520 - Arrange by Volume

MATHS / 3D GEOMETRY

DESCRIPTION

LEVEL 1   Students arrange the blocks in ascending order according to their volume without calculations.

LEVEL 2   Students arrange the blocks in ascending order according to their volume using calculations. Students also consider what blocks of equal volume have in common.

LEVEL 3   Students arrange the blocks in ascending order according to their volume without calculating the volume of the blocks but calculating (or just comparing) the differences of the volumes of any two blocks.

HINT   Calculate the difference of the volumes of two blocks that have two heights in common but the third heights differ by 1 unit.

SOLUTIONS / EXAMPLES

LEVEL 1   Some possible reasonings are listed below.

- Two blocks 123 (or 132) form a regular prism of height 4, hence one block 123 (or 132) has the same volume as block 222.

- When two blocks have two heights in common but the third heights differ, the block with the larger third height has larger volume, because a polyhedra can be cut off from it to get the other block.

- The blocks 133 and 113 form a regular prism of height 4 together, and block 133 has larger volume than block 113 by the previous reasoning. Hence block 133 has larger volume than block 222 and block 113 has smaller volume than block 222.

LEVEL 2   To calculate the volume of the blocks see exercices [515 - Simple Volumes](#) and [516 - Truncated Volumes](#).

For some blocks, we get the same value. According to the calculated volumes, the ascending order of the blocks is as follows:

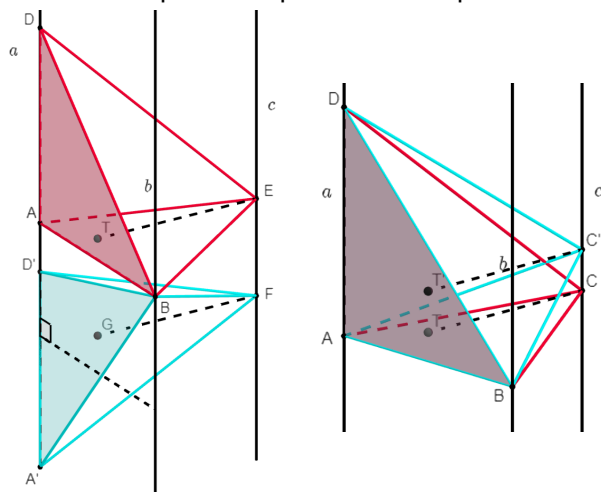| Volume	Blocks
$V = 4\sqrt{3} \approx 6.928$	111
$V = \frac{16}{3}\sqrt{3} \approx 9.234$	112
$V = \frac{20}{3}\sqrt{3} \approx 11.547$	122, 113

$V = 8\sqrt{3} \approx 13.856$	123, 132, 222
$V = \frac{28}{3}\sqrt{3} \approx 16.166$	133, 223
$V = \frac{32}{3}\sqrt{3} \approx 18.475$	233
$V = 12\sqrt{3} \approx 20.785$	333

LEVEL 3   When two blocks have two heights in common but the third heights differ by 1 unit, then a polyhedra can be cut off from the larger block to get the smaller one, see the diagram below. The polyhedra cut off is a pyramid with a triangle as a base. On the figures, the triangle ABD is considered as the base triangle, the edge AD has length 1 and the altitude of the triangle ABD is the distance of the lines a and b, which is 4 units. The height of the pyramid from the base to the apex C is the distance of the line c from the plane of the lines a and b, which equals $2\sqrt{3}$. It follows that the volume of the pyramid is $V = \frac{1}{3}Ah = \frac{1}{3} \times \frac{1 \times 4}{2} \times 2\sqrt{3} = \frac{4\sqrt{3}}{3}$.
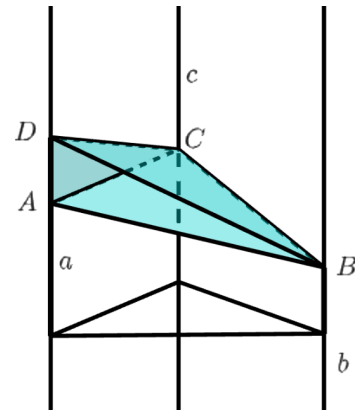
The above reasoning holds in general for every pyramid defined by the following 4 points: let a, b and c be the lines through the vertical edges of a Logifaces block and let two points chosen on one of these lines such that the distance of the points is 1, and let the two other points chosen on the two other lines. See the figure that illustrates that the altitude of the base triangle and the height of the pyramid always have the same length, regardless of the particular position of the points B and C.

The above calculations imply that increasing one height of a Logifaces block by 1 unit increases the volume by $\frac{4\sqrt{3}}{3}$, and decreasing one height by 1 unit decreases the volume by $\frac{4\sqrt{3}}{3}$. Every two blocks can be obtained from each other through such steps, hence the blocks with equal sums of heights have the same volume and the blocks with larger sums of heights have larger volumes.

In fact, the volume is always $\frac{4\sqrt{3}}{3}$ times the sum of the three heights.

PRIOR KNOWLEDGE

Formula of the volume of prisms and pyramids

RECOMMENDATIONS / COMMENTS

This task is suitable for differentiation, as each level is more difficult than the previous one.

Exercices 515 - Simple Volumes and 516 - Truncated Volumes are recommended before this exercise.