

Grade: 9-12 (AS)

Duration: 45 min

Tools: one 16 pcs Set / group

Group work

Keywords: Faces, Favourable outcome, Total outcome

## 621 - Fitting Faces



### MATHS / PROBABILITY



LOGIFACES  
METHODOLOGY  
Erasmus+

## TEACHER

Logifaces

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### DESCRIPTION

Students select a block first and choose one other block at random, then consider which block should be selected to get the largest or smallest possible probability for the following questions.

What is the probability that the chosen block

LEVEL 1 cannot be fit to the first block?

LEVEL 2 can be fit to the first block by at least one face?

LEVEL 3 can be fit to two or more faces of the first block?

Fitting the base faces is not permitted, as all base faces are congruent equilateral triangles. Fitting by a face means that the whole face fits together. Students are encouraged to use a strategic and systematic method to avoid unnecessary calculations, for example by distributing the task between the group members.

### SOLUTIONS / EXAMPLES

The traditional model of probability is used in the solution, because all elementary events are equally likely. Hence the probability of an event is calculated using the formula

$$Pr(event) = \frac{\text{number of favourable outcomes}}{\text{total number outcomes}}$$

Total number of outcomes is 15 in every case.

A possible way to find the number of favourable outcomes is to count the blocks that can be fitted to a particular face.

The blocks sorted by the top face are listed below in the table (see also exercise [406 - Different Triangles](#)). The notation 112(2) means that there are two blocks of type 112.

top face				
number of blocks in the 16 pcs Set	2	8	2	4
list of blocks in the 16 pcs Set	111, 333	112(2), 122, 223(2), 233(3)	113, 133	123(2), 132(2)

The blocks sorted by the vertical faces are listed below in the table.

Note that the orientation of the faces matters. For example, the vertical faces of 12 and 21 are reflections of each other. The block 123 has a vertical face 12 but does not have a vertical face 21.

vertical face by heights	11	12	21	13	31	22	23	32	33
number of blocks in the 16 pcs Set	4	5	5	4	4	3	7	7	4
list of blocks in the 16 pcs Set	111, 112(2), 113	112(2), 122, 123(2)	112(2), 122, 132(2)	113, 133, 132(2)	113, 133, 123(2)	122, 223(2)	223(2), 233(3), 123(2)	223(2), 233(3), 132(2)	133, 233(3), 333

To count the number of favourable outcomes, note that an axially symmetric face (11, 22, 33) can be fitted to the same type of face, but an asymmetric face fits to its reflection (for example, the vertical face 12 fits to the face 21). The asymmetric faces are marked in the table by a darker background.

#### LEVEL 1

**HINT** First listing the blocks that do not fit by the top face helps reducing the possible cases. The favourable outcomes and the probabilities are listed in the table below.

block	111	112 (2)	122	113	133	223(2)	233 (3)	123 (2)	132 (2)	333
blocks that do not fit	122, 133, 223(2), 233(3), 123(2), 132(2)	133, 333	111, 113, 133	122, 223(2), 233(3), 333	111, 112(2), 122, 223(2)	111, 113, 133, 333	111, 113	111, 333	111, 333	112(2), 122, 113, 223(2), 123(2), 132(2)
favourable outcome	11	2	3	7	6	4	2	2	2	10
probability	$\frac{11}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{7}{15}$	$\frac{6}{15}$	$\frac{4}{15}$	$\frac{2}{15}$	$\frac{2}{15}$	$\frac{2}{15}$	$\frac{10}{15}$

Selecting the blocks 112, 233, 123 or 132 gives the smallest probability.  
Selecting the block 111 gives the largest probability.

#### LEVEL 2

**HINT** Listing separately the following three types helps find all favourable outcomes:

- I. the blocks that fit both to the top face and to some vertical faces
- II. the blocks that fit only to the top face
- III. the blocks that fit only to some vertical faces.

**NOTE** At first sight it seems easier to list the favourable outcomes by the following two types: the blocks that fit to the top face, the blocks that fit to some vertical faces. But listing in such a way requires more attention, because some blocks are counted twice, see exercise [616 - Pick a Pair](#).

The probabilities are listed in the table below. The notation fav.o. stands for the number of favourable outcomes.

block	111	112(2)	122	113	133	223(2)	233(3)	123(2)	132(2)	333
I. both top and vertical face	-	112(1), 122	112(2), 223(2)	133	113	122, 223(1), 233(3)	223(2), 233(2)	132(2)	123(2)	-
II. only top face	333	223(2), 233(3)	233(3)	-	-	112(2)	112(2), 122	123(1)	132(1)	111
III. only vertical face	112(2), 113	111, 113, 123(2), 132(2)	123(2), 132(2)	111, 112(2), 123(2), 132(2)	123(2), 132(2), 233(3), 333	123(2), 132(2)	133, 123(2), 132(2), 333	112(2), 122, 113, 133, 223(2), 233(3)	112(2), 122, 113, 133, 223(2), 233(3)	133, 233(3)
fav. o.	4	13	11	8	9	11	13	13	13	5
prob.	$\frac{4}{15}$	$\frac{13}{15}$	$\frac{11}{15}$	$\frac{8}{15}$	$\frac{9}{15}$	$\frac{11}{15}$	$\frac{13}{15}$	$\frac{13}{15}$	$\frac{13}{15}$	$\frac{5}{15}$

Selecting the block 111 gives the smallest probability.

Selecting the blocks 112, 233, 123 or 132 gives the largest probability.

### LEVEL 3

#### HINT

Listing separately the following two types helps find all favourable outcomes:

- I. the blocks that fit both to the top face and to some vertical faces
- II. the blocks that fit to at least two vertical faces but not to the top face

The probabilities are listed in the table below.

block	111	112(2)	122	113	133	223(2)	233(3)	123(2)	132(2)	333
I. top and vertical face	-	112(1), 122	112(2), 223(2)	133	113	122, 223(1), 233(3)	223(2), 233(2)	132(2)	123(2)	-
II. only vertical faces	112(2), 113	-	-	-	-	-	-	-	-	133, 233(3)
fav. o.	3	2	4	1	1	5	4	2	2	4
prob.	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{5}{15}$	$\frac{4}{15}$	$\frac{2}{15}$	$\frac{2}{15}$	$\frac{4}{15}$

Selecting the block 113 or 133 gives the smallest probability.

Selecting the block 223 gives the largest probability.

#### PRIOR KNOWLEDGE

The traditional model of probability

#### RECOMMENDATIONS / COMMENTS

This exercise is suitable for differentiation and group work. The groups can be encouraged to first make a plan and then divide the calculation tasks between the members.

Exercise [617 - Can you Match them?](#) can be considered as a warm-up exercise to this one.

Exercise [622 - Addition Rule of Probability](#) is recommended after this exercise to practise the addition rule of probability.