 7–8 minutes to analyse and discuss Lin's work with a partner and then complete the second set of questions.

Pause to discuss students' analyses and at least one of the division problems. Students should understand that, up until reaching the decimal point, long division works the same for $62 \div 5 = 12.4$ as it does for $657 \div 3 = 219$. In $62 \div 5$, however, there is a remainder of 2 ones, and we need to convert to the next smaller place value (tenths), change the 2 ones into 20 tenths, and then divide these into 5 equal groups of 4 tenths.

Prepare students to do the last set of questions by setting up the long division of $5 \div 4$ for all to see. Discuss the placement of the decimal point. Reiterate that we can write extra zeros at the end of the dividend, following the decimal point, so that remainders can be worked with. Have students work through this problem and the others. Follow with a whole-class discussion.

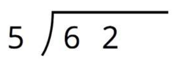
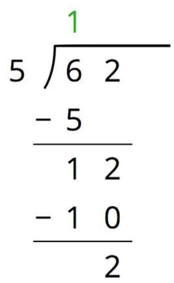
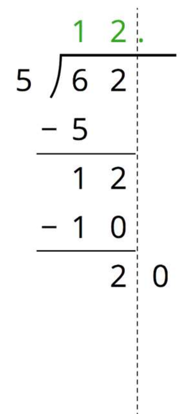
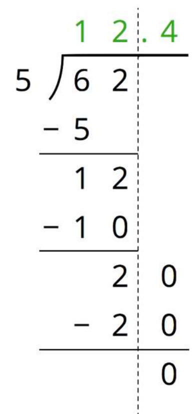
Representation: Access for Perception. Display and read Lin's method for calculating the quotient of $62 \div 5$ aloud. Students who both listen to and read the information will benefit from extra processing time.
Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

Students may be perplexed by the repeating decimals in the last question and think that they have made a mistake. Ask them to compare their work with a partner's, and then clarify during discussion that some decimals do repeat. Because this work comes into focus later in KS3, the goal here is simply for students to observe that not all decimals terminate.

Student Task Statement

Here is how Lin calculated $62 \div 5$.

Lin set up the numbers for long division.	She subtracted 5 times 1 from the 6, which leaves a remainder of 1. She wrote the 2 from 62 next to the 1, which made 12, and subtracted 5 times 2 from 12.	Lin drew a vertical line and a decimal point, separating the ones and tenths place. 12 – 10 is 2. She wrote 0 to the right of the 2, which made 20.	Lastly, she subtracted 5 times 4 from 20, which left no remainder. At the top, she wrote 4 next to the decimal point.
			

1. Discuss with your partner:
   - Lin put a 0 after the remainder of 2. Why? Why does this 0 not change the value of the quotient?
   - Lin subtracted 5 groups of 4 from 20. What value does the 4 in the quotient represent?
   - What value did Lin find for $62 \div 5$?

2. Use long division to find the value of each expression. Then pause so your teacher can review your work.

   a. $126 \div 8$

   b. $90 \div 12$

3. Use long division to show that:

   a. $5 \div 4$, or $\frac{5}{4}$, is 1.25.

   b. $4 \div 5$, or $\frac{4}{5}$, is 0.8.

   c. $1 \div 8$, or $\frac{1}{8}$, is 0.125.

   d. $1 \div 25$, or $\frac{1}{25}$, is 0.04.

4. Noah said we cannot use long division to calculate 10 ÷ 3 because there will always be a remainder.

   a. What do you think Noah meant by “there will always be a remainder”?

   b. Do you agree with him? Explain your reasoning.

Student Response

1. Answers vary. Sample response:

   a. Lin put a 0 after the 2 because she could not take any more 5’s from 2 but wanted to continue the calculation to the tenths place. The 0 represents 0 tenths and 20 tenths has the same value as 2 ones.

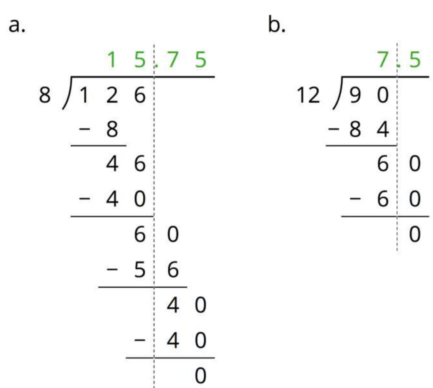
   b. The 4 in the quotient represents 4 tenths. 20 tenths is 5 equal groups of 4 tenths, and so 4 tenths is added to the quotient.

   c. The value of 62 ÷ 5 is 12.4. There was no remainder after putting 1 ten, 2 ones, and 4 tenths into 5 equal groups.

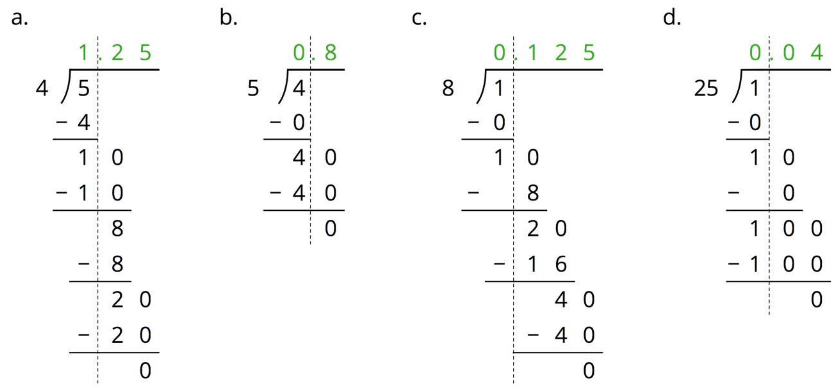
2.

   a. $126 \div 8 = 15.75$

   b. $90 \div 12 = 7.5$



3.



4.

a. Write a decimal point and a 0 because 3 is larger than 1. There are 3 threes in 10 with a remainder of 1. Then write another 0 after the decimal point resulting in 10 again. 3 threes can continue to be taken out, but there is *always* 1 remaining.

b. Yes, after writing another 0 after the decimal point, there is one remaining after taking out all of the threes.

Activity Synthesis

Focus the whole-class discussion on the third and fourth sets of questions. Ask a few students to show their long division for all to see and to explain their steps. Some ideas to bring to uncover:

- Problems like 1 ÷ 25 are challenging because the first step is 0: there are *zero* groups of 25 in 1. This means that we need to introduce a decimal and put a 0 to the right of the decimal. But one 0 is not enough. It is not until we add the second 0 to the right of the decimal that we can find 4 groups of 25 in 100. Because we moved two places to the right of the decimal, these 4 groups are really 0.04, which is the quotient of 1 by 25.
- Problems like 1 ÷ 3 are not fully treated until later in KS3. At this point, we can observe that the long division process will go on and on because there is always a remainder of 1.

Some questions to ask students that highlight these points include:

- When you found 1 ÷ 8, what was your first step? (I put a 0 above the dividend 1 and added a decimal because I cannot take any 8’s from 1.)
- When you found 1 ÷ 25, what were your first few steps? (I put a 0 above the dividend 1 and added a decimal because I cannot take any 25’s from 1. I needed to add another 0 to the right of the decimal because I still cannot take any 25’s from 10.)

- Why do we not write 0’s in advance to the right of the decimal? (We do not know in advance if we will need these 0’s or how many of them we might need, so we usually write them as needed.)

Another valuable link to make is to connect the values for the decimals in Problem 4 to percentages. For example, the fact that $\frac{4}{5} = 0.8$ means that $\frac{4}{5}$ of a quantity is 80% of that quantity. Similarly $\frac{1}{25}$ of a quantity is the same as 4% of that quantity.

Writing, Listening, Conversing: Stronger and Clearer Each Time. Use this routine to help students improve their written responses to the question, “Noah said we cannot use long division to calculate 10 ÷ 3 because there will always be a remainder. Do you agree or disagree with this statement?” Give students time to meet with 2–3 partners to share and get feedback on their responses. Provide listeners with prompts that will help their partners strengthen their ideas and clarify their language (e.g., “Can you explain why there is always a remainder?”, “What number always remains?”, and “Could you try to explain this using a different example?”). Give students 1–2 minutes to revise their writing based on the feedback they received. Communicating their reasoning with a partner will help students understand situations in which long division does not work
Design Principle(s): Optimise output (for explanation); Cultivate conversation

Lesson Synthesis

In this lesson, we saw that the quotient of two whole numbers can result in a decimal value. We can observe how this happens with base-ten blocks and by calculating with long division.

- When using base-ten blocks to divide, how can we work with a remainder of ones? (Ungroup the ones into tenths.)
- When calculating with long division, how can we keep dividing when there is a remainder? (Put a decimal point after the dividend and follow it with zeros. This allows you to bring down a zero and continue the calculation.)
- How do we divide a whole number that is smaller than the divisor, for instance 4 ÷ 5? (Start by writing a 0 for the quotient and follow it with a decimal point. In the example of 4 ÷ 5, the 0 means there is not enough ones to divide equally into 5 groups. Then, ungroup the number into ten of the next smaller unit so that it can be divided. In this example, 4 ones can be ungrouped into 40 tenths, which can then be divided by 5.)

11.4 Calculating Quotients

Cool Down: 5 minutes

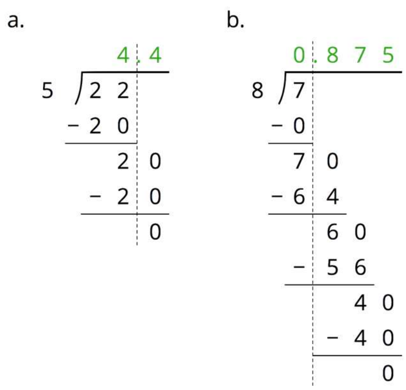
Student Task Statement

Use long division to find each quotient. Show your calculation and write your answer as a decimal.

1. 22 ÷ 5

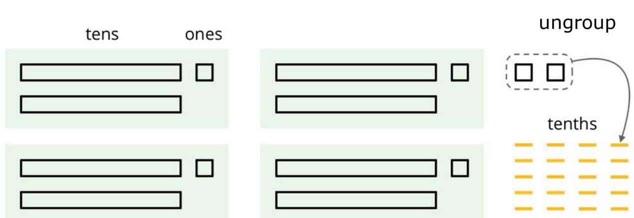
2. 7 ÷ 8

Student Response



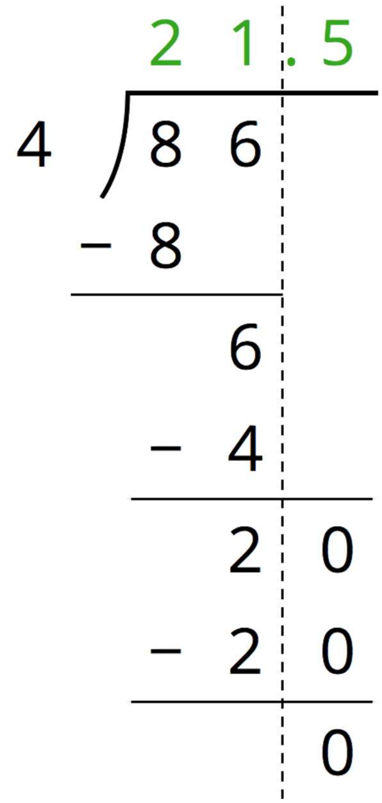
Student Lesson Summary

Dividing a whole number by another whole number does not always produce a whole-number quotient. Let's look at 86 ÷ 4, which we can think of as dividing 86 into 4 equal groups.



We can see in the base-ten diagram that there are 4 groups of 21 in 86 with 2 ones left over. To find the quotient, we need to distribute the 2 ones into the 4 groups. To do this, we can ungroup or decompose the 2 ones into 20 tenths, which enables us to put 5 tenths in each group.

Once the 20 tenths are distributed, each group will have 2 tens, 1 one, and 5 tenths, so $86 \div 4 = 21.5$.



We can also calculate 86 ÷ 4 using long division.

The calculation shows that, after removing 4 groups of 21, there are 2 ones remaining. We can continue dividing by writing a 0 to the right of the 2 and thinking of that remainder as 20 tenths, which can then be divided into 4 groups.

To show that the quotient we are working with now is in the tenth place, we put a decimal point to the right of the 1 (which is in the ones place) at the top. It may also be helpful to draw a vertical line to separate the ones and the tenths.

There are 4 groups of 5 tenths in 20 tenths, so we write 5 in the tenths place at the top. The calculation likewise shows $86 \div 4 = 21.5$.

Lesson 11 Practice Problems

1. **Problem 1 Statement**

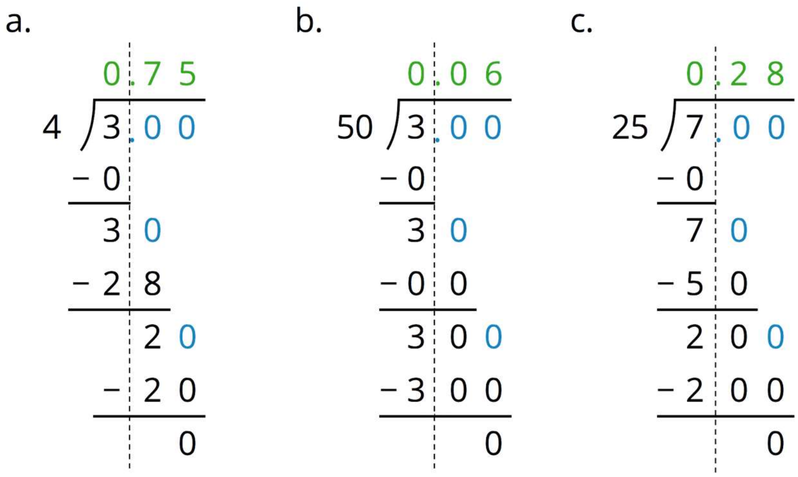
   Use long division to show that the fraction and decimal in each pair are equal.

   $\frac{3}{4}$ and 0.75

   $\frac{3}{50}$ and 0.06

   $\frac{7}{25}$ and 0.28

Solution



2. **Problem 2 Statement**

Mai walked $\frac{1}{8}$ of a 30-mile walking trail. How many miles did Mai walk? Explain or show your reasoning.

Solution

3.75 miles. Reasoning varies. Sample reasoning: $\frac{1}{8}$ of 30 is $30 \div 8 = 3.75$

3. **Problem 3 Statement**

Use long division to find each quotient. Write your answer as a decimal.

a. $99 \div 12$

b. $216 \div 5$

c. $1,988 \div 8$

Solution

a.

```
         8.2 5
12  ) 9 9.0 0
    - 9 6
        3 0
      - 2 4
          6 0
        - 6 0
            0
```

b.

```
          4 3.2
5  ) 2 1 6.0
   - 2 0
       1 6
     - 1 5
         1 0
       - 1 0
           0
```

c.

```
        2 4 8.5
8  ) 1 9 8 8.0
   - 1 6
       3 8
     - 3 2
         6 8
       - 6 4
           4 0
         - 4 0
             0
```

4. **Problem 4 Statement**

Tyler reasoned: "$\frac{9}{25}$ is equivalent to $\frac{18}{50}$ and to $\frac{36}{100}$, so the decimal of $\frac{9}{25}$ is 0.36."

a. Use long division to show that Tyler is correct.

b. Is the decimal of $\frac{18}{50}$ also 0.36? Use long division to support your answer.

Solution

a.

```
        0.3 6
2 5 ) 9.0 0
    - 0
      9 0
    - 7 5
      1 5 0
    - 1 5 0
          0
```

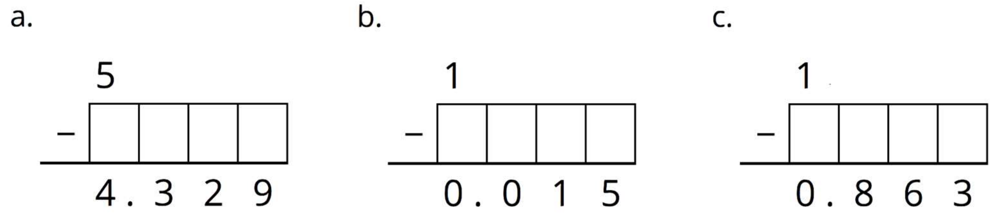
b.

```
           0.3 6
5 0 ) 1 8.0 0
    -     0
      1 8 0
    - 1 5 0
        3 0 0
    -   3 0 0
            0
```

Yes, the decimal of $\frac{18}{50}$ is also 0.36.

5. **Problem 5 Statement**

Complete the calculations so that each shows the correct difference.

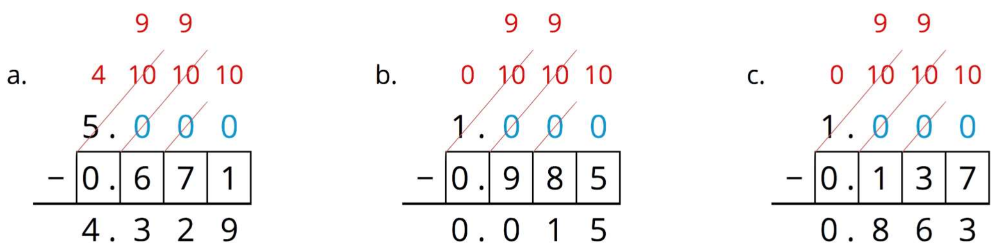


Solution

a. 0.671

b. 0.985

c. 0.137



Problem 6 Statement

Use the equation $124 \times 15 = 1860$ and what you know about fractions, decimals, and place value to explain the value reached when you evaluate $(1.24) \times (0.15)$.

Solution

1.24 is $124 \times (0.01)$ and 0.15 is $15 \times (0.01)$. So $(1.24) \times (0.15)$ can be written as $124 \times 15 \times (0.01) \times (0.01)$, which is $(1860) \times (0.0001)$ or 0.186.