

Parametrizacions de superfícies

1. Esfera de radi R i centre a l'origen de coordenades:

$$\boxed{x^2 + y^2 + z^2 = R^2} \quad \begin{cases} x = R \sin v \cos u & 0 \leq u < 2\pi \\ y = R \sin v \sin u & 0 \leq v \leq \pi \\ z = R \cos v \end{cases}$$

$$T_u \times T_v = -(R^2 \sin^2 v \cos u, R^2 \sin^2 v \sin u, R^2 \sin v \cos v)$$

$$\|T_u \times T_v\| = R^2 \sin v$$

2. El·lipsoide de semieixos a, b, c i centre a l'origen de coordenades:

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1} \quad \begin{cases} x = a \sin v \cos u & 0 \leq u < 2\pi \\ y = b \sin v \sin u & 0 \leq v \leq \pi \\ z = c \cos v \end{cases}$$

$$T_u \times T_v = -(bc \sin^2 v \cos u, ac \sin^2 v \sin u, ab \sin v \cos v)$$

$$\|T_u \times T_v\| = abc \sin v \sqrt{\sin^2 v \left(\frac{\cos^2 u}{a^2} + \frac{\sin^2 u}{b^2} \right) + \frac{\cos^2 v}{c^2}}$$

3. Paraboloid el·líptic centrat a l'eix Z :

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = z} \quad \begin{cases} x = a r \cos t & 0 \leq t < 2\pi \\ y = b r \sin t & r \geq 0 \\ z = r^2 \end{cases}$$

$$T_t \times T_r = (2b r^2 \cos t, 2a r^2 \sin t, -ab r)$$

$$\|T_t \times T_r\| = 2ab r \sqrt{r^2 \left(\frac{\cos^2 t}{a^2} + \frac{\sin^2 t}{b^2} \right) + \frac{1}{4}}$$

4. Cilindre el·líptic centrat a l'eix Z :

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \quad \begin{cases} x = a \cos t & 0 \leq t < 2\pi \\ y = b \sin t & z \in \mathbb{R} \\ z = z \end{cases}$$

$$T_t \times T_z = (b \cos t, a \sin t, 0)$$

$$\|T_t \times T_z\| = ab \sqrt{\frac{\cos^2 t}{a^2} + \frac{\sin^2 t}{b^2}}$$

5. Con el·líptic centrat a l'eix Z :

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = z^2} \quad \begin{cases} x = a z \cos t & 0 \leq t < 2\pi \\ y = b z \sin t & z \in \mathbb{R} \\ z = z \end{cases}$$

$$T_t \times T_z = (b z \cos t, a z \sin t, -ab z)$$

$$\|T_t \times T_z\| = ab|z| \sqrt{\frac{\cos^2 t}{a^2} + \frac{\sin^2 t}{b^2} + 1}$$