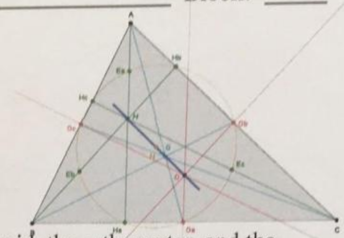
Group 1

Your Name: SOLUTIONS **Partner's Name:** ________________ **Block:** _____



Geometry – Quarter 2 Project
Euler Line

In 1765, Swiss mathematician, Leonhard Euler proved that the centroid, the orthocenter, and the circumcenter of a triangle are all collinear. The line containing these three points is called the Euler Line. Complete the steps outlined here to investigate the Euler Line.

Show all work neatly, with formulas, to receive credit for each step. Hint: Use a pencil.

Plot and label the three vertices of ΔABC on the graph provided. A(-2, -1), B(1,5), C(4,1)

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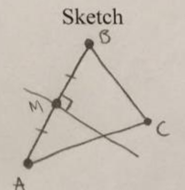
(1) Finding the Circumcenter:
Find an equation for each perpendicular bisector of the triangle.

(1a) Perpendicular Bisector of $\overline{AB}$ A($\overset{x_1}{-2}$, $\overset{y_1}{-1}$), B($\overset{x_2}{1}$,$\overset{y_2}{5}$), C(4,1)

Sketch



Slope of $\overline{AB}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{5+1}{1+2}$

$m = \frac{6}{3}$

$m = 2$

Slope of Perpendicular Bisector

$* \; m_\perp = -\frac{1}{2}$

Midpoint of $\overline{AB}$

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M = \left(\frac{-2+1}{2}, \frac{-1+5}{2}\right)$

$* \; M = \left(\frac{-1}{2}, 2\right)$

Equation of line in point-slope form

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{2}(x + \frac{1}{2})$

Equation of line in slope intercept form

$y = mx + b$

$y - 2 = -\frac{1}{2}(x + \frac{1}{2})$

$y - 2 = -\frac{1}{2}x - \frac{1}{4}$

$\;\;+2 \qquad\qquad +2$

$y = -\frac{1}{2}x + 1\frac{3}{4}$

$y = -\frac{1}{2}x + 1.75$

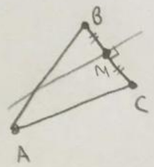
$y = -\frac{1}{2}x + \frac{7}{4}$

Group 1

$x_1\ y_1 \qquad x_2\ y_2$

(1b) Perpendicular Bisector of $\overline{BC}$ A(-2, -1), B(1,5), C(4,1)

Sketch



Slope of $\overline{BC}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{1-5}{4-1}$

$m = \frac{-4}{3}$

Slope of Perpendicular Bisector

$* \; m_\perp = \frac{3}{4}$

Midpoint of $\overline{BC}$

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M = \left(\frac{1+4}{2}, \frac{5+1}{2}\right)$

$* \; M = \left(\frac{5}{2}, 3\right)$

Equation of line in point-slope form

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{3}{4}\left(x - \frac{5}{2}\right)$

Equation of line in slope intercept form

$y = mx + b \qquad y - 3 = \frac{3}{4}\left(x - \frac{5}{2}\right)$

$y - 3 = \frac{3}{4}x - \frac{15}{8}$

$+3 \qquad +3$

$y = \frac{3}{4}x - \frac{15}{8} + \frac{24}{8}$

$y = \frac{3}{4}x + 1\frac{1}{8}$

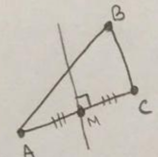
$y = \frac{3}{4}x + 1.125$

$y = \frac{3}{4}x + \frac{9}{8}$

$x_1\ y_1 \qquad x_2\ y_2$

(1c) Perpendicular Bisector of $\overline{AC}$ A(-2, -1), B(1,5), C(4,1)

Sketch



Slope of $\overline{AC}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{1+1}{4+2}$

$m = \frac{2}{6} = \frac{1}{3}$

Slope of Perpendicular Bisector

$* \; m_\perp = -3$

Midpoint of $\overline{AC}$

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M = \left(\frac{-2+4}{2}, \frac{-1+1}{2}\right)$

$M = \left(\frac{2}{2}, \frac{0}{2}\right)$

$* \; M = (1, 0)$

Equation of line in point-slope form

$y - y_1 = m(x - x_1)$

$y - 0 = -3(x - 1)$

Equation of line in slope intercept form

$y = mx + b$

$y - 0 = -3(x - 1)$

$y = -3x + 3$

(1d) When you are sure of your perpendicular bisector lines, draw all three lines IN PENCIL on your graph. When you are certain that these are correct, then use colored pencils or markers to carefully mark these lines on the graph. Use the same color for all the perpendicular bisectors. Add this color to your key on the graph.

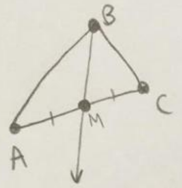
Group 1

(2) Finding the Centroid:

Find the equation of each median of the triangle.

(2a) Median from B to midpoint of $\overline{AC}$ A(-2, -1), B(1,5), C(4,1)

Sketch



Midpoint of $\overline{AC}$

$M = (1,0)$

Slope of median from B to midpoint of $\overline{AC}$

$\overset{x_1\ y_1}{(1,5)}$ $\overset{x_2\ y_2}{(1,0)}$

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{0-5}{1-1} = \frac{-5}{0} = \phi$

undefined

Equation of line in point-slope form

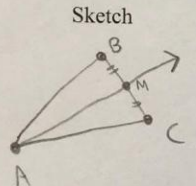
$y-y_1=m(x-x_1)$ $x=1$

Equation of line in slope intercept form

$y=mx+b$ $x=1$

(2b) Median from A to midpoint of $\overline{BC}$ A(-2, -1), B(1,5), C(4,1)

Sketch



Midpoint of $\overline{BC}$

$M = \left(\frac{5}{2}, 3\right)$

Slope of median from A to midpoint of $\overline{BC}$

$\overset{x_1\ y_1}{(-2,-1)}$ $\overset{x_2\ y_2}{\left(\frac{5}{2},3\right)}$

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{3+1}{\frac{5}{2}+\frac{2\cdot 2}{1\cdot 2}} = \frac{4}{\frac{9}{2}} = 4 \div \frac{9}{2} = 4 \cdot \frac{2}{9} = \boxed{\frac{8}{9}}$

$= .\overline{8}$

Equation of line in point-slope form

$y-y_1=m(x-x_1)$

point A $y+1 = \frac{8}{9}(x+2)$

OR

midpoint $y-3 = \frac{8}{9}\left(x-\frac{5}{2}\right)$

Equation of line in slope intercept form

$y=mx+b$

$y+1 = \frac{8}{9}(x+2)$

$y+1 = \frac{8}{9}x + \frac{16}{9}$

$-1 \qquad -1$

$y = \frac{8}{9}x + \frac{16}{9} - \frac{9}{9}$

$y = \frac{8}{9}x + \frac{7}{9}$

$y-3 = \frac{8}{9}\left(x-\frac{5}{2}\right)$

$y-3 = \frac{8}{9}x - \frac{20}{9}$

$+3 \qquad +3$

$y = \frac{8}{9}x - \frac{20}{9} + \frac{27}{9}$

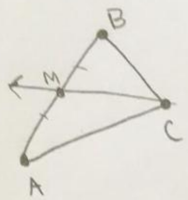
$y = \frac{8}{9}x + \frac{7}{9}$

$y = .\overline{8}x + .\overline{7}$

Group 1

(2c) Median from C to midpoint of $\overline{AB}$ A(-2, -1), B(1,5), C(4,1)

Sketch



Midpoint of $\overline{AB}$

$M = \left(-\frac{1}{2}, 2\right)$

Slope of median from C to midpoint of $\overline{AB}$

$\overset{x_1,\ y_1}{(4,1)}$ $\overset{x_2\ \ y_2}{\left(-\frac{1}{2},2\right)}$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{2-1}{-\frac{1}{2}-4} = \frac{1}{-\frac{1}{2}-\frac{8}{2}} = \frac{1}{-\frac{9}{2}} = 1 \cdot \frac{-2}{9} = -\frac{2}{9}$$

$$= -.\overline{2}$$

Equation of line in point-slope form

$y - y_1 = m(x - x_1)$

point C OR point M

$y-1 = -\frac{2}{9}(x-4)$ $y-2 = -\frac{2}{9}\left(x+\frac{1}{2}\right)$

Equation of line in slope intercept form

$y = mx + b$

$y-1 = \frac{-2}{9}(x-4)$

$y-1 = \frac{-2}{9}x + \frac{8}{9}$

$+1 \quad +1$

$y = -\frac{2}{9}x + \frac{17}{9}$ $y = -\frac{2}{9}x + 1\frac{8}{9}$

$y-2 = -\frac{2}{9}\left(x+\frac{1}{2}\right)$

$y-2 = \frac{-2}{9}x - \frac{1}{9}$

$+2 \quad +2$

$y = \frac{-2}{9}x + 1\frac{8}{9}$

(2d) When you are sure of your median lines, draw all three lines IN PENCIL on your graph. When you are certain that these are correct, then use colored pencils or markers to carefully mark these lines on the graph. Use the same color for all the medians. Add this color to your key on the graph.

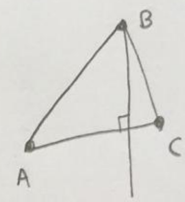
Group 1

(3) Finding the Orthocenter

Find the equation of each altitude of the triangle.

(3a) Altitude from B to $\overline{AC}$

sketch



Slope of $\overline{AC}$: $\frac{1}{3}$

Slope of line perpendicular to $\overline{AC}$: -3

Coordinates of B: $(1,5)$

Equation of line in point-slope form: $y-5=-3(x-1)$

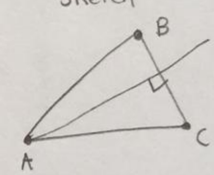
$y-y_1=m(x-x_1)$

Equation of line in slope intercept form: $y=-3x+8$

$y=mx+b$ $\quad$ $y-5=-3x+3$ $\quad$ $+5 \quad +5$

(3b) Altitude from A to $\overline{BC}$

sketch



Slope of $\overline{BC}$: $-\frac{4}{3}$

Slope of line perpendicular to $\overline{BC}$: $\frac{3}{4}$

Coordinates of A: $(-2,-1)$

Equation of line in point-slope form: $y+1=\frac{3}{4}(x+2)$

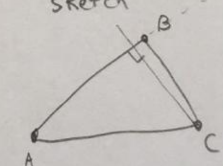
$y-y_1=m(x-x_1)$

Equation of line in slope intercept form: $y=\frac{3}{4}x+\frac{1}{2}$

$y=mx+b$ $\quad$ $y+1=\frac{3}{4}x+\frac{3}{2}$ $\quad$ $-1 \quad -1$

(3c) Altitude from C to $\overline{AB}$

sketch



Slope of $\overline{AB}$: 2

Slope of line perpendicular to $\overline{AB}$: $-\frac{1}{2}$

Coordinates of C: $(4,1)$

Equation of line in point-slope form: $y-1=-\frac{1}{2}(x-4)$

$y-y_1=m(x-x_1)$

Equation of line in slope intercept form: $y=-\frac{1}{2}x+3$

$y=mx+b$ $\quad$ $y-1=-\frac{1}{2}x+2$ $\quad$ $+1 \quad +1$

(3d) When you are sure of your altitude lines, draw all three lines IN PENCIL on your graph. When you are certain that these are correct, then use colored pencils or markers to carefully mark these lines on the graph. Use the same color for all the altitudes. Add this color to your key on the graph.

Group 1 $A(-2,-1)$ $B(1,5)$ $C(4,1)$

(4) Coordinate Points of the Circumcenter, Median, and Orthocenter

(4a) Use the graph to determine the **circumcenter** of ΔABC (this is the point of concurrency of the perpendicular bisectors).

$\left(\frac{1}{2}, \frac{3}{2}\right)$ $\left(\frac{1}{2}, 1.5\right)$

Circumcenter = $\left(\frac{1}{2}, 1\frac{1}{2}\right)$

On the graph paper: Use the same color as you did for all the perpendicular bisectors. Plot and label this point on the graph. Add this point to your key on the graph.

(4b) Use the graph to determine the **centroid** of ΔABC (this is the point of concurrency of the medians).

$(1, 1.\overline{6})$ $\left(1, 1\frac{2}{3}\right)$

Centroid $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = \left(\frac{-2+1+4}{3}, \frac{-1+5+1}{3}\right)$

$= \left(\frac{3}{3}, \frac{5}{3}\right) = \left(1, \frac{5}{3}\right)$

Centroid = $\left(1, \frac{5}{3}\right)$

If you can't identify the exact point, use the centroid formula.

On the graph paper: Use the same color as you did for all the medians. Plot and label this point on the graph. Add this point to your key on the graph.

(4c) Use the graph to determine the **orthocenter** of ΔABC (this is the point of concurrency of the altitudes).

Orthocenter = $(2,2)$

On the graph paper: Use the same color as you did for all the altitudes. Plot and label this point on the graph. Add this point to your key on the graph.

Group 1

(5) Slopes Between the Points

(5a) Find the slope of the line between the **circumcenter** and the **centroid** of ΔABC.

$x_1\ y_1$ $\left(\frac{1}{2}, \frac{3}{2}\right)$ $x_2\ y_2$ $\left(1, \frac{5}{3}\right)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\left(\frac{5}{3} - \frac{3}{2}\right)}{\left(1 - \frac{1}{2}\right)} = \frac{1}{3}$

Slope = $\frac{1}{3}$

(5b) Find the slope of the line between the **orthocenter** and the **centroid** of ΔABC.

$x_1\ y_1$ $(2, 2)$ $x_2\ y_2$ $\left(1, \frac{5}{3}\right)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\left(\frac{5}{3} - 2\right)}{(1 - 2)} = \frac{1}{3}$

Slope = $\frac{1}{3}$

(5c) From steps 5a and 5b, we see that the circumcenter, centroid, and orthocenter of ΔABC

are collinear.

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(6) Euler Line Equation: The Euler Line passes through the circumcenter, centroid, and orthocenter of a triangle. Find the equation of this line.

$m = \frac{1}{3}$ point: $(2,2)$

a) Equation of Euler Line in point-slope form: $y - 2 = \frac{1}{3}(x - 2)$

$y - y_1 = m(x - x_1)$

b) Equation of Euler Line in slope intercept form: $y = \frac{1}{3}x + \frac{4}{3}$

$y = mx + b$ $y - 2 = \frac{1}{3}x - \frac{2}{3}$
$+2$ $+2 \cdot \frac{3}{3}$

c) Graph the Euler Line. Use a new color to mark this line on the graph. Add this color and line to your key.

Group 1

Rubric	30 points possible
Triangle graphed neatly and labeled with correct vertices	1
Equations of three perpendicular bisectors with necessary slopes and points identified (Neat work shown)	7
Equation of three medians with necessary slopes and points identified (Neat work shown)	7
Equations of three altitudes with necessary slopes and points identified (Neat work shown)	7
Graphs of perpendicular bisectors, medians, and altitudes (neat, accurate, colored, keyed)	3
Coordinates of circumcenter, centroid, and orthocenter identified	1
Circumcenter, centroid, and orthocenter labeled neatly on graph	1
Slope of line through circumcenter and centroid / slope of line through centroid and orthocenter (Neat work shown)	1
Equation of Euler line (Neat work shown)	1
Graph of Euler line (Neat, accurate, colored, keyed)	1