Lesson 8: Percentage increase and decrease with equations

Goals

- Explain (orally and in writing) how to calculate the original amount given the new amount and a percentage for increase or decrease.

- Generate algebraic expressions that represent a situation involving percentage increase or decrease, and justify (orally) the reasoning.

Learning Targets

- I can solve percentage increase and decrease problems by writing an equation to represent the situation and solving it.

Lesson Narrative

In this lesson, students represent situations involving percentage increase and percentage decrease using equations. They write equations like $y = 1.06x$ to represent growth of a bank account, and use the equation to answer questions about different starting amounts. They write equations like $t - 0.25t = 12$ or $0.75t = 12$ to represent the initial price t of a T-shirt that was £12 after a 25% discount. The focus of this unit is writing equations and understanding their connection to the context. In a later unit on solving equations the focus will be more on using equations to solve problems about percentage increase and percentage decrease.

Building On

- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Addressing

- Use proportional relationships to solve multistep ratio and percentage problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Building Towards

- Use proportional relationships to solve multistep ratio and percentage problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Instructional Routines

- Stronger and Clearer Each Time

- Compare and Connect

- Discussion Supports

- Number Talk

- Think Pair Share

Student Learning Goals

Let's use equations to represent increases and decreases.

8.1 Number Talk: From 100 to 106

Warm Up: 5 minutes

In this lesson, students will be finding percentage increase and percentage decrease by multiplying by an appropriate factor. For example, to find a 36% increase in a quantity, we can multiply that quantity by 1.36. This warm-up prompts them to think about the scale factor needed to multiply one number to get another.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Allow students to share their answers with a partner and note any discrepancies. Follow with a whole-class discussion.

Student Task Statement

How do you get from one number to the next using multiplication or division?

From 100 to 106

From 100 to 90

From 90 to 100

From 106 to 100

Student Response

Answers vary. Sample responses:

- Multiply by 1.06
- Multiply by 0.90
- Divide by 0.90
- Divide by 1.06

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate _________’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to _________’s strategy?”
- “Do you agree or disagree? Why?”

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

8.2 Interest and Depreciation

10 minutes

In this activity, students are calculating percentage increase and decrease in the context of interest and depreciation. They may have encountered these contexts before this unit, but be sure that everyone understands the basic idea of interest and depreciation before students begin work.

Students have enough experience to solve these problems using various strategies and representations. Look for students writing different but correct expressions in terms of x.

Instructional Routines

- Compare and Connect
- Think Pair Share

Launch

Remind students what interest is, give examples of when interest is applied (savings accounts, credit cards, etc.), and discuss depreciation value of an item.

Students in groups of 2. Give students 3–5 minutes of quiet work time, followed by partner then whole-class discussion.

Representation: Internalise Comprehension. Represent the same information through different modalities by using tables. If students are unsure where to begin, suggest that they draw a table to help organise the information provided.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

If students have a hard time organising their information, suggest that they use a table.

Students might struggle to write an expression in terms of x. Have students describe in words how they calculated the previous problems with numbers and use that to come up with an expression.

Student Task Statement

1. Money in a particular savings account increases by about 6% after a year. How much money will be in the account after one year if the initial amount is £100? £50? £200? £125? x pounds? If you get stuck, consider using diagrams or a table to organise your work.

2. The value of a new car decreases by about 15% in the first year. How much will a car be worth after one year if its initial value was £1 000? £5 000? £5 020? x pounds? If you get stuck, consider using diagrams or a table to organise your work.

Student Response

1.
£106, since $100 \times 1.06 = 106$.
£53. Since 6% of 100 is 6, we know 6% of 50 is 3. Increasing £50 by £3 gives £53.
£212. Since 6% of 100 is 6, we know 6% of 200 is 12. Increasing £200 by £12 gives £212.
£132.50. If we deposit £25, we would have £26.50 after one year, because it is half the amount we would have if we deposit £50. If we deposit £125, we will have £132.50 because 106+26.50=132.50.
$1.06x$ or another equivalent expression

2. £850. After one year, the car is worth 85% of its original value, because $100 - 15 = 85$. The car would be worth £ because $1\,000 \times 0.85 = 850$.
£4 250, since $5\,000 \times 0.85 = 4\,250$.

£4 267, since $5\,020 \times 0.85 = 4\,267$.
$0.85x$ or another equivalent expression

Activity Synthesis

Select students with different ways of writing correct expressions in terms of x to share their expression. Connect both problems back to the previous work with the distributive property by asking students:

- How else can the first problem be written? ($x + 0.06x$ or $(1 + 0.06)x$ or $1.06x$)
- How else can the second problem be written? ($x - 0.15x$ or $(1 - 0.15)x$ or $0.85x$)

Representing: Compare and Connect. Use this routine when students present their equations that represent the first problem. Ask students to say what they notice about the different ways of writing correct expressions. For each expression, draw students' attention to the meaning of each variable, and to the different ways the percentage increase is represented. This will support students' mathematical language use as they make sense of strategies used to solve problems about percentage increase or decrease.
Design Principle(s): Maximise meta-awareness

8.3 Matching Equations

5 minutes

In this activity, students match equations that represent a percentage increase situation to the situations they represent.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of two. 2 minutes of quiet think time followed by 2 minutes of partner discussion.

Student Task Statement

Match an equation to each of these situations. Be prepared to share your reasoning.

1. The water level in a reservoir is now 52 metres. If this was a 23% increase, what was the initial depth?
2. The snow is now 52 inches deep. If this was a 77% decrease, what was the initial depth?

$0.23x = 52$

$0.77x = 52$

$1.23x = 52$

$1.77x = 52$

Student Response

1. $1.23x = 52$
2. $0.23x = 52$

Are You Ready for More?

An astronaut was exploring the moon of a distant planet, and found some glowing goo at the bottom of a very deep crater. She brought a 10 gram sample of the goo to her laboratory. She found that when the goo was exposed to light, the total amount of goo increased by 100% every hour.

1. How much goo will she have after 1 hour? After 2 hours? After 3 hours? After n hours?
2. When she put the goo in the dark, it shrank by 75% every hour. How many hours will it take for the goo that was exposed to light for n hours to return to the original size?

Student Response

1. After 1 hour, there will be 20 grams. After 2 hours, there will be 40 grams. After 3 hours, there will be 80 grams. After n hours, there will be 10×2^n.
2. $\frac{n}{2}$ hours. A 75% decrease is $\frac{1}{4}$ as much, so for every hour, the amount decreases to $\frac{1}{4}$ of what was there at the beginning of the hour. For example, after 2 hours of light exposure, there will be 40 grams of goo, but after only one hour in the dark, it will be back to 10 grams.

Activity Synthesis

Ask one or more students to share which equation they matched with each situation, and resolve any discrepancies. Once the matches are agreed upon, ask students how they would solve the equation to find the amount without actually solving the equations.

Speaking: Discussion Supports. Use this routine to support whole-class discussion. Provide sentence frames for students to use when they share the equation they matched to each situation: "Situation ____ matches with equation ____ because ____." Call students attention to how the percentage increase is represented in the equation and the situation.
Design Principle(s): Support sense-making; Optimise output (for explanation)

8.4 Representing Percentage Increase and Decrease: Equations

Optional: 15 minutes

The purpose of this activity is for students to use equations to represent situations of percentage increase and decrease. Additionally, students identify the original and new amount in the double number lines to reinforce what they learned in earlier lessons (that the original amount pertains to 100%).

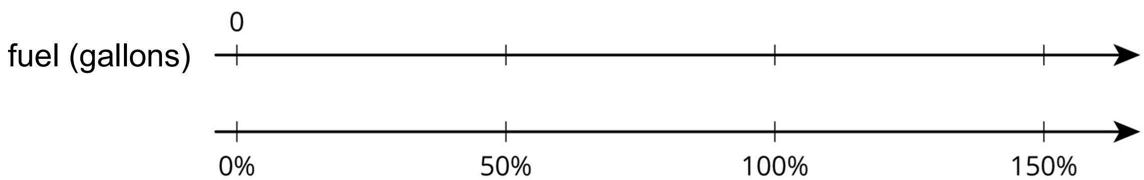
As students work on the task, look for students who created various equations for the last question.

Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

Launch

Show students the double number line from the activity in the previous lesson.



Help them make connections to the first problem. Tell them that they can go back to the previous lesson to see the double number lines for the others if that helps.

Arrange students in groups of 2. Give 5–8 minutes of quiet work time. After 5 minutes allow students to work with a partner or to continue to work alone.

Representation: Internalise Comprehension. Represent the same information through different modalities by using double number line diagrams. If students are unsure where to begin, suggest that they draw a double number line diagram to help organise the information provided.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

Students may continue to struggle to recognise the original amount and new amount with the proper percentages on the double number line. Remind them that the original amount always corresponds to 100%.

Student Task Statement

1. The fuel tank in dad's car holds 12 gallons. The fuel tank in mum's truck holds 50% more than that. How much fuel does the truck's tank hold? Explain why this situation can be represented by the equation $(1.5) \times 12 = t$. Make sure that you explain what t represents.

2. Write an equation to represent each of the following situations.

    a. A movie theatre decreased the size of its popcorn bags by 20%. If the old bags held 15 cups of popcorn, how much do the new bags hold?

b. After a 25% discount, the price of a T-shirt was £12. What was the price before the discount?

c. Compared to last year, the population of Boom Town has increased by 25%.The population is now 6 600. What was the population last year?

Student Response

1. 18 gallons. Let t represent the amount of gas that mum's truck holds. Dad's car holds 12 gallons, and mum's truck holds 50% more or 0.5×12 more, so it holds $12 + 0.5 \times 12 = 1.5 \times 12$. So $1.5 \times 12 = t$.

2.

a. $(0.8) \times 15 = p$, where p is the amount of popcorn the new bag holds

b. $(0.75) \times t = 12$, where t is the price before the discount

c. $(1.25) \times p = 6{,}600$, where p is the population last year

Activity Synthesis

Select students to share the values they identified as original amount and the new amount for a few problems. Discuss how 100% always corresponds to the original value and when there is an increase in the value the new value corresponds to a percentage greater than the original 100%.

Select students to share the different equations they came up with. Discuss how the distributive property is useful for finding the percentage that corresponds with the new value instead of the percentage of the change.

Discuss how solving problems about percentage change may require either multiplying or dividing numbers. It can be confusing, but it helps to first express the relationship as an equation and then think about how you can find the unknown number. Looking at the examples below, the first two require multiplication, but the others require division.

Using the structure A% of B is C:

- $(1.5) \times 12 = c$
- $(0.80) \times 15 = c$
- $a \times (1\,200) = 1\,080$
- $a \times (1.50) = 1.75$
- $(0.75) \times b = 12$
- $(1.25) \times b = 61\,600$

Writing, Speaking: Stronger and Clearer Each Time. To begin the whole-class discussion, use this routine to give students a structured opportunity to revise and refine their response to

the first question, "Explain why this situation can be represented by the equation $(1.5) \times 12 = t$." Give students time to meet with 2–3 partners, to share and get feedback on their initial explanations. Provide listeners with prompts for feedback such as, “What does t represent?” “Where is 50% in the equation?” “Can you say that another way?” etc. Invite students to go back and refine their written explanation based on the peer feedback they receive. This will help students understand how situations of percentage increase or decrease can be represented by an equation.
Design Principle(s): Optimise output (for generalisation); Cultivate conversation

Lesson Synthesis

Students should feel confident calculating percentage increase/decrease using a method of their choice. Ask students:

- “What are some ways we have learned to solve percentage increase or percentage decrease problems?” (double number lines, tables, equations)
- “Which representation do you prefer to use? Why?”

8.5 Tyler's Savings Bond

Cool Down: 5 minutes

Student Task Statement

Tyler's mum purchased a savings bond for Tyler. The value of the savings bond increases by 4% each year. One year after it was purchased, the value of the savings bond was £156.

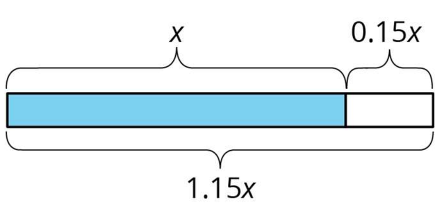
Find the value of the bond when Tyler's mum purchased it. Explain your reasoning.

Student Response

Sample reasoning: Represent the situation using the equation $1.04x = 156$, where x represents the value of the savings bond when Tyler's mum purchased it. The solution is $x = 156 \div 1.04 = 150$, so the bond was originally worth £150.

Student Lesson Summary

We can use equations to express percentage increase and percentage decrease. For example, if y is 15% more than x,



we can represent this using any of these equations:

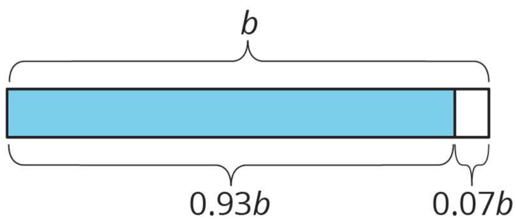
$y = x + 0.15x$

$y = (1 + 0.15)x$

$y = 1.15x$

So if someone makes an investment of x pounds, and its value increases by 15% to £1 250, then we can write and solve the equation $1.15x = 1\,250$ to find the value of the initial investment.

Here is another example: if a is 7% less than b,



we can represent this using any of these equations:

$a = b - 0.07b$

$a = (1 - 0.07)b$

$a = 0.93b$

So if the amount of water in a tank decreased 7% from its starting value of b to its ending value of 348 gallons, then you can write $0.93b = 348$.

Often, an equation is the most efficient way to solve a problem involving percentage increase or percentage decrease.

Lesson 8 Practice Problems

Problem 1 Statement

A pair of designer trainers was purchased for £120. Since they were purchased, their price has increased by 15%. What is the new price?

Solution

£138

Problem 2 Statement

Elena's aunt bought her a £150 savings bond when she was born. When Elena is 20 years old, the bond will have earned 105% in interest. How much will the bond be worth when Elena is 20 years old?

Solution

£307.50

Problem 3 Statement

In a video game, Clare scored 50% more points than Tyler. If c is the number of points that Clare scored and t is the number of points that Tyler scored, which equations are correct? Select **all** that apply.

a. $c = 1.5t$

b. $c = t + 0.5$

c. $c = t + 0.5t$

d. $c = t + 50$

e. $c = (1 + 0.5)t$

Solution ["A", "C", "E"]

Problem 4 Statement

Draw a diagram to represent each situation:

a. The number of miles driven this month was a 30% decrease of the number of miles driven last month.

b. The amount of paper that the copy shop used this month was a 25% increase of the amount of paper they used last month.

Solution

Answers vary. Sample responses:

a. A bar model showing 10 equal pieces labelled “number of miles driven last month” on the top with one below it that is just 7 pieces long and is labelled, “number of miles driven this month.”

b. A bar model showing 4 equal pieces labelled “amount of paper they used last month” on the top with one below it that is 5 pieces long and is labelled, “amount of paper they used this month.”

Problem 5 Statement

Which decimal is the best estimate of the fraction $\frac{29}{40}$?

a. 0.5

b. 0.6

c. 0.7

d. 0.8

Solution C

Problem 6 Statement

Could 7.2 inches and 28 inches be the diameter and circumference of the same circle? Explain why or why not.

Solution

No, since $7.2 \times \pi \approx 22.6$.