Lesson 8: Triangles with three common measurements

Goals

- Describe, compare, and contrast (orally and in writing) triangles that share three common measurements of angles or sides.
- Justify (orally and using other representations) whether triangles are identical copies or are "different" triangles.
- Recognise that examining which side lengths and angle measures are adjacent can help determine whether triangles are identical copies.

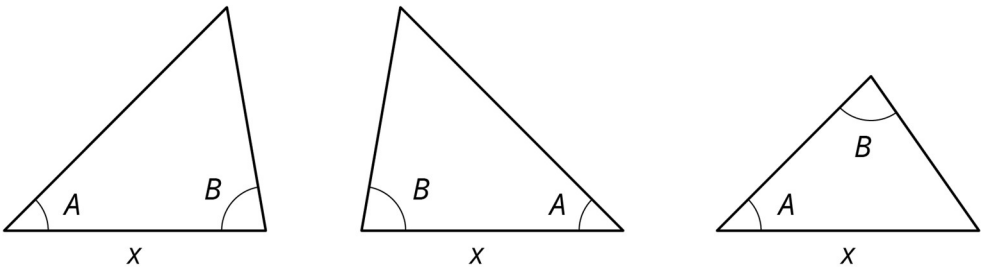
Learning Targets

- I understand that changing which sides and angles are next to each other can make different triangles.

Lesson Narrative

In this lesson, students examine sets of triangles in which all the triangles share 3 common measurements of angles or sides. Students learn to recognise when triangles are "identical copies" that are oriented differently on the page, and when they are different triangles (meaning triangles that are not identical copies). This prepares them for trying to draw more than one triangle given 3 measurements in the next lesson.

For example, suppose a triangle has angles that measure A and B and a side length that measures x. Here are 3 triangles that have these measurements:



This example shows 2 "different triangles" (triangles that are not identical copies). The first two triangles are identical copies, so they are the same, but the third is not, so it is different from the other two.

Students see that the configuration of which sides and angles are adjacent to each other can help them decide whether triangles are identical copies or different triangles (not identical copies). In the example, the first two figures have angles A and B adjacent to side x. However, in the third figure angle B is no longer adjacent to side x. Here students can see that a good way to try to make a different triangle with the same 3 measurements is to change which sides and angles are adjacent.

Students do not need to memorise how many different kinds of triangles are possible given different combinations of angles and sides, and they do not need to know criteria such as angle-side-angle for determining if two triangles are identical copies.

Addressing

- Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measurements of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Instructional Routines

- Collect and Display
- Discussion Supports
- Think Pair Share

Required Materials

Geometry toolkits
tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Student Learning Goals

Let's contrast triangles.

8.1 3 Sides; 3 Angles

Warm Up: 10 minutes

The purpose of this warm-up is to begin looking at the different triangles that can be drawn when three measurements are specified. The first set of triangles in this activity all share the same 3 side lengths. The second set of triangles all share the same 3 angle sizes. Later in this lesson, students will look at sets of triangles that share some combination of side lengths and angles.

Launch

Provide access to geometry toolkits. Give students 1 minute of quiet think time, followed by a whole-class discussion.

Anticipated Misconceptions

Some students may say that all the triangles in the second set are "the same shape." This statement can result from two very different misconceptions. Listen to the students' reasoning and explain as needed:

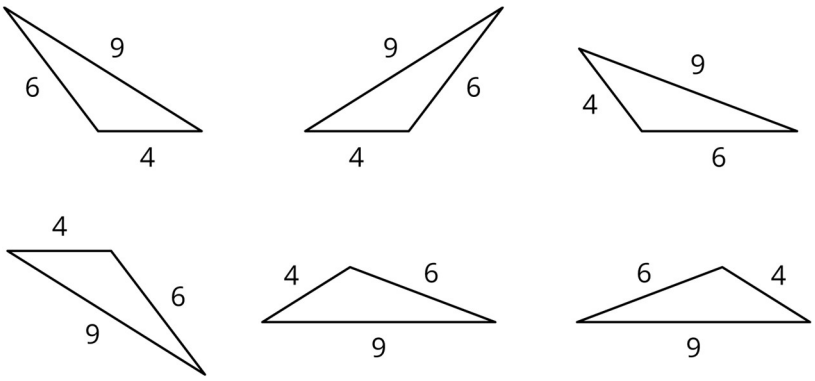
1. Just because they are all in the same category "triangles" doesn't mean they are all the same shape. If we can take two shapes and position one exactly on top of the other, so all the sides and corners line up, then they are identical copies.

2. These triangles are scaled copies of each other, but that does not make them "the same" because their side lengths are still different. Only scaled copies made using a scale factor of 1 are identical copies.

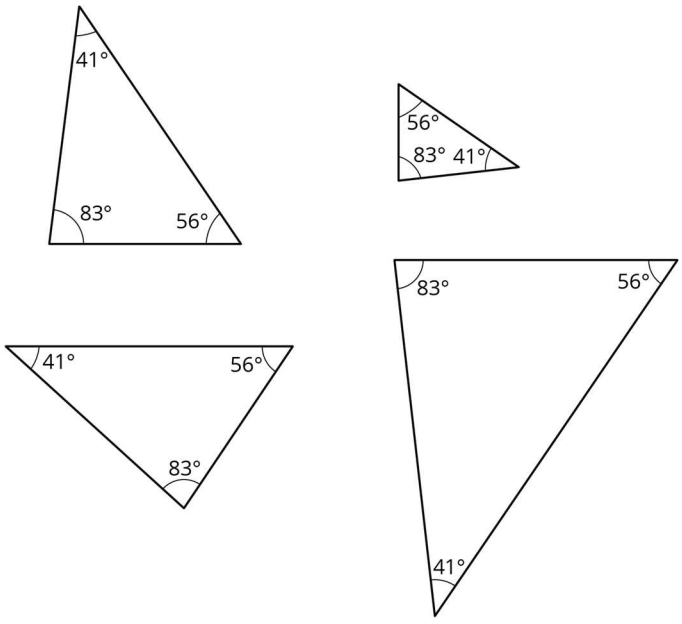
Student Task Statement

Examine each set of triangles. What do you notice? What is the same about the triangles in the set? What is different?

Set 1:



Set 2:



Student Response

1. All of the side lengths and angles are the same size. These triangles are identical copies. The triangles face different directions.

2. These triangles all have the same angles, but different side lengths. They could be scaled copies that are oriented differently.

Activity Synthesis

Invite students to share things they notice, things that are the same and things that are different about the triangles. Record and display these ideas for all to see.

If these discussion points do not come up in students' explanations make them explicit:

In the first set:

- All the triangles are identical copies, just in different orientations.
- They have the same 3 side lengths.
- They have the same 3 angle sizes (can be checked with tracing paper or a protractor).

In the second set:

- The triangles are not identical copies.
  - Note: Students may recognise that these triangles are scaled copies of each other, since they have the same angle sizes. However, this is the first time students have seen scaled copies in different orientations, and it is not essential to this lesson that students recognise that these triangles are scaled copies.
- They have the same 3 angle sizes.
- They have different side lengths (can be checked with tracing paper or a ruler).

The goal is to make sure students understand that the second set has 3 different triangles (because they are different sizes) and the first set really only shows 1 triangle in many different orientations. Tracing paper may be helpful to convince students of this.

8.2 2 Sides and 1 Angle

15 minutes

In this activity, students examine different orientations of triangles that all share 2 sides lengths and one angle size. They recognise that some of these triangles are identical copies and others are different triangles (not identical copies).

In the coming lessons, students are asked to draw their own triangles. On their own, students often have trouble thinking about triangles where the three given conditions are not included adjacent to one another. For example, when given two sides and an angle, many students will immediately think of putting the given angle between the two sides, but struggle with visualising putting the angle anywhere else. This task is important for helping students view this as a viable option.

Instructional Routines

- Collect and Display
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2–3 minutes of quiet work time followed by time to discuss their explanations with a partner. Follow with a whole-class discussion. Provide access to geometry toolkits.

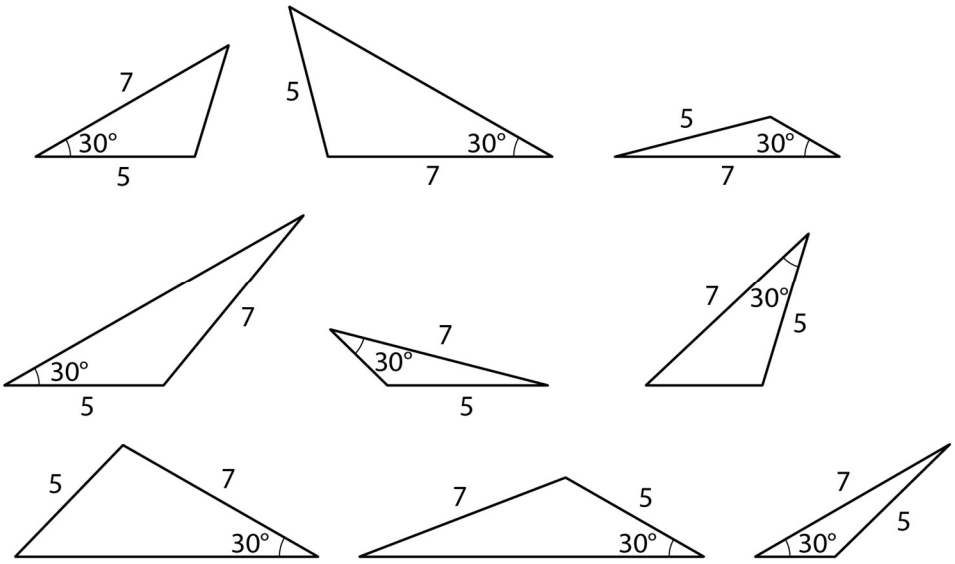
Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide students with a printed copy of the triangles for them to cut out and rearrange to determine the number of different triangles. *Supports accessibility for: Conceptual processing Conversing: Collect and Display.* As students discuss their explanations with a partner, listen for and collect vocabulary, gestures, and diagrams students use to identify and describe the similarities and differences between them. Capture student language that reflects a variety of ways to describe the differences between triangles and the relative position of sides and angles. Write the students' words on a visual display and update it throughout the remainder of the lesson. Remind students to borrow language from the display as needed. This will help students read and use mathematical language during their paired and whole-class discussions.
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

Anticipated Misconceptions

Some students may say that there are 9 different triangles, because they do not recognise that some of them are identical copies oriented differently. Prompt them to use tracing paper to compare the triangles.

Student Task Statement

Examine this set of triangles.



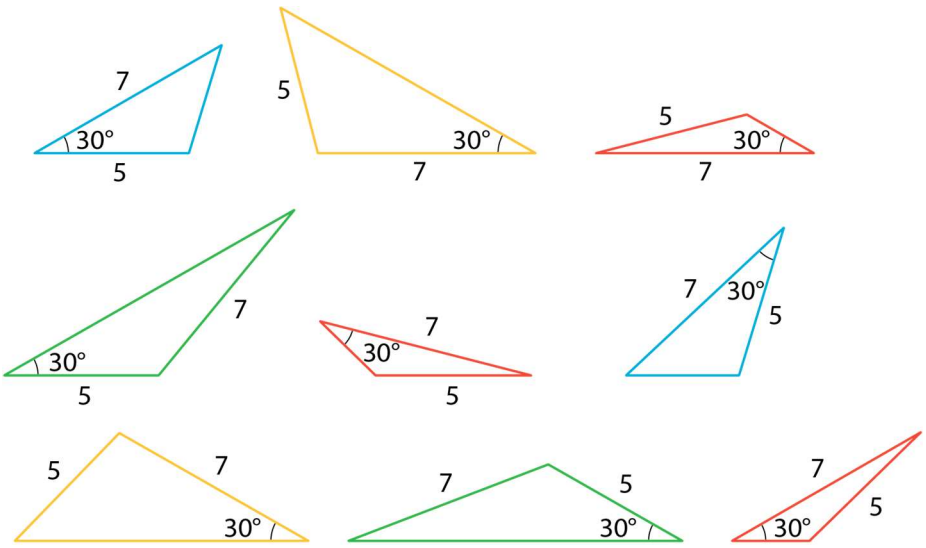
1. What is the same about the triangles in the set? What is different?

2. How many different triangles are there? Explain or show your reasoning.

Student Response

1. All of the triangles have two sides that are the same length and one angle that is the same size. The triangles are oriented differently and the two sides and one angle are in a different order.

2. There are 4 different triangles.



Sample Explanation:

- The triangles marked in blue have the common measurements in the order 7 cm, 30°, 5 cm.
- The triangles marked in green have the common measurements in the order 30°, 5 cm, 7 cm.
- The triangles marked in both yellow and red have the common measurements in the order 30°, 7 cm, 5 cm, but the yellow triangles are larger and the red triangles are smaller.

Activity Synthesis

Select students to share the similarities and differences between the triangles in the set.

Trace a few of the triangles from the set and show how you can turn, flip, or move some of them to line up while others cannot be lined up. Ask students what this means about all the triangles in the set (they are not all identical to each other). Explain that, "While there are certainly times when the position of a triangle is important ('I wouldn't want my roof upside down!'), for this unit in geometry, we will consider shapes the same if they are identical copies."

To highlight the differences among the triangles, ask students:

- "Is there only one possible triangle that could be created from the given conditions?" (No, there were 4.)
- "How would you explain what is different about these four triangles?" (Some have the 30° angle between the two sides of known length and others have the 30° angle next to the side of unknown length.)

Explain to students that it seems the order in which the conditions are included in the triangle (for example, is the angle between the two sides or not?) matters in creating different triangles. Emphasise that the three required pieces (2 sides and 1 angle) do not have to all be put next to one another. When they are asked to draw triangles with three or more conditions, they should consider the way in which the conditions are arranged in their drawing. For example, think about whether the given angle must go between the two sides or not.

8.3 2 Angles and 1 Side

10 minutes

This activity is similar to what students did in the previous activity; however, here the conditions given are 2 angles and 1 side.

Instructional Routines

- Discussion Supports

Launch

Keep students in the same groups. Tell students that this activity is similar to the previous one, and they should pay close attention to what they find different here. Provide access to geometry toolkits. Give students 2–3 minutes of quiet work time followed by time to discuss their explanations with a partner. Follow with a whole-class discussion.

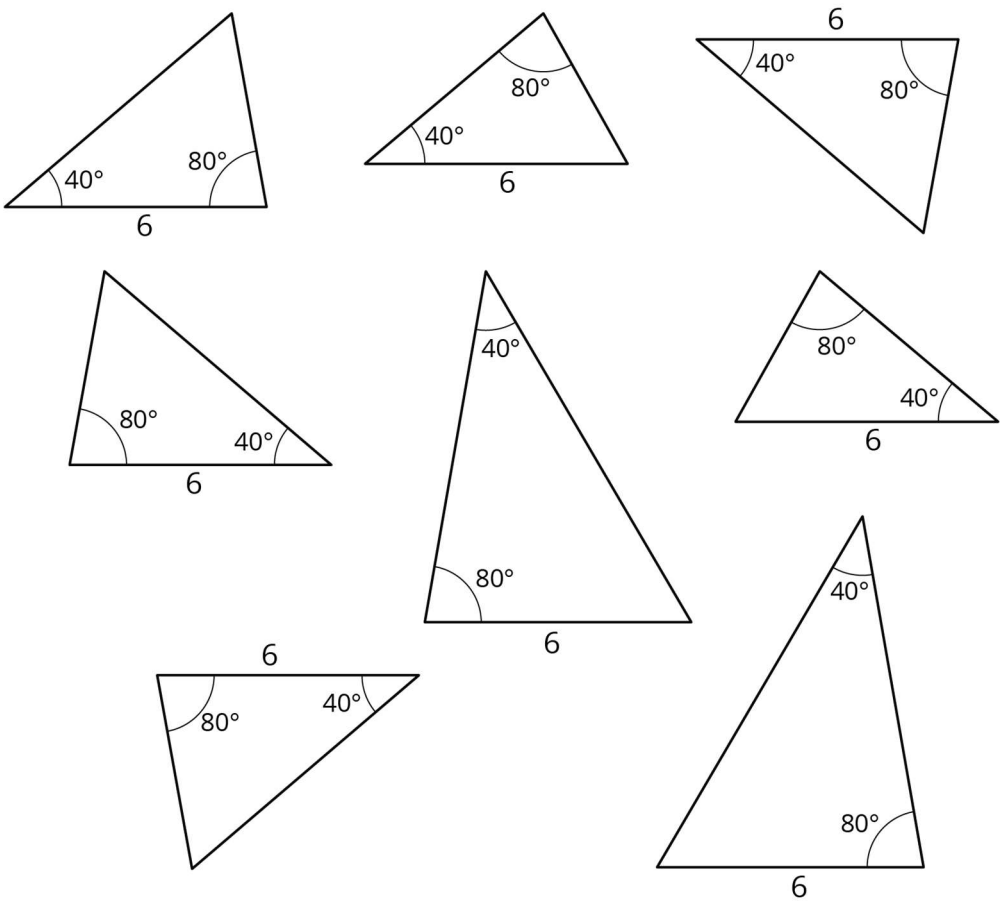
Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "Both _____ and _____ are alike because . . .", " _____ and _____ are different because . ." *Supports accessibility for: Language; Social-emotional skills*

Anticipated Misconceptions

Some students may say there are only 2 different triangles in this set, because they do not notice the slight size difference between the smaller two groups of triangles. Prompt them to look at where the 80° angle is located in comparison to the 6 cm side.

Student Task Statement

Examine this set of triangles.



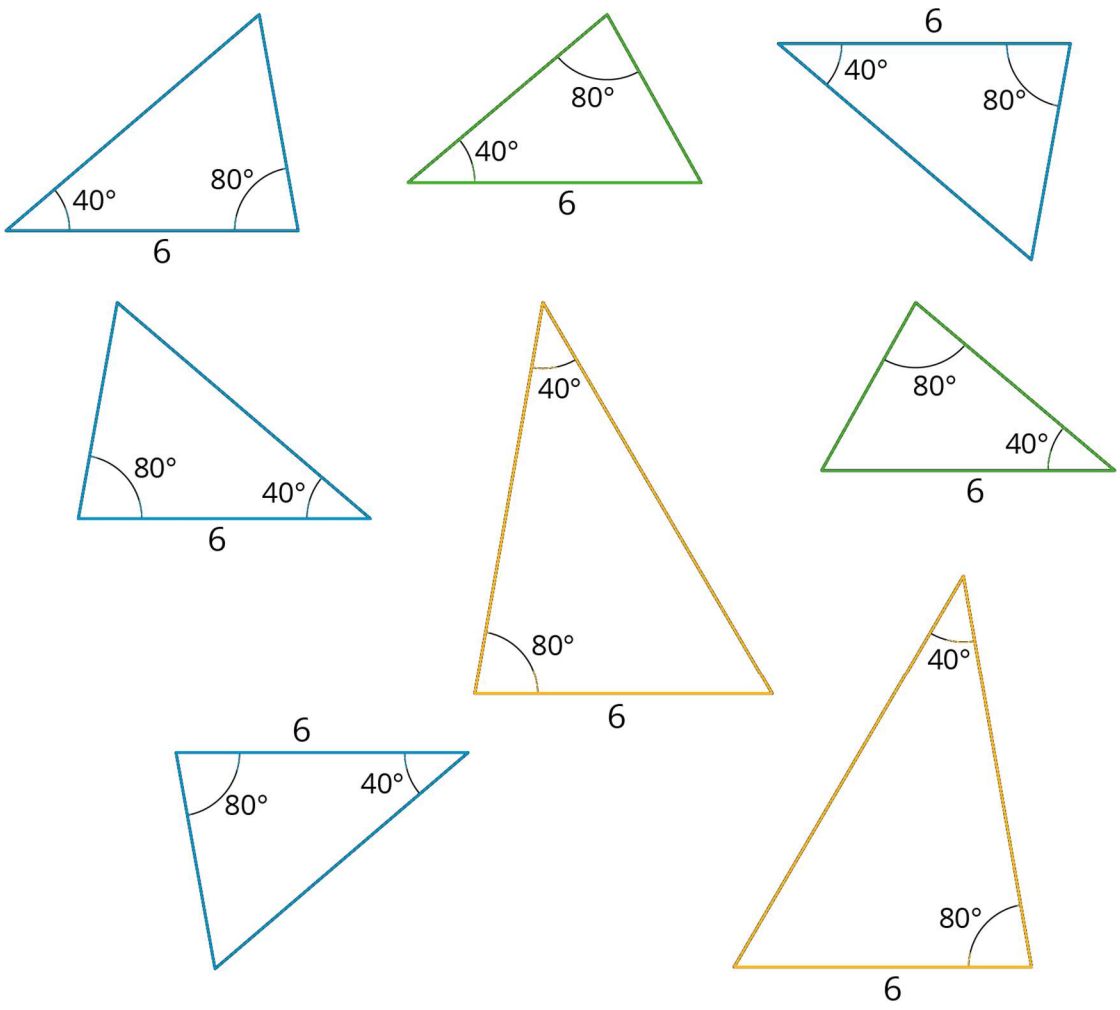
1. What is the same about the triangles in the set? What is different?

2. How many different triangles are there? Explain or show your reasoning.

Student Response

1. All of the triangles have one side that is the same length and two angles that have the same size. The triangles are oriented differently and the one side and two angles are in a different order.

2. There are 3 different triangles.



Sample explanation:

- The triangles marked in blue have the common measurements in the order 40°, 6 cm, 80°.
- The triangles marked in yellow have the common measurements in the order 40°, 80°, 6 cm.
- The triangles marked in green have the common measurements in the order 80°, 40°, 6 cm.

Activity Synthesis

Ask a few students to share how many different triangles they think are in this set. Select students to share the similarities and differences between triangles in the set. If necessary, trace a few triangles from the set and show how you can turn, flip, or move some of them to line up while others cannot be lined up. Ask students what this means about all the triangles in the set (they are not all identical to each other).

To highlight the differences among the triangles ask students:

- "What differences do you see between the triangles in this activity and the triangles in the previous activity?" (The given conditions here were 2 angles and 1 side, the previous activity was 2 sides and 1 angle.)
- "What similarities do you see between the triangles in this activity and the triangles in the previous activity?" (These triangles have all the same conditions but in a different order, and they made different triangles as was seen in the previous activity.)

If time permits, consider asking students to use a protractor to measure the unlabelled angle from each of the three different triangles. Discuss what they notice about the third angle. (It's the same size in every triangle.)

Explain to students that here we see another example of different triangles that can be made using the same conditions (2 angles and 1 side) in different orders (side between the two angles, side next to the 40 degree angle and side next to the 80 degree angle). Tell them that in upcoming lessons we will continue to investigate what they noticed here with the addition of drawing the different triangles.

Speaking: Discussion Supports. Use this routine to amplify mathematical uses of language to communicate about similar triangles. As students share the similarities and differences they noticed about the triangles, invite other students to press for details, challenge an idea, elaborate on an idea, or give an example of their own process. Revoice student ideas to demonstrate mathematical language when necessary. This will help students produce and make sense of the language needed to communicate their own ideas.
Design Principle(s): Optimise output (for explanation)

Lesson Synthesis

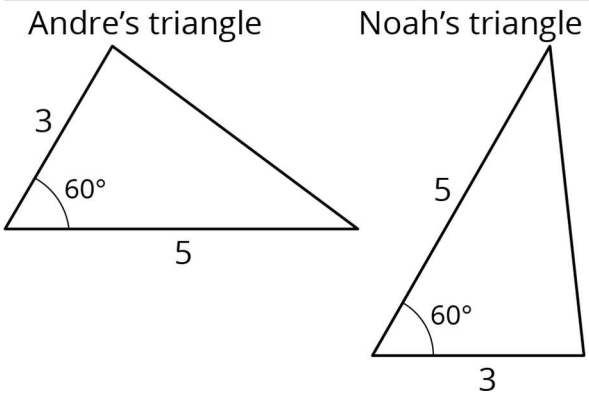
- For what we have done today, what does it mean for two triangles to be "different?" (They are not identical copies.)
- If you have a drawing of two triangles, how can you tell if they are identical copies? (If I trace one triangle and can move the tracing to perfectly line up with the other, then they are identical copies.)
- When trying to draw different triangles with the same set of conditions, what are some things to try? (Change the order of the conditions in the triangle.)

8.4 Comparing Andre and Noah's Triangles

Cool Down: 5 minutes

Student Task Statement

Andre and Noah each drew a triangle with side lengths of 5 cm and 3 cm and an angle that measures 60°, and then they showed each other their drawings.



1. Did Andre and Noah draw different triangles? Explain your reasoning.

2. Explain what Andre and Noah would have to do to draw another triangle that is different from what either of them has already drawn.

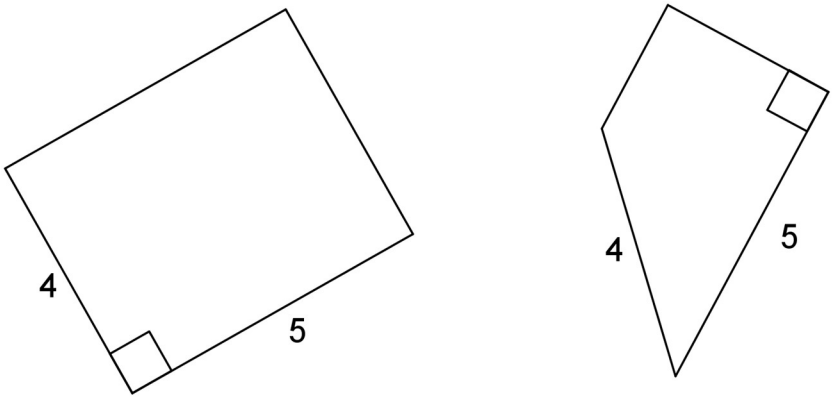
Student Response

1. These are both the same triangle. In both cases, the 60° angle is between the 3 cm and 5 cm sides. If you trace one triangle, flip it and turn it, it can line up exactly with the other triangle.

2. To draw a different triangle, they should try putting the 60° angle next to the side of unknown length, instead of between the two known sides.

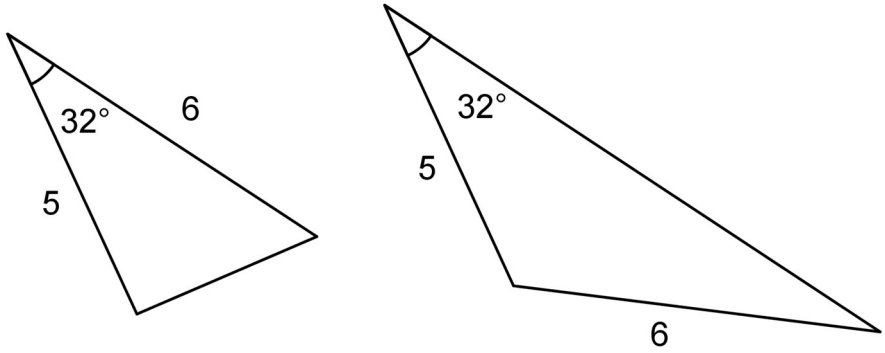
Student Lesson Summary

Both of these quadrilaterals have a right angle and side lengths 4 and 5:



However, in one case, the right angle is *between* the two given side lengths; in the other, it is not.

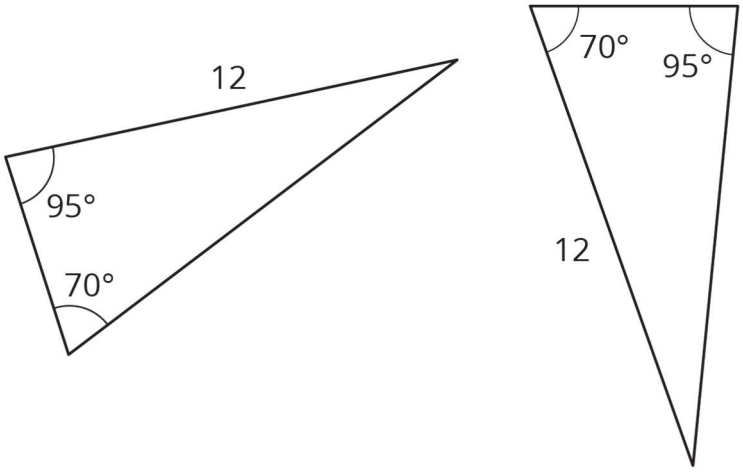
If we create two triangles with three equal measurements, but these measurements are not next to each other in the same order, that usually means the triangles are different. Here is an example:



Lesson 8 Practice Problems

1. **Problem 1 Statement**

   Are these two triangles identical? Explain how you know.

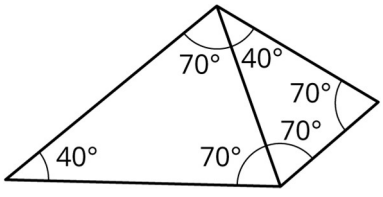


   Solution

   No, these two triangles are not identical. They have two of the same angle sizes and one side length is the same, but the sides and angles are arranged differently in each triangle. In the triangle on the left, the side marked 12 is adjacent to the 95° angle. In the triangle on the right, the side marked 12 is adjacent to the 70° angle.

2. **Problem 2 Statement**

   Are these triangles identical? Explain your reasoning.



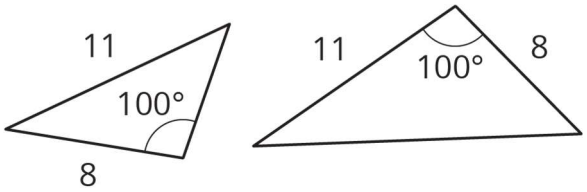
Solution

No, they are not identical. Although they have the same angle measurements, two of the side lengths are different.

3. **Problem 3 Statement**

Tyler claims that if two triangles each have a side length of 11 units and a side length of 8 units, and also an angle measuring 100°, they must be identical to each other. Do you agree? Explain your reasoning.

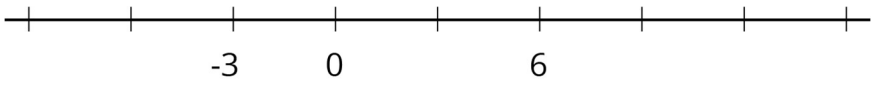
Solution



No, it is possible to build two different triangles with these measurements.

4. **Problem 4 Statement**

The markings on the number line are equally spaced. Label the other markings on the number line.



Solution

-9, -6, -3, 0, 3, 6, 9, 12, 15

5. **Problem 5 Statement**

A passenger on a ship dropped his camera into the ocean. If it is descending at a rate of -4.2 metres per second, how long until it hits the bottom of the ocean, which is at -1 875 metres?

Solution

It will take about 446 seconds, which is about 7 and a half minutes.

6. **Problem 6 Statement**

Apples cost £1.99 per pound.

a. How much do $3\frac{1}{4}$ pounds of apples cost?

b. How much do x pounds of apples cost?

c. Clare spent £5.17 on apples. How many pounds of apples did Clare buy?

Solution

a. £6.47 (this number is rounded to the nearest cent)

b. $1.99x$

c. About 2.6 pounds. $1.99x = 5.17$, so $x \approx 2.598$. Most grocery store scales round to the nearest tenth.

7. **Problem 7 Statement**

Diego has a glue stick with a diameter of 0.7 inches. He sets it down 3.5 inches away from the edge of the table, but it rolls onto the floor. How many rotations did the glue stick make before it fell off of the table?

Solution

3.5 ÷ 2.2 rotations (about 1.6)