Lesson 5: Coordinate moves

Goals

- Draw and label a diagram of a line segment rotated 90 degrees clockwise or anti-clockwise about a given centre.
- Generalise (orally and in writing) the process to reflect any point in the coordinate grid.
- Identify (orally and in writing) coordinates that represent a transformation of one shape to another.

Learning Targets

- I can apply transformations to points on a grid if I know their coordinates.

Lesson Narrative

Students continue to investigate the effects of transformations. The new feature of this lesson is the coordinate grid. In this lesson, students use coordinates to describe shapes and their images under transformations in the coordinate grid. Reflections over the x-axis and y-axis have a very nice structure captured by coordinates. When we reflect a point like (2,5) over the x-axis, the distance from the x-axis stays the same but instead of lying 5 units *above* the x-axis the image lies 5 units *below* the x-axis. That means the image of (2,5) when reflected over the x-axis is (2,-5). Similarly, when reflected over the y-axis, (2,5) goes to (-2,5), the point 2 units to the left of the y-axis.

Building On

- Verify experimentally the properties of rotations, reflections, and translations:

Addressing

- Describe the effect of enlargements, translations, rotations, and reflections on two-dimensional shapes using coordinates.

Building Towards

- Describe the effect of enlargements, translations, rotations, and reflections on two-dimensional shapes using coordinates.

Instructional Routines

- Compare and Connect
- Discussion Supports

Required Materials

Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Student Learning Goals

Let's transform some shapes and see what happens to the coordinates of points.

5.1 Translating Coordinates

Warm Up: 5 minutes

The purpose of this warm-up is to remind students how the coordinate grid works and to give them an opportunity to see how one might describe a translation when the shape is plotted on the coordinate grid.

There are many ways to express a translation because a translation is determined by two points P and Q once we know that P is translated to Q. There are many pairs of points that express the *same* translation. This is different from reflections which are determined by a unique line and rotations which have a unique centre and a specific angle of rotation.

Launch

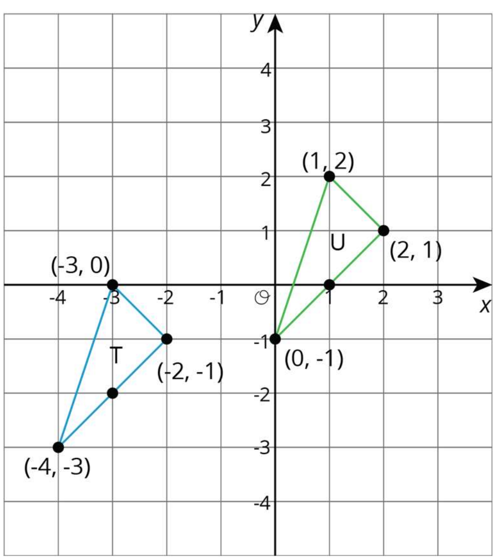
Ask students how they describe a translation. Is there more than one way to describe the same translation? After they have thought about this for a minute, give them 2 minutes of quiet work time followed by a whole-class discussion.

Anticipated Misconceptions

Students may think that they need more information to determine the translation. Remind them that specifying one point tells you the distance and direction all of the other points move in a translation.

Student Task Statement

Select all of the translations that take triangle T to triangle U. There may be more than one correct answer.



1. Translate (-3,0) to (1,2).

2. Translate (2,1) to (-2,-1).

3. Translate (-4,-3) to (0,-1).

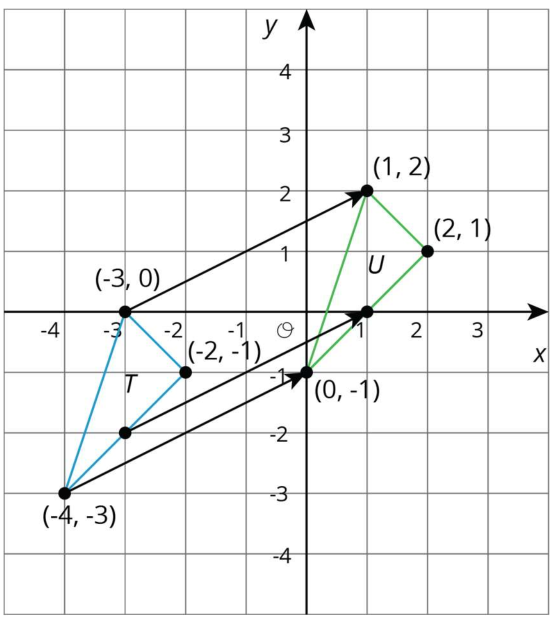
4. Translate (1,2) to (2,1).

Student Response

These are both correct: (-3,0) to (1,2) and (-4,-3) to (0,-1)

Activity Synthesis

Remind students that once you name a starting point and an ending point, that completely determines a translation because it specifies a distance and direction for *all* points in the grid. Appealing to their experiences with tracing paper may help. In this case, we might describe that distance and direction by saying "all points go up 2 units and to the right 4 units." Draw the arrow for the two correct descriptions and a third one not in the list, like this:



Point out that each arrow does, in fact, go up 2 and 4 to the right.

5.2 Reflecting Points on the Coordinate Grid

15 minutes (there is a digital version of this activity)

While the warm-up focuses on studying translations using a coordinate grid, the goal of this activity is for students to work through multiple examples of specific points reflected over the x-axis and then generalise to describe where a reflection takes *any* point. They also consider reflections over the y-axis with slightly less scaffolding. In the next activity, students will study 90 degree rotations on a coordinate grid, rounding out this preliminary investigation of how transformations work on the coordinate grid.

Watch for students who identify early the pattern for how reflections over the x-axis or y-axis influence the coordinates of a point. Make sure that they focus on explaining *why* the pattern holds as the goal here is to understand reflections better using the coordinate grid. The rule is less important than understanding how it is essential to see the coordinate grid and state the rule.

Instructional Routines

- Compare and Connect

Launch

Tell students that they will have 5 minutes of quiet think time to work on the activity, and tell them to pause after the second question.

Select 2–3 students to share their strategies for the first 2 questions. You may wish to start with students who are measuring distances of points from the x-axis or counting the number of squares a point is from the x-axis and then counting out the same amount to find the reflected point. These strategies work, but overlook the structure of the coordinate grid. To help point out the role of the coordinate grid, select a student who noticed the pattern of changing the sign of the y-coordinate when reflecting over the x-axis.

After this initial discussion, give 2–3 minutes of quiet work time for the remaining questions, which ask them to generalise how to reflect a point over the y-axis.

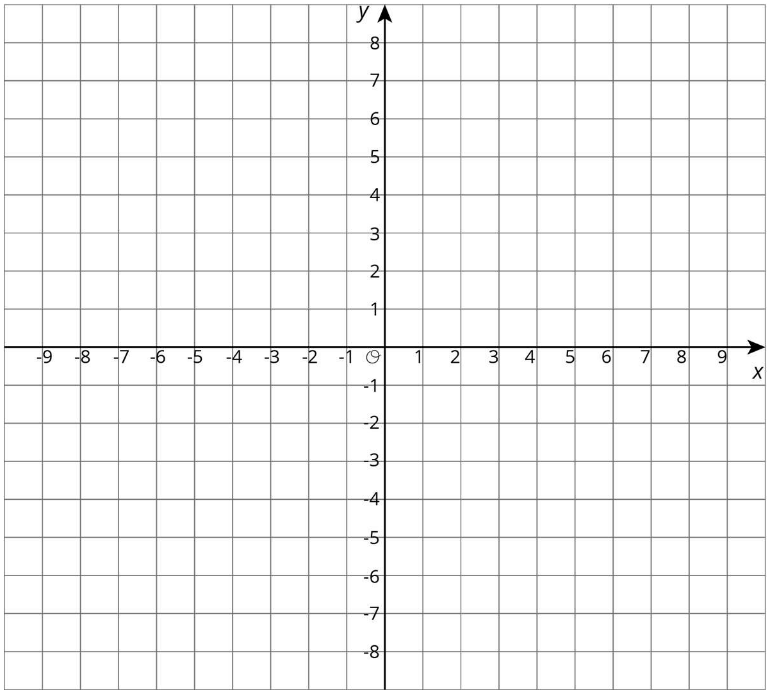
Classes using the digital version have an applet for graphing and labelling points.

Representation: Internalise Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. For example, to get students started, provide a smaller bank of points and only the first two instructions. Once students have successfully completed the four steps for each, present the remaining questions, one at a time.
Supports accessibility for: Conceptual processing; Organisation Speaking: Compare and Connect. Use this routine when students present their strategies for reflecting points using the x-axis as the line of reflection before continuing on. Ask students to consider what is the same and what is different about the strategies. Draw students' attention to the different ways students reasoned to find the reflected coordinates. These exchanges strengthen students' mathematical language use and reasoning of reflections along the x-axis and y-axis.
Design Principle(s): Maximise meta-awareness

Anticipated Misconceptions

If any students struggle getting started because they are confused about where to plot the points, refer them back to the warm-up activity and practise plotting a few example points with them.

Student Task Statement



1. Here is a list of points
   $A = (0.5,4)$ $B = (-4,5)$ $C = (7,-2)$ $D = (6,0)$ $E = (0,-3)$

On the **coordinate grid**:

   a. Plot each point and label each with its coordinates.

   b. Using the x-axis as the line of reflection, plot the image of each point.

   c. Label the image of each point with its coordinates.

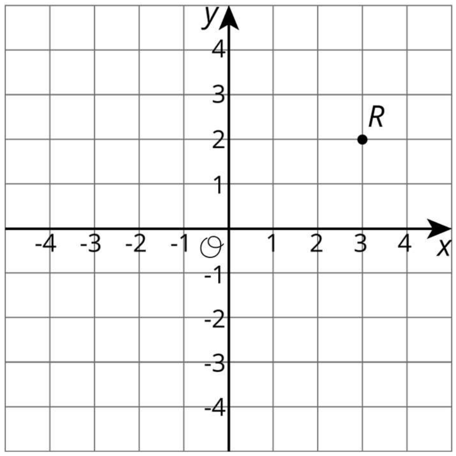
   d. Include a label using a letter. For example, the image of point A should be labelled A'.

2. If the point (13,10) were reflected using the x-axis as the line of reflection, what would be the coordinates of the image? What about (13,-20)? (13,570)? Explain how you know.

3. The point R has coordinates (3,2).

   a. Without graphing, predict the coordinates of the image of point R if point R were reflected using the y-axis as the line of reflection.

   b. Check your answer by finding the image of R on the graph.



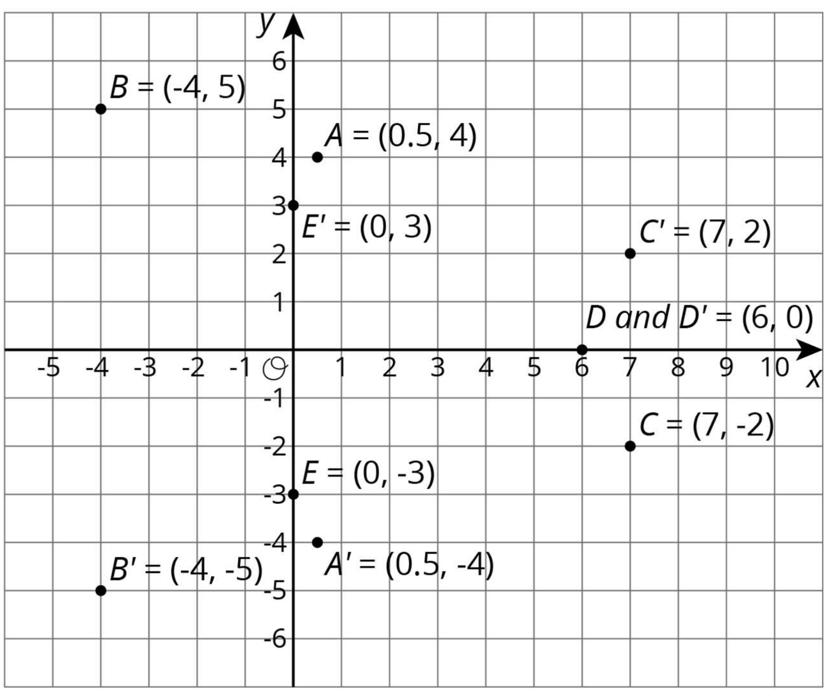
c. Label the image of point R as R'.

d. What are the coordinates of R'?

4. Suppose you reflect a point using the y-axis as line of reflection. How would you describe its image?

Student Response

1. The picture shows the points A, B, C, D, E and also their reflections over the x-axis: $A' =$ (0.5,-4), $B' =$ (-4,-5), $C' =$ (7,2), $D' =$ (6,0), $E' =$ (0,3).



2. Using the x-axis as line of reflection, the reflection of (13,10) is (13,-10), the reflection of (13,-20) is (13,20) and the reflection of (13,570) is (13,-570). Using the x-axis as line of reflection does not move points horizontally but it does move points which are

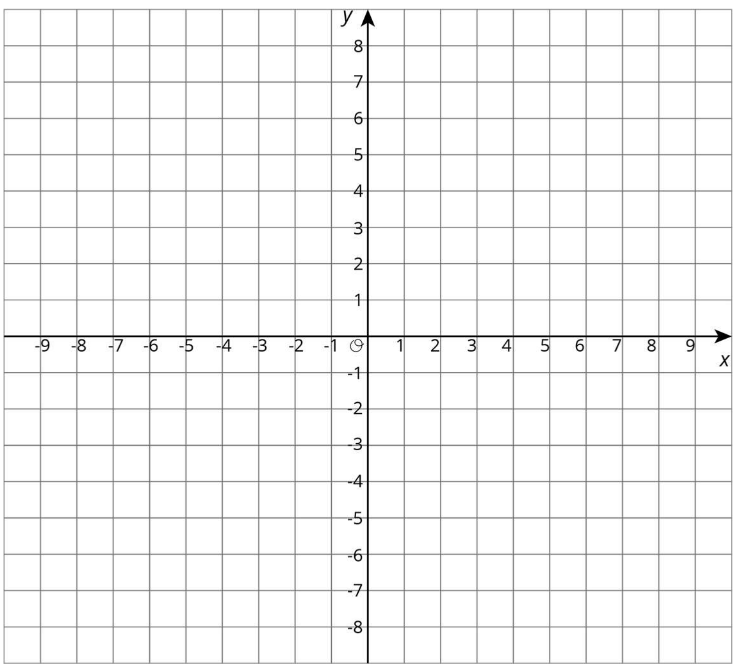
not on the x-axis vertically. In coordinates, the x-coordinate of the point stays the same while the y-coordinate changes sign.

3. Using the y-axis as line of reflection does not move points vertically but it does move points that are not on the y-axis horizontally. In coordinates, the y-coordinate of the point stays the same while the x-coordinate changes sign. The point R has coordinates (3,2). When I reflect it over the y-axis it will go to (-3,2): the x-coordinate changes sign but the y-coordinate remains the same.

4. The point will have the same y-coordinate but the x-coordinate will change signs. The distance from the y-axis does not change and the y-coordinate does not change.

Activity Synthesis

To facilitate discussion, display a blank coordinate grid.



Questions for discussion:

- "When you have a point and an axis of reflection, how do you find the reflection of the point?"
- "How can you use the coordinates of a point to help find the reflection?"
- "Are some points easier to reflect than others? Why?"
- "What patterns have you seen in these reflections of points on the coordinate grid?"

The goal of the activity is *not* to create a rule that students memorise. The goal is for students to notice the pattern of reflecting over an axis changing the sign of the coordinate (without having to graph). The coordinate grid can sometimes be a powerful tool for understanding and expressing structure and this is true for reflections over both the x-axis and y-axis.

5.3 Transformations of a Line Segment

15 minutes (there is a digital version of this activity)

This activity concludes looking at how the different basic transformations (translations, rotations, and reflections) behave when applied to points on a coordinate grid. In general, it is difficult to use coordinates to describe rotations. But when the centre of the rotation is (0,0) and the rotation is 90 degrees (clockwise or anti-clockwise), there is a straightforward description of rotations using coordinates.

Unlike translations and reflections over the x or y axis, it is more difficult to visualise where a 90 degree rotation takes a point. Tracing paper is a helpful tool, as is an index card.

Instructional Routines

- Discussion Supports

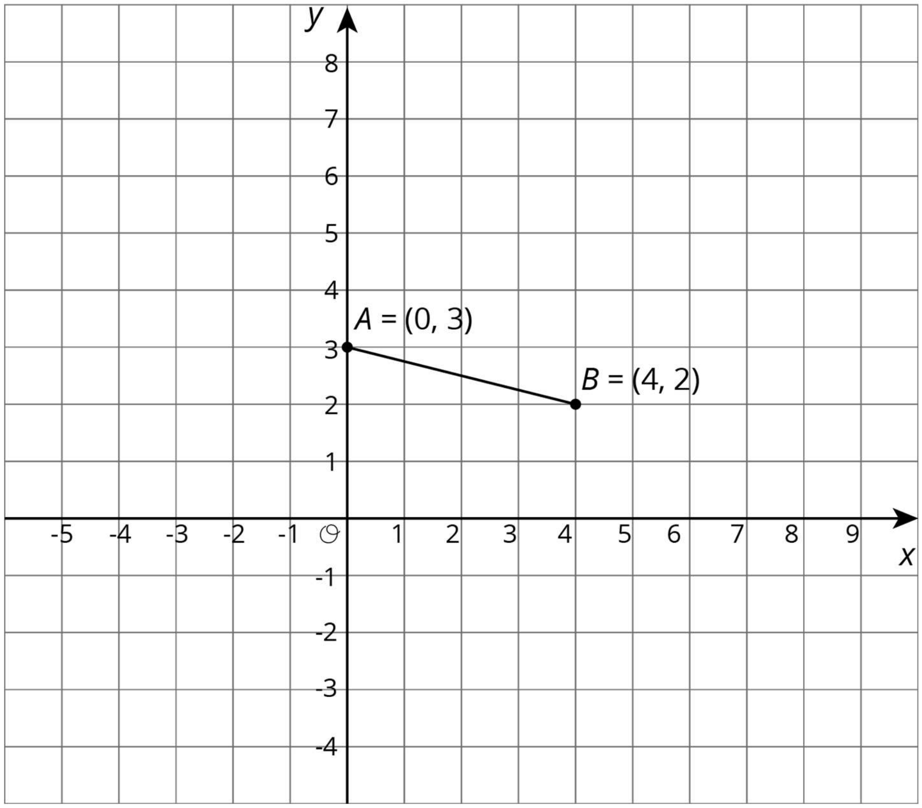
Launch

Demonstrate how to use tracing paper in order to perform a 90 degree rotation. It is helpful to put a small set of perpendicular axes (a + sign) on the piece of tracing paper and place their intersection point at the centre of rotation. One of the small axes can be lined up with the line segment being rotated and then the rotation is complete when the other small axis lines up with the line segment.

An alternative method to perform rotations would be with the corner of an index card, which is part of the geometry toolkit.

Students using the digital version will see the line segment being rotated by the computer as they manipulate the sliders.

Student Task Statement



Apply each of the following transformations to line segment AB.

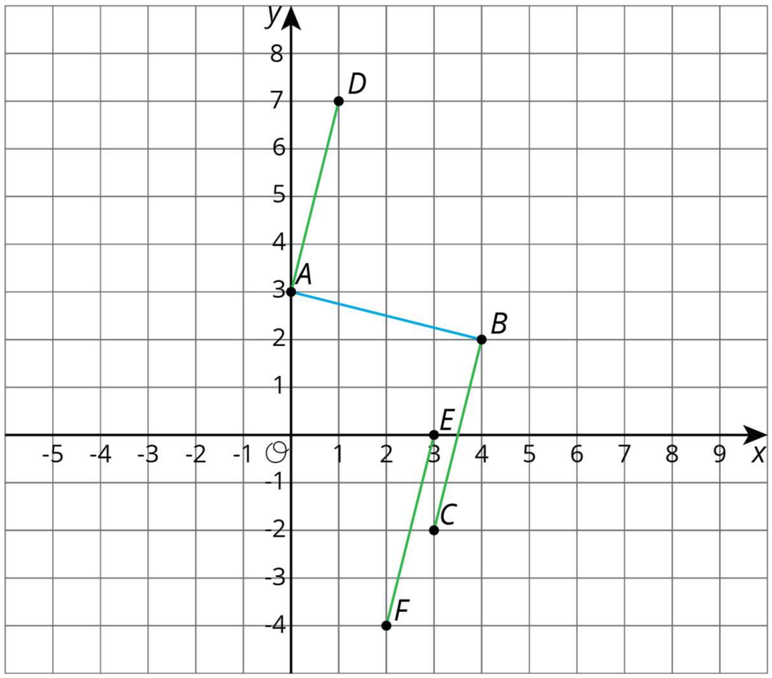
1. Rotate line segment AB 90 degrees anti-clockwise around centre B. Label the image of A as C. What are the coordinates of C?

2. Rotate line segment AB 90 degrees anti-clockwise around centre A. Label the image of B as D. What are the coordinates of D?

3. Rotate line segment AB 90 degrees clockwise around $(0,0)$. Label the image of A as E and the image of B as F. What are the coordinates of E and F?

4. Compare the two 90-degree anti-clockwise rotations of line segment AB. What is the same about the images of these rotations? What is different?

Student Response



1. C = (3,-2)

2. D = (1,7)

3. E = (3,0), F = (2,-4)

4. Answers vary. Sample response. The two anti-clockwise rotations of AB are in different locations. The points A and B move different distances with the different rotations. One rotation can be mapped to the other by a translation.

Are You Ready for More?

Suppose EF and GH are line segments of the same length. Describe a sequence of transformations that moves EF to GH.

Student Response

Answers vary. For example, translate EF so that E lands on G ,and then rotate EF with centre G until (the image of) F lands on H.

Activity Synthesis

Ask students to describe or demonstrate how they found the rotations of line segment AB. Make sure to highlight these strategies:

- Using tracing paper to enact a rotation through a 90 degree angle.

- Using an index card: Place the corner of the card at the centre of rotation, align one side with the point to be rotated, and find the location of the rotated point along an adjacent side of the card. (Each point's distance from the corner needs to be equal.)
- Using the structure of the coordinate grid: All grid lines are perpendicular, so a 90 degree rotation with centre at the intersection of two grid lines will take horizontal grid lines to vertical grid lines and vertical grid lines to horizontal grid lines.

The third strategy should only be highlighted if students notice or use this in order to execute the rotation, with or without tracing paper. This last method is the most accurate because it does not require any technology in order to execute, relying instead on the structure of the coordinate grid.

If some students notice that the three rotations of line segment AB are all parallel, this should also be highlighted.

Engagement: Develop Effort and Persistence. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class. *Supports accessibility for: Language; Social-emotional skills; Attention Speaking: Discussion Supports.* To support students in explaining the similarities and differences of the line segment rotations for the last question, provide sentence frames for students to use when they are comparing line segments, points, and rotations. For example, "____ is similar to _____ because _____" or "_____ is different than _____ because ______." Revoice student ideas using mathematical language use as needed.
Design Principle(s): Support sense-making; Optimise output for (comparison)

Lesson Synthesis

By this point, students should start to feel confident applying translations, reflections over either axis, and rotations of 90 degrees clockwise or anti-clockwise to a point or shape in the coordinate grid.

To highlight working on the coordinate grid when doing transformations, ask:

- "What are some advantages to knowing the coordinates of points when you are doing transformations?"
- "What changes did we see when reflecting points over the x-axis? y-axis?"
- "How do you perform a 90 degree clockwise rotation of a point with centre (0,0)?"

Time permitting, ask students to apply a few transformations to a point. For example, where does (1,2) go when

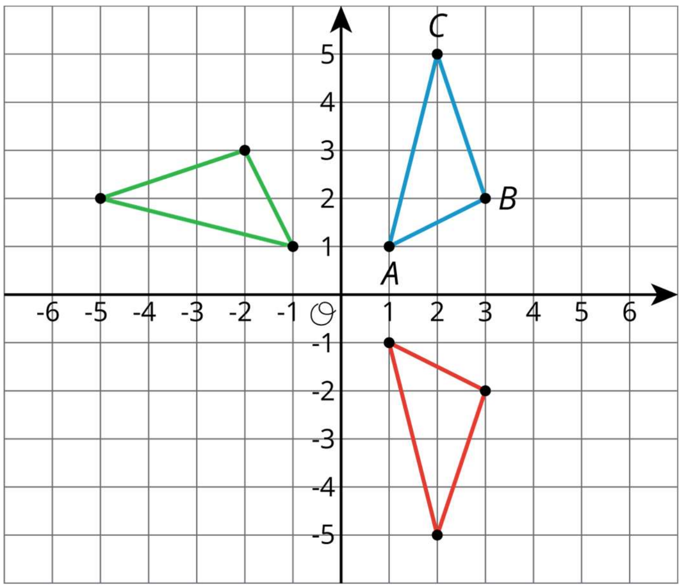
- reflected over the x-axis? (1,-2)
- reflected over the y-axis? (-1,2)
- rotated 90 degrees clockwise with centre (0,0)? (2,-1)

5.4 Rotation or Reflection

Cool Down: 5 minutes

Student Task Statement

One of the triangles pictured is a rotation of triangle ABC and one of them is a reflection.



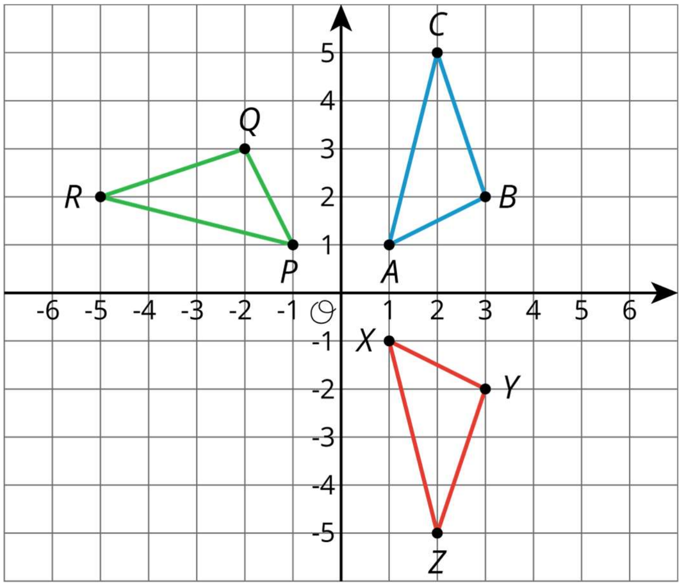
1. Identify the centre of rotation, and label the rotated image PQR.

2. Identify the line of reflection, and label the reflected image XYZ.

Student Response

1. The centre of the rotation taking $\triangle ABC$ to $\triangle PQR$ is (0,0), and the rotation is 90 degrees in a anti-clockwise direction.

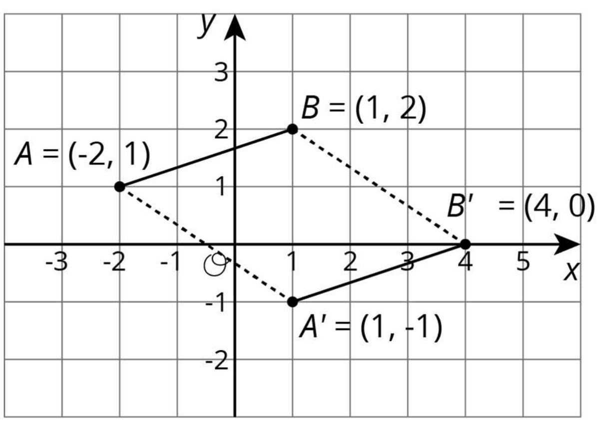
2. A reflection over the x-axis takes $\triangle ABC$ to $\triangle XYZ$.



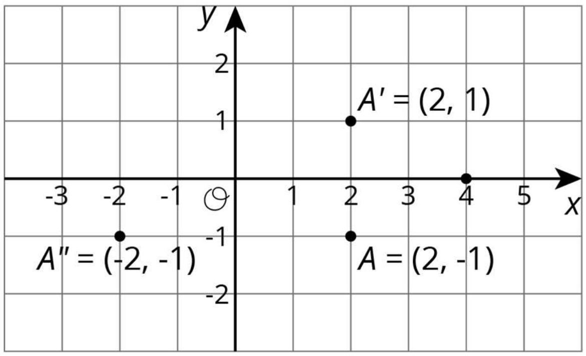
Student Lesson Summary

We can use coordinates to describe points and find patterns in the coordinates of transformed points.

We can describe a translation by expressing it as a sequence of horizontal and vertical translations. For example, line segment AB is translated right 3 and down 2.

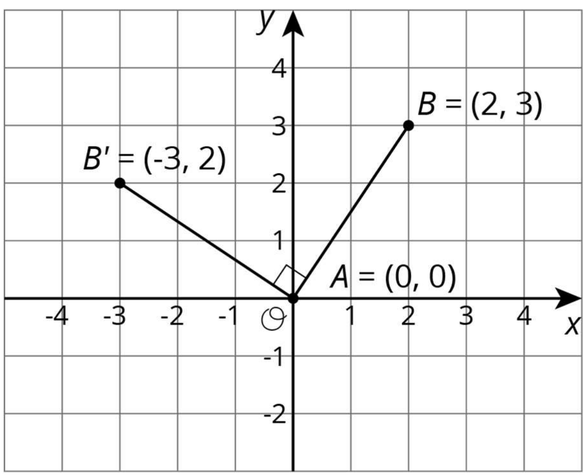


Reflecting a point across an axis changes the sign of one coordinate. For example, reflecting the point A whose coordinates are (2,-1) across the x-axis changes the sign of the y-coordinate, making its image the point A' whose coordinates are (2,1). Reflecting the point A across the y-axis changes the sign of the x-coordinate, making the image the point A'' whose coordinates are (-2,-1).



Reflections across other lines are more complex to describe.

We don’t have the tools yet to describe rotations in terms of coordinates in general. Here is an example of a 90° rotation with centre (0,0) in an anti-clockwise direction.



Point A has coordinates (0,0). Line segment AB was rotated 90° anti-clockwise around A. Point B with coordinates (2,3) rotates to point B' whose coordinates are (-3,2).

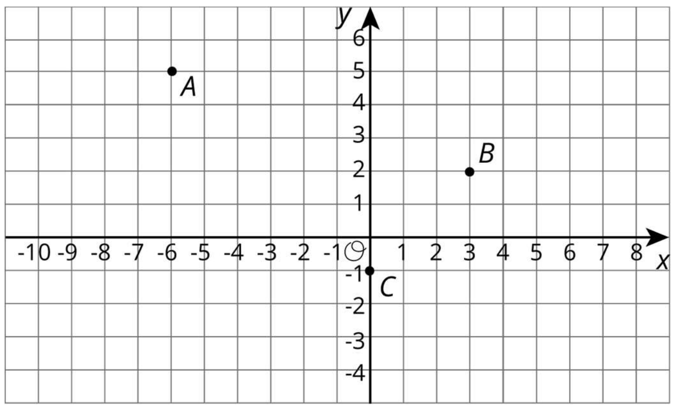
Glossary

- coordinate grid

Lesson 5 Practice Problems

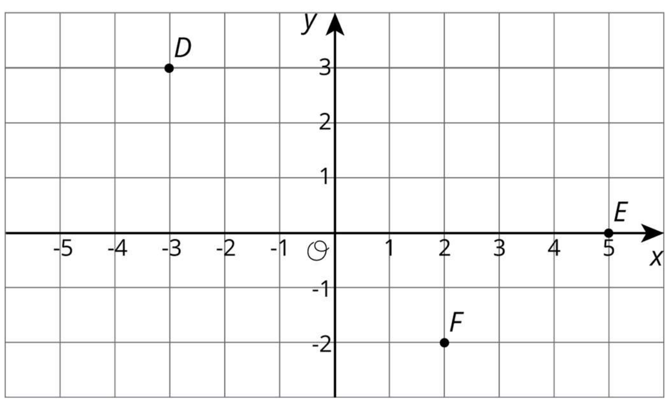
1. **Problem 1 Statement**

    a. Here are some points.

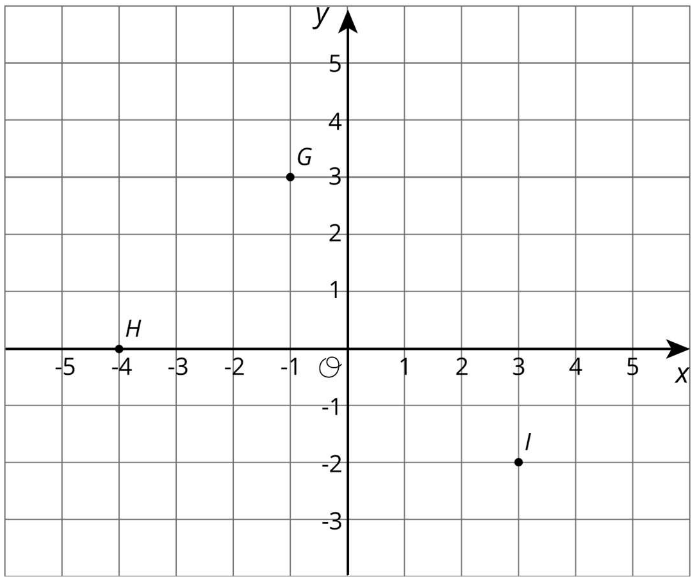
    What are the coordinates of A, B, and C after a translation to the right by 4 units and up 1 unit? Plot these points on the grid, and label them A', B' and C'.

    b. Here are some points.

    What are the coordinates of D, E, and F after a reflection over the y axis? Plot these points on the grid, and label them D', E' and F'.

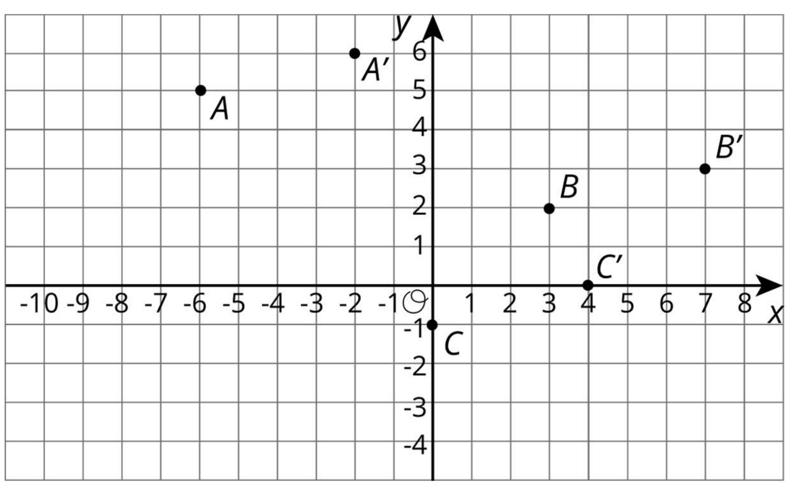
    c. Here are some points.



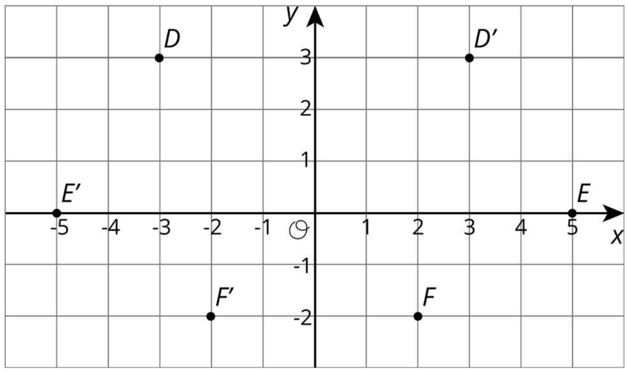
What are the coordinates of G, H, and I after a rotation about (0,0) by 90 degrees clockwise? Plot these points on the grid, and label them G', H' and I'.

Solution

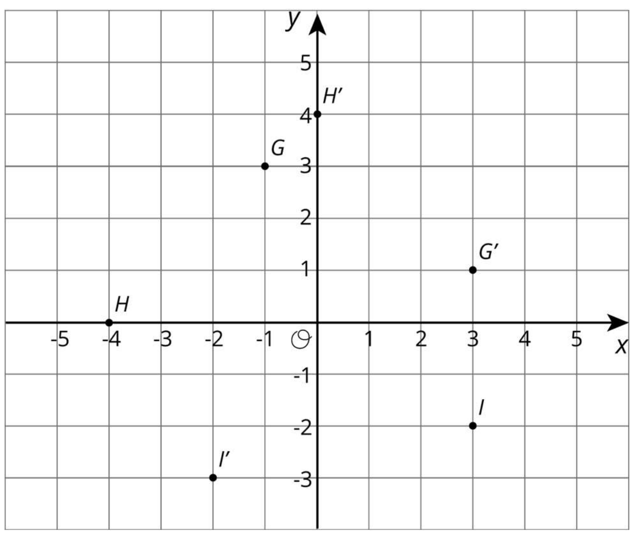
a. $A' = (-2,6), B' = (7,3), C' = (4,0)$



b. $D' = (3,3), E' = (-5,0), F' = (-2,-2)$

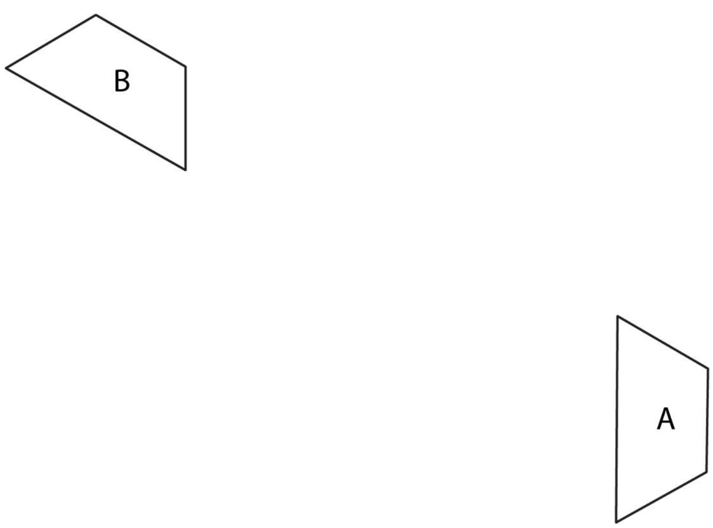


c. $G' = (3,1), H' = (0,4), I' = (-2,-3)$



2. **Problem 2 Statement**

Describe a sequence of transformations that takes trapezium A to trapezium B.

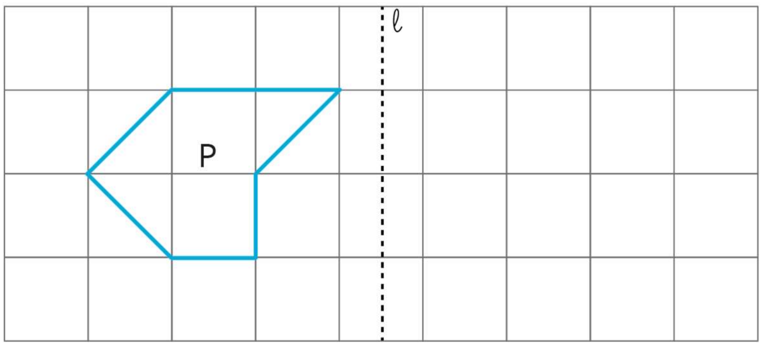


Solution

Answers vary. Sample response: Translate A up, then rotate it 60 degrees anti-clockwise (with centre of rotation the bottom vertex), and then translate it left.

3. **Problem 3 Statement**

Reflect polygon P using line ℓ.



Solution