Lesson 2: Side lengths and areas

Goals

- Comprehend the term “square root of a” (in spoken language) and the notation $\sqrt{a}$ (in written language) to mean the side length of a square whose area is a square units.
- Create a table and graph that represents the relationship between side length and area of a square, and use the graph to estimate the side lengths of squares with non-integer side lengths.
- Determine the exact side length of a square and express it (in writing) using square root notation.

Learning Targets

- I can explain what a square root is.
- If I know the area of a square, I can express its side length using square root notation.
- I understand the meaning of expressions like $\sqrt{25}$ and $\sqrt{3}$.

Lesson Narrative

In this lesson, students learn about square roots. The warm-up helps them see a single line segment as it relates to two different figures: as a side length of a triangle and as a radius of a circle. In the next activity, they use this insight to estimate the side length of a square via a geometric construction that relates the side length of the square to a point on the number line, and verify their estimate using techniques from the previous lesson. Once students locate the side length of the square as a point on the number line, they are formally introduced to **square roots** and square root notation:

$\sqrt{a}$ is the length of a side of a square whose area is a square units.

In the final activity, students use the graph of the function $y = x^2$ to estimate side lengths of squares with integer areas but non-integer side lengths.

Building On

- Classify two-dimensional figures into categories based on their properties.

Addressing

- Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
- Use functions to model relationships between quantities.
- Know that there are numbers that are not rational, and approximate them by rational numbers.

Building Towards

- Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
- Know that there are numbers that are not rational, and approximate them by rational numbers.

Instructional Routines

- Collect and Display
- Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Materials

Four-function calculators
Geometry toolkits
tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Student Learning Goals

Let's investigate some more squares.

2.1 Notice and Wonder: Intersecting Circles

Warm Up: 5 minutes

The purpose of this warm-up is to get students in the habit of seeing the same line segment being a part of two different figures. In this case, the sides of the triangle are also radii of the circles. This primes them to see the sides of the square in the next activity as the radius of a circle.

Instructional Routines

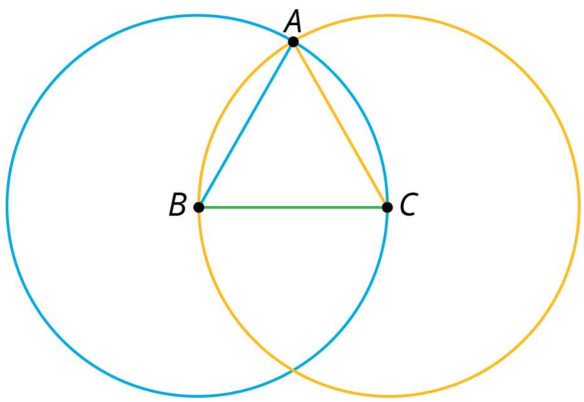
- Notice and Wonder

Launch

Arrange students in groups of 2. Display the diagram for all to see. Give students 1 minute of quiet work time to identify at least one thing they notice and at least one thing they wonder about the diagram. Ask students to give a signal when they have noticed or wondered about something. When the minute is up, give students 1 minute to discuss their observations and questions with their partner. Follow with a whole-class discussion.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Responses vary. Sample responses:

I notice that:

- There are two circles that intersect.
- The colour of two of the sides of the triangle match the colour of the circles and the third is a mixture.
- Segment AB is a radius of circle B.
- Segment AC is a radius of circle C.
- Segment BC is a radius of both circles.
- The three radii form an equilateral triangle.

I wonder:

- Why it is coloured the way it is.
- If BC is the radius of one of the circles.
- If triangle ABC is equilateral.
- If this diagram is going to help me with anything.

Activity Synthesis

A segment can be a part of more than one figure. In this case, the sides of the triangle are also the radii of the circles. Being able to see parts of a figure in more than one way is helpful for solving problems.

2.2 One Square

15 minutes

The purpose of this activity is for students to estimate the side length of a square via a geometric construction that relates the side length of the square to a point on the number line, and verify their estimate using techniques from the previous lesson. Once students connect the side length to a point on the number line, they learn that this number has a name and a special notation to denote it: *square root* and the square root symbol. While this is the students' first formal introduction to square roots, they will have many opportunities to deepen their understanding of square roots and practise using square root notation in later activities and lessons.

Instructional Routines

- Collect and Display
- Think Pair Share

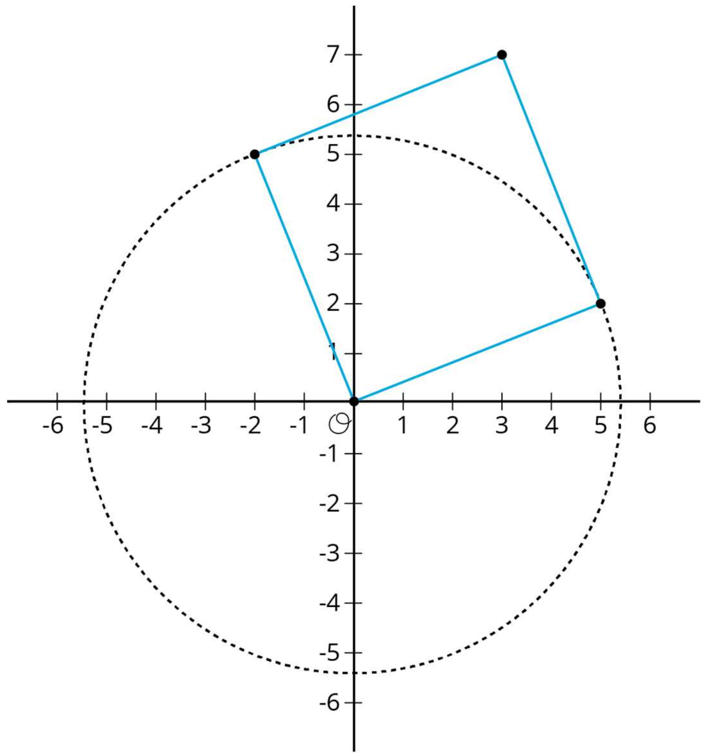
Launch

Students in groups of 2. Give 1 minute quiet work time on the first problem and have students check in with a partner. Students continue to work. Follow with a whole-class discussion.

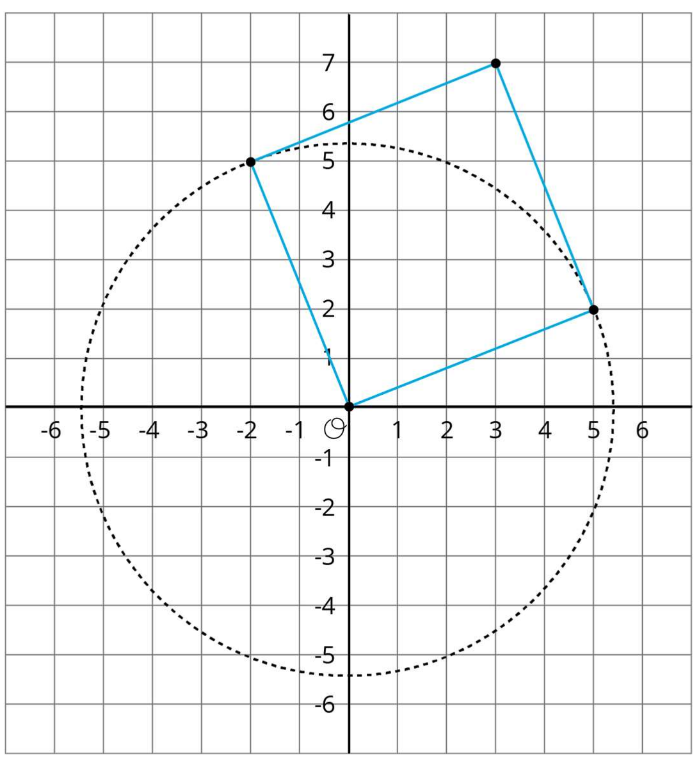
Conversing, Reading: Collect and Display. As students work in pairs on the task, circulate and listen as they discuss how to estimate the area of the square. Record and display the words and phrases students use, as well as any helpful sketches or diagrams. As students review the language collected in the visual display, encourage students to clarify the meaning of a word or phrase. For example, a phrase such as "the side is the same as the radius" can be clarified by restating it as "the side length of the square is equal to the radius of the circle." Encourage students to refer back to the visual display during whole-class discussions throughout the lesson and unit. This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

1. Use the circle to estimate the area of the square shown here:



2. Use the grid to check your answer to the first problem.



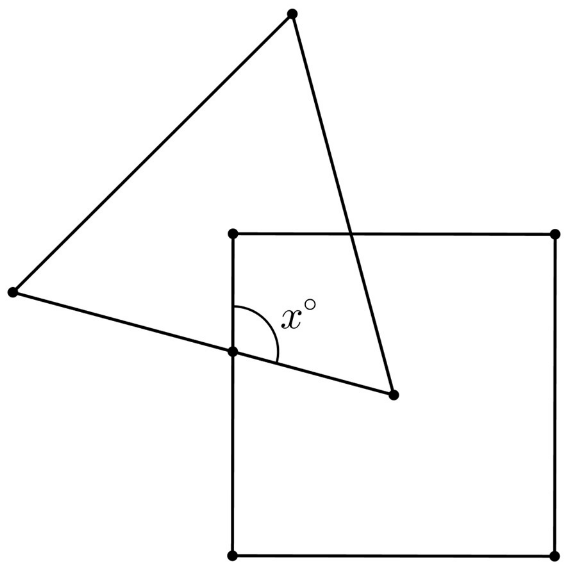
Student Response

1. Answers vary. Sample response: Approximately 28 square units. The radius of the circle is about 5.3 units, which means the area of the square is 5.3^2 units.

2. 29 square units.

Are You Ready for More?

One vertex of the equilateral triangle is in the centre of the square, and one vertex of the square is in the centre of the equilateral triangle. What is x?



Student Response

Draw a segment to connect the centre of the square to the centre of the equilateral triangle. This segment cuts the 90 degree angle at the centre of the equilateral triangle in half because a line from a vertex of an equilateral triangle through its centre is a line of symmetry. In the same way, the segment cuts the 60 degree angle at the centre of the square in half. So this segment creates a new triangle with angles 45 degrees, x degrees, and 30 degrees. Since the angles in a triangle must sum to 180 degrees, x must be equal to 105 degrees.

Activity Synthesis

Select students to share how they determined the areas in each problem and how their answers compared.

Ask, "What do you think the actual side length of the square is? That is, what is the number that when squared is equal to 29?" If not brought up in students' explanations, point out that while 5.3^2 is only 28.09, 5.35^2 is 28.6225, a number closer to 29, and ask what number they might try to square next.

Tell students that in the previous lesson and this lesson, we have seen squares that have areas that are whole numbers, but the side lengths are not whole numbers. In this activity, we saw that we can locate a point on the number line (the x-axis is a number line) that corresponds to the side length, and that this number is a *square root*, which we write like this:

$\sqrt{29}$

This is the way we write the exact length of the side of a square with area 29 square units. So $\left(\sqrt{29}\right)^2 = 29$.

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: square root.
Supports accessibility for: Memory; Language

2.3 The Sides and Areas of Tilted Squares

15 minutes (there is a digital version of this activity)

In this activity, students continue to develop their understanding of square roots. Students first find the areas of three squares, estimate the side lengths using tracing paper, and then write the exact side lengths. Then they make a table of side-area pairs. They then graph the ordered pairs from the table and use the graph to estimate the values of squares with non-integer side lengths, such as the ones they drew in the previous activity.

Instructional Routines

- Discussion Supports
- Think Pair Share

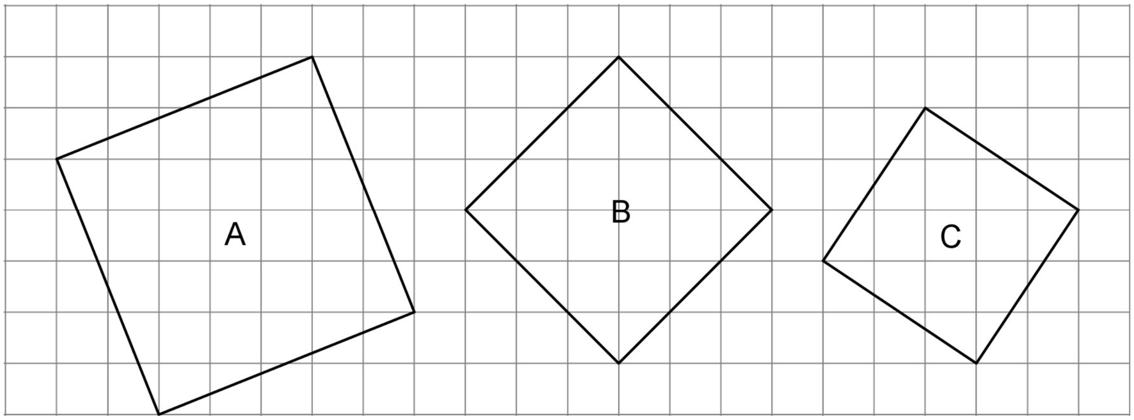
Launch

Arrange students in groups of 2. Provide access to calculators. Remind students about the meaning and use of square root notation. Have students work together on the first problem and check each other's work. Have them make their graphs independently and then check with their partners that they look the same. Follow with a whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts. Check in with students after the first 2-3 minutes of work time. Check to make sure students have attended to all parts of the original figures.
Supports accessibility for: Organisation; Attention

Student Task Statement

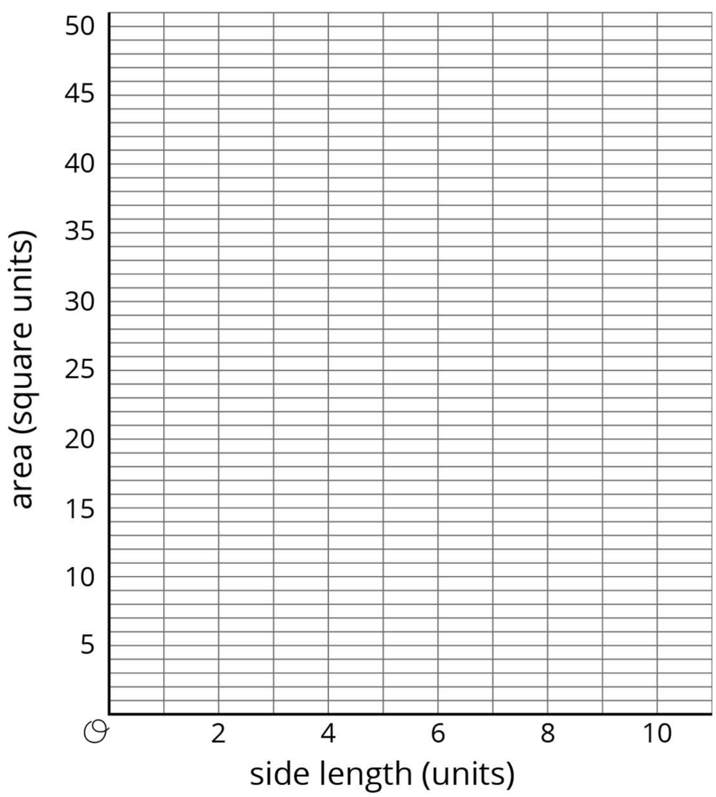
1. Find the area of each square and estimate the side lengths using your geometry toolkit. Then write the exact lengths for the sides of each square.



2. Complete the tables with the missing side lengths and areas.

side length, s	0.5		1.5		2.5		3.5	
area, a		1		4		9		16
side length, s	4.5		5.5		6.5		7.5	
area, a		25		36		49		64

3. Plot the points, (s, a), on the coordinate grid shown here.



4. Use this graph to estimate the side lengths of the squares in the first question. How do your estimates from the graph compare to the estimates you made initially using your geometry toolkit?

5. Use the graph to approximate $\sqrt{45}$.

Student Response

1. Answers for estimates vary. Sample response:
   A: Area is 29 square units and side length is between 5 and 6 units. $s = \sqrt{29}$
   B: Area is 18 square units and side length is between 4 and 5 units. $s = \sqrt{18}$
   C: Area is 13 square units and side length is between 3 and 4 units. $s = \sqrt{13}$

2. Complete the table with the missing side lengths and areas.

side length, s	0.5	1	1.5	2	2.5	3	3.5	4
area, a	0.25	1	2.25	4	6.25	9	12.25	16
side length, s	4.5	5	5.5	6	6.5	7	7.5	8
area, a	20.25	25	30.25	36	42.25	49	56.25	64

3. All of the points (s, a) lie on the graph of the equation $y = x^2$.

4. Answers vary, but should be between the correct adjacent values in the table. Estimates should be a little more precise than when made with tracing paper.

5. $\sqrt{45}$ is between 6.5 and 7.

Activity Synthesis

Invite students to share their graphs from the second problem and display the graph for all to see. Ask: “What relationship does the graph display?” (The relationship between the side length and area of a square.) Select 2–3 students to share their answers to the last problem, including how they used the graph to estimate s for the tilted square they drew earlier.

Speaking: Discussion Supports. Use this routine to support whole-class discussion. After a student shares their response to the last problem, ask students to restate and/or revoice what they heard using mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. *Design Principle(s): Support sense-making*

Lesson Synthesis

The purpose of this discussion is to check that students understand the definition of square roots as they relate to side lengths of squares. Here are possible questions for discussion:

- "What does it mean when we write $\sqrt{100} = 10$ in terms of squares and side lengths?" (It means that a square with area 100 has side lengths of 10.)
- "If $\sqrt{17}$ is a side length of a square, what does that mean about the area?" (The area is 17 square units.)
- "Look at the graph of area as a function of side length. Should the points on the graph be connected? What would that mean in terms of squares and side lengths?" (Yes, the points should be connected because it's possible to have any positive value as a side length and then squaring that value gives the area of the square with that specific side length.)

2.4 What Is the Side Length?

Cool Down: 5 minutes

Student Task Statement

Write the exact value of the side length of a square with an area of

1. 100 square units.
2. 95 square units.
3. 36 square units.
4. 30 square units.

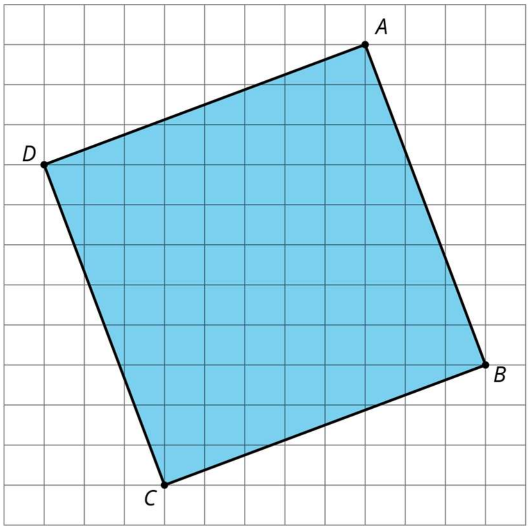
If the exact value is not a whole number, estimate the length.

Student Response

1. The side length is exactly 10 units since $10^2 = 100$. (Students might also note that $\sqrt{100} = 10$.)
2. The exact value is $\sqrt{95}$ units. It will be a little bit less than 10.
3. The side length is exactly 6 units since $6^2 = 36$. (Students might also note that $\sqrt{36} = 6$.)
4. The exact value is $\sqrt{30}$ units. It will be between 5 and 6 units.

Student Lesson Summary

We saw earlier that the area of square ABCD is 73 units^2.



What is the side length? The area is between $8^2 = 64$ and $9^2 = 81$, so the side length must be between 8 units and 9 units. We can also use tracing paper to trace a side length and compare it to the grid, which also shows the side length is between 8 units and 9 units. But we want to be able to talk about its *exact* length. In order to write "the side length of a square whose area is 73 square units," we use the **square root** symbol. "The square root of 73" is written $\sqrt{73}$, and it means "the length of a side of a square whose area is 73 square units."

We say the side length of a square with area 73 units^2 is $\sqrt{73}$ units. This means that

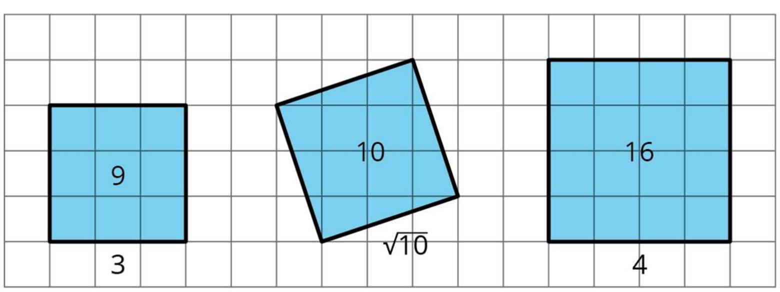
$\left(\sqrt{73}\right)^2 = 73$

All of these statements are also true:

$\sqrt{9} = 3$ because $3^2 = 9$

$\sqrt{16} = 4$ because $4^2 = 16$

$\sqrt{10}$ units is the side length of a square whose area is 10 units^2, and $\left(\sqrt{10}\right)^2 = 10$



Glossary

- square root

Lesson 2 Practice Problems

1. **Problem 1 Statement**

   A square has an area of 81 square feet. Select **all** the expressions that equal the side length of this square, in feet.

   a. $\frac{81}{2}$

   b. $\sqrt{81}$

   c. 9

   d. $\sqrt{9}$

   e. 3

   Solution ["B", "C"]

2. **Problem 2 Statement**

   Write the exact value of the side length, in units, of a square whose area in square units is:

   a. 36

   b. 37

   c. $\frac{100}{9}$

   d. $\frac{2}{5}$

   e. 0.0001

   f. 0.11

   Solution

   a. 6

   b. $\sqrt{37}$

   c. $\frac{10}{3}$

   d. $\sqrt{\frac{2}{5}}$

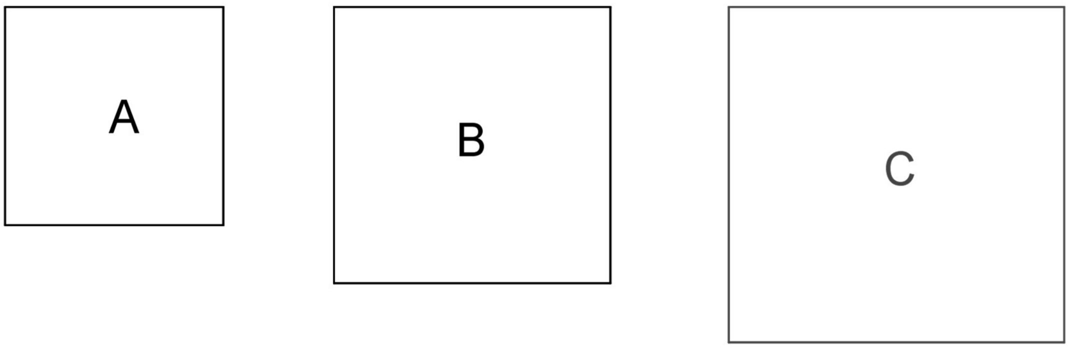
   e. 0.01

f. $\sqrt{0.11}$

3. **Problem 3 Statement**

Square A is smaller than Square B. Square B is smaller than Square C.

The three squares' side lengths are $\sqrt{26}$, 4.2, and $\sqrt{11}$.



What is the side length of Square A? Square B? Square C? Explain how you know.

Solution

Square A: $\sqrt{11}$ units, Square B: 4.2 units, Square C: $\sqrt{26}$. I know this because $\sqrt{11}$ is between 3 and 4 and $\sqrt{26}$ is between 5 and 6, so $\sqrt{11} < 4.2 < \sqrt{26}$ and the side length of A is less than the side length of B is less than the side length of C.

4. **Problem 4 Statement**

Find the area of a square if its side length is:

a. $\frac{1}{5}$ cm

b. $\frac{3}{7}$ units

c. $\frac{11}{8}$ inches

d. 0.1 metres

e. 3.5 cm

Solution

a. $\frac{1}{25}$ cm^2

b. $\frac{9}{49}$ square units

c. $\frac{121}{64}$ square inches

d. 0.01 square metres

e. 12.25 cm^2

5. **Problem 5 Statement**

Here is a table showing the areas of the seven largest countries.

a. How much larger is Russia than Canada?

b. The Asian countries on this list are Russia, China, and India. The American countries are Canada, the United States, and Brazil. Which has the greater total area: the three Asian countries, or the three American countries?

country	area (in km^2)
Russia	1.71×10^7
Canada	9.98×10^6
China	9.60×10^6
United States	9.53×10^6
Brazil	8.52×10^6
Australia	6.79×10^6
India	3.29×10^6

Solution

a. 7.12×10^6 km^2

b. The Asian countries (2.999×10^7 vs. 2.803×10^7)

6. **Problem 6 Statement**

Select **all** the expressions that are equivalent to 10^{-6}.

a. $\frac{1}{1\,000\,000}$

b. $\frac{-1}{1\,000\,000}$

c. $\frac{1}{10^6}$

d. $10^8 \times 10^{-2}$

e. $\left(\frac{1}{10}\right)^6$

f. $\frac{1}{10\times10\times10\times10\times10\times10}$

Solution ["A", "C", "E", "F"]