Lesson 15: Distinguishing volume and surface area

Goals

- Compare and contrast (orally and in writing) problems that involve surface area and volume of prisms.
- Decide whether to calculate the surface area or volume of a prism to solve a problem in a real-world situation, and justify (orally) the decision.
- Estimate measurements of a prism in a real-world situation, and explain (orally) the estimation strategy.

Learning Targets

- I can decide whether I need to find the surface area or volume when solving a problem about a real-world situation.

Lesson Narrative

This is the first of two lessons where students apply their knowledge of surface area and volume to solve real-world problems. The purpose of this first lesson is to help students distinguish between surface area and volume and to choose which of the two quantities is appropriate for solving a problem. They solve problems that require finding the surface area or volume of a prism, or both.

Addressing

- Solve real-life and mathematical problems involving angles, area, surface area, and volume.
- Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and prisms.

Instructional Routines

- Compare and Connect
- Discussion Supports
- Poll the Class

Required Materials

Pre-printed slips, cut from copies of the blackline master

Card sort

Card Sort: Surface area or volume How much wood is needed to make triangular shaped stacking blocks?	Card Sort: Surface area or volume How much glass is needed to build a greenhouse?
Card Sort: Surface area or volume How long it would take to fill a rectangular swimming pool?	Card Sort: Surface area or volume How long would it take to paint the outside of a barn?
Card Sort: Surface area or volume How many yards of fabric are needed to sew a pillowcase?	Card Sort: Surface area or volume How much cardboard is needed to make a cereal box?
Card Sort: Surface area or volume How long would it take to dig soil out to form a rectangular foundation for a new building?	Card Sort: Surface area or volume How much wood is needed to build a birdhouse?

Required Preparation

Make 1 copy of the Card Sort: Surface Area or Volume blackline master for every 2 students, and cut them up ahead of time.

Student Learning Goals

Let's work with surface area and volume in context.

15.1 The Science Competition

Warm Up: 5 minutes

The purpose of this warm-up is for students to reason about two objects that have the same volume but different surface areas within a context.

Instructional Routines

- Poll the Class

Launch

Give students 1 minute of quiet think time followed by a whole-class discussion.

Student Task Statement

Mai's science teacher told her that when there is more ice touching the water in a glass, the ice melts faster. She wants to test this statement so she designs her science project to determine if crushed ice or ice cubes will melt faster in a drink.

She begins with two cups of warm water. In one cup, she puts a cube of ice. In a second cup, she puts crushed ice with the same volume as the cube. What is your hypothesis? Will the ice cube or crushed ice melt faster, or will they melt at the same rate? Explain your reasoning.

Student Response

The crushed ice will melt faster because there are more parts touching the warm water. The increased surface area will make the crushed ice melt faster.

Activity Synthesis

Poll the students on their hypotheses. Record and display their responses for all to see. If all students agree the crushed ice will melt the fastest, ask them to share their reasoning. If there are different hypotheses, ask students to explain their choice and ask questions of one another. Continue the discussion until the students reach an agreement on the crushed ice. Important ideas to highlight during the discussion are:

- Since the crushed ice has more places touching the warm water, it should melt faster.
- The centre of the cube is surrounded by cold ice and will not melt until it is touching the water.
- The *surface area* of the crushed ice is greater, so it will melt faster than the ice cube even though they have the same volume.

If any of these ideas are not mentioned by students, bring them to their attention at the end of the discussion.

15.2 Revisiting the Box of Chocolates

10 minutes

In this activity students are presented with a prism that was used in a previous lesson to calculate volume. Here, they calculate the surface area of the prism. This provides students with the opportunity to work with complex shapes to find surface area in a given context.

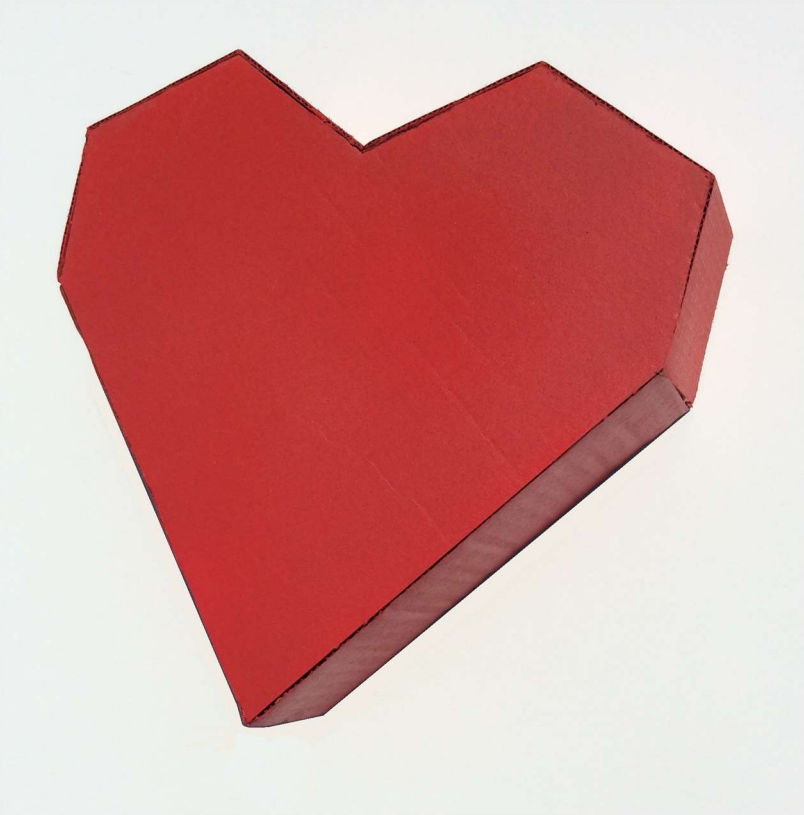
Instructional Routines

- Compare and Connect

Launch

Display the image for all to see throughout the activity. Tell students that they calculated the volume of this heart-shaped box in a previous lesson and today they are going to

calculate a different measurement. Ask students what additional information they need to find the total amount of cardboard in the box. When students recognise that they need the lengths of the diagonal sides of the box give them the measurements for those sides (2.2 inches for the sides around the top and 6.4 inches for the sides around the bottom). Give students 2–3 minutes of quiet work time followed by a whole-class discussion.



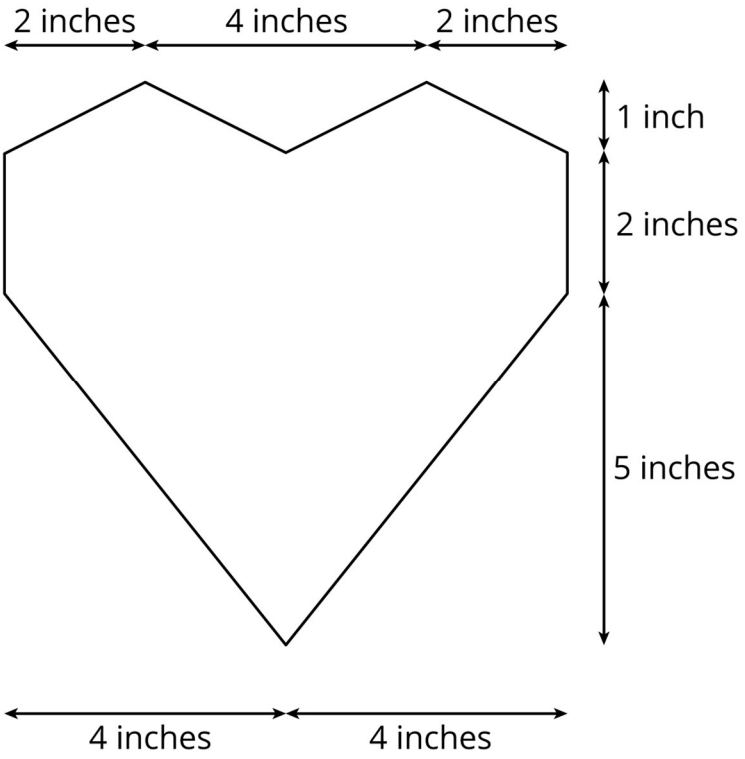
Representation: Internalise Comprehension. Activate or supply background knowledge by reviewing the heart-shaped box volume calculations. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Students who are familiar with actual heart-shaped boxes of chocolate may want to double the lateral area to represent the way the top and bottom pieces nest together.

Student Task Statement

The other day, you calculated the volume of this heart-shaped box of chocolates.



The depth of the box is 2 inches. How much cardboard is needed to create the box?

Student Response

131.2 in^2. The area of the heart base is 40 in^2 as we solved the other day. The perimeter of the base is 25.6 in, so the area of the long rectangle of the prism is 51.2 in^2, because $25.6 \times 2 = 51.2$. Therefore, the total surface area is found with $40 + 40 + 51.2 = 131.2$.

Activity Synthesis

Select students to share their solutions and methods for calculating the surface area. Consider asking some of the following questions:

- "How did you figure out that you had to calculate the surface area of the box?"
- "What method did you use to calculate the surface area?"
- "If the chocolate maker wants to make a set of two boxes that are each half of a heart and could be put together to make a box that looks like this one, would the total amount of cardboard used to make the two boxes be different than making the one box?" (Yes, it would increase.)
- "How could the chocolate maker reduce the surface area of the one box without reducing the volume?" (If he made it a triangle or some other shape with fewer segments instead of a heart.)

Representing, Conversing: Compare and Connect. Use this routine to help students consider audience when preparing a visual display of their work. Ask students to prepare a visual display that shows how they calculated the surface area of the box. Students should consider how to display their calculations so that another student can interpret them. Some students may wish to add notes or details to their drawings to help communicate their thinking. Invite students exchange their displays with 2–3 other students, to review and compare their calculations. These exchanges strengthen students' mathematical language use and reasoning related to surface area.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

15.3 Card Sort: Surface Area or Volume

15 minutes

The purpose of this activity is for students to sort cards with questions that have a context referring to either volume or surface area of a prism. In previous lessons, students focused on determining volume or surface area and the two concepts were never presented side by side. Here, students are asked to sort questions with a context to determine if it makes more sense to think about surface area or volume when answering the question. After sorting, students think about what information they need to answer a question and estimate reasonable measurements to calculate the answer to their question.

Instructional Routines

- Compare and Connect

Launch

Arrange students in groups of 2. Distribute pre-printed slips, cut from copies of the blackline master. Give groups 2–3 minutes of quiet work time. Once each group has sorted their cards, poll the class as to where they placed each card. Display the answers for all to see for the rest of the activity. Ask students to come to agreement on any differences.

After the class has come to agreement on the sorted cards, students can either choose one card or be assigned a specific card to examine further. Give students 2–3 minutes of quiet work time to complete the rest of the task followed by time to discuss their work with a partner. Follow with a whole-class discussion.

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, “I noticed ____ so I knew it was …”.
Supports accessibility for: Language; Social-emotional skills

Student Task Statement

Your teacher will give you cards with different shapes and questions on them.

1. Sort the cards into two groups based on whether it would make more sense to think about the surface area or the volume of the shape when answering the question. Pause here so your teacher can review your work.

2. Your teacher will assign you a card to examine more closely. What additional information would you need to be able to answer the question on your card?

3. Estimate reasonable measurements for the shape on your card.

4. Use your estimated measurements to calculate the answer to the question.

Student Response

The first three cards in the first column of blackline master are volume, the rest of the cards refer to surface area.

Are You Ready for More?

A cake is shaped like a square prism. The top is 20 centimetres on each side, and the cake is 10 centimetres tall. It has frosting on the sides and on the top, and a single candle on the top at the exact centre of the square. You have a knife and a 20-centimetre ruler.

1. Find a way to cut the cake into 4 fair portions, so that all 4 portions have the same amount of cake and frosting.

2. Find another way to cut the cake into 4 fair portions.

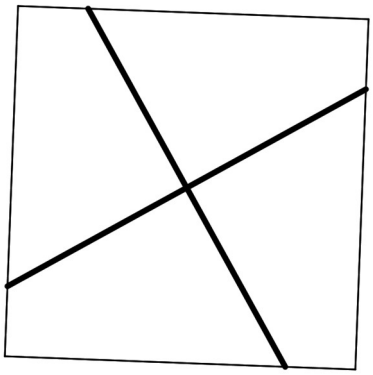
3. Find a way to cut the cake into 5 fair portions.

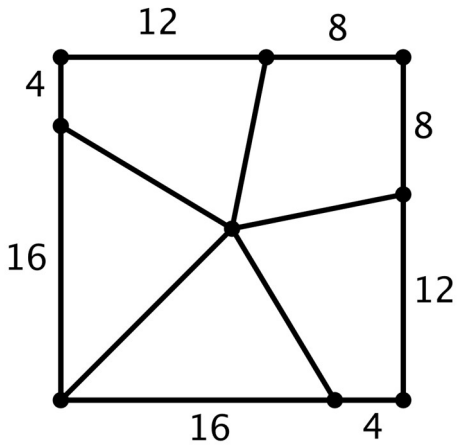
Student Response

1. The simplest way is to cut the cake in half on one diagonal, and then on the other diagonal.

2. The key is to pick points that are equally spaced from the corners, as shown.

3. The perimeter of the cake is 80 cm, and $80 \div 5 = 16$. Find 5 points around the perimeter of the cake that are each 16 centimetres apart, and connect each of them to the candle in the centre with a cut. Each piece gets 16 cm of the edge of the cake, so the same amount of frosting. Through decomposing into triangles, we can show that each piece of the cake has a volume of 800 cubic centimetres. (Note that this solution presumes that the frosting has no thickness, which is not likely to be true, but is a handy simplifying assumption. In reality, people who love frosting should still take a corner piece.)





Activity Synthesis

Select students to share their explanations of how they determined if a question referenced volume or surface area. To highlight the differences and similarities ask students:

- "How do you know volume is being asked about in that question?"
- "How do you know surface area is being asked about in that question?"

The goal is to ensure students can verbally describe their reasoning because they are asked to write about the similarities and differences in the cool down. You may wish to refer back to the poll responses collected in the middle of the activity.

Invite students to share the measurements they came up with and how they calculated an answer to their question. Ask other students who had the same question but used different measurements to share and compare the reasonableness of each answer.

Consider asking some of the following questions:

- "How did you determine which information you needed to answer the question? What information might not be needed?"
- "How did you decide on the exact measurements to use?"

- "What is different about the measurements used in these questions?"
- "Do they all make sense?"
- "Are there other measurements that could have been used?"

Representing, Speaking, and Listening: Compare and Connect. Use this routine when students share their strategies for determining whether the question referenced volume or surface area. Ask students to consider what is the same and what is different about each approach. Draw students' attention to the relationship between the language of the question and whether the problem examines volume or surface area. These exchanges can strengthen students' mathematical language use and reasoning based on volume and surface area.
Design Principle(s): Maximise meta-awareness; Support sense-making

15.4 A Wheelbarrow of Concrete

Optional: 5 minutes

This optional activity reinforces work students have done in previous activities with regards to surface area and volume. Students work with a contextual problem to determine the surface area and volume of an object.

Instructional Routines

- Discussion Supports

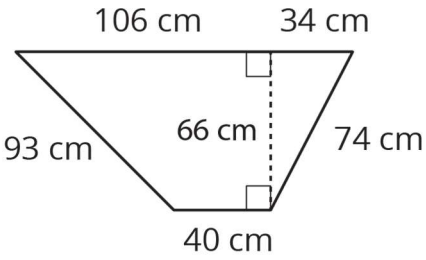
Launch

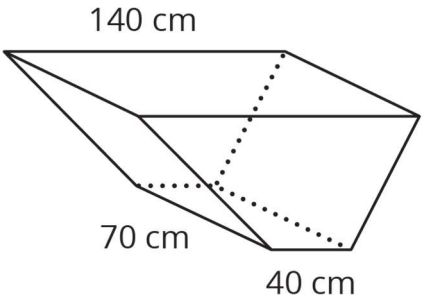
Arrange students in groups of 2. If desired, display the image of a wheelbarrow for all to see.

Give students 1–2 minutes of quiet work time followed by time to discuss their answers with their partner. Follow with a whole-class discussion.

Student Task Statement

A wheelbarrow is being used to carry wet concrete. Here are its dimensions.





1. What volume of concrete would it take to fill the tray?

2. After dumping the wet concrete, you notice that a thin film is left on the inside of the tray. What is the area of the concrete coating the tray? (Remember, there is no top.)

Student Response

1. The volume is 415 800 cm^3. The area of the base is 5 940 cm^2 and can be found by cutting it into two triangles and a rectangle, because $40 \times 66 + \frac{1}{2} \times 34 \times 66 + \frac{1}{2} \times 66 \times 66 = 5\,940$. So, the volume is 415 800 cm^3, because $5\,940 \times 70 = 415\,800$.

2. The surface area inside the wheelbarrow is 26 370 cm^2. The perimeter of the base is 347 cm, but there is no top to the wheelbarrow, so we only need a length of 207 cm for the long rectangle, because $347 - 106 - 34 = 207$. So the area of the long rectangular side should be 14 490 cm^2, because $207 \times 70 = 14\,490$). So, the total area is the area of the bases added to the area of the long rectangle. $5\,940 + 5\,940 + 14\,490 = 26\,370$.

Activity Synthesis

Reveal the solution to each problem and give students a few minutes to resolve any discrepancies with their partners.

Speaking: Discussion Supports. Use this routine to scaffold students as they discuss their strategies and solutions for calculating the volume of concrete in and surface area of the wheelbarrow. Provide sentence frames for students to use with their partner, such as:

"First I _____. Then I _____, because . . ." and "The volume is _____ because . . . ." Encourage students to consider what details are important to share and to think about how they will explain their reasoning using mathematical language.
Design Principle(s): Optimise output (for explanation); Cultivate conversation

Lesson Synthesis

- "When is it better to know surface area than volume?" (When you are covering an object, when you want to know how much is exposed to the environment, etc.)
- "When is it better to know volume than surface area?" (When you are filling up the object, when you need to know how much is already inside, etc.)
- "If you cut an object in half, how does that affect the volume and surface area?" (The volume remains unchanged, but the surface area will increase.)

15.5 Surface Area Differences

Cool Down: 5 minutes

Student Task Statement

Describe some similarities and differences between a situation that involves calculating surface area and a situation that involves calculating volume.

Student Response

Answers vary. Sample response: Volume refers to how much of something fits inside an object. Surface area refers to how much of something is needed to cover the outside of an object.

Student Lesson Summary

Sometimes we need to find the volume of a prism, and sometimes we need to find the surface area.

Here are some examples of quantities related to volume:

- How much water a container can hold
- How much material it took to build a solid object

Volume is measured in cubic units, like in^3 or m^3.

Here are some examples of quantities related to surface area:

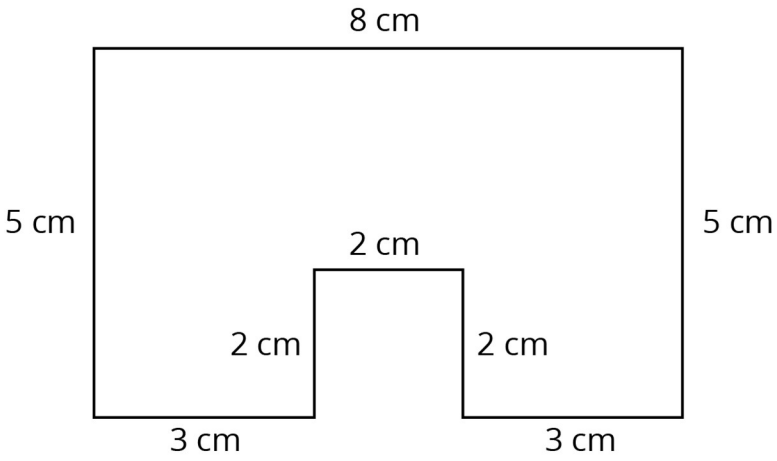
- How much fabric is needed to cover a surface
- How much of an object needs to be painted

Surface area is measured in square units, like in^2 or m^2.

Lesson 15 Practice Problems

1. **Problem 1 Statement**

   Here is the base of a prism.



   a. If the height of the prism is 5 cm, what is its surface area? What is its volume?

   b. If the height of the prism is 10 cm, what is its surface area? What is its volume?

   c. When the height doubled, what was the percentage increase for the surface area? For the volume?

   Solution

   a. $SA = 222 \text{ cm}^2, V = 180 \text{ cm}^3$

   b. $SA = 372 \text{ cm}^2, V = 360 \text{ cm}^3$

   c. The surface area increased by about 67.6%. The volume increased by 100% (doubled).

2. **Problem 2 Statement**

   Select **all** the situations where knowing the volume of an object would be more useful than knowing its surface area.

   a. Determining the amount of paint needed to paint a barn.

   b. Determining the monetary value of a piece of gold jewellery.

   c. Filling an aquarium with buckets of water.

   d. Deciding how much wrapping paper a gift will need.

   e. Packing a box with watermelons for shipping.

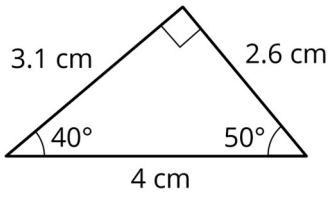
   f. Charging a company for ad space on your race car.

g. Measuring the amount of fuel left in the tank of a tractor.

Solution ["B", "C", "E", "G"]

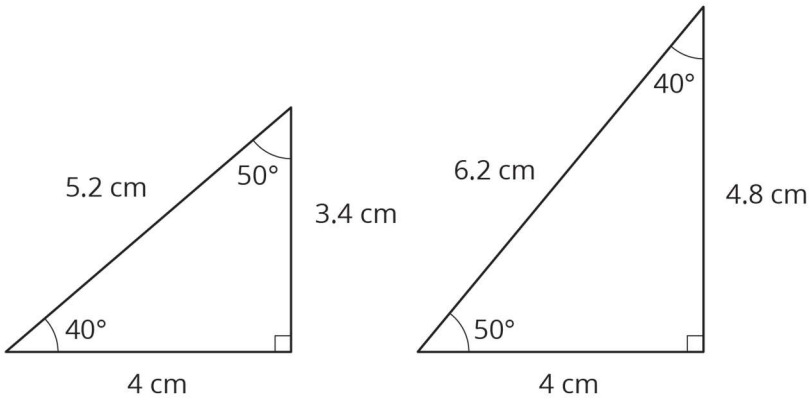
3. **Problem 3 Statement**

Han draws a triangle with a 50° angle, a 40° angle, and a side of length 4 cm as shown. Can you draw a different triangle with the same conditions?



Solution

Answers vary. Sample response: Yes, if we rearrange the angles and side, there are more possibilities.



4. **Problem 4 Statement**

Angle H is half as large as angle J. Angle J is one fourth as large as angle K. Angle K is 240 degrees. What is the size of angle H?

Solution

30°

5. **Problem 5 Statement**

The Colorado state flag consists of three horizontal stripes of equal height. The side lengths of the flag are in the ratio 2: 3. The diameter of the gold-coloured disk is equal to the height of the centre stripe. What percentage of the flag is gold?

Solution

Approximately 5.82% (the exact proportion is $\frac{\pi}{54}$ or equivalent)