Lesson 10: Introducing graphs of proportional relationships

Goals

- Compare and contrast (orally) graphs of relationships.
- Generalise (orally and in writing) that a proportional relationship can be represented in the coordinate grid by a line that includes the "origin" or by a collection of points that lie on such a line.
- Justify (orally) that a table and a graph represent the same relationship.

Learning Targets

- I know that the graph of a proportional relationship lies on a line through $(0,0)$.

Lesson Narrative

This lesson introduces an important way of representing a proportional relationship: its graph. Students plot points on the graph from tables, and, by the end of the lesson, start to see that the graph of a proportional relationship always lies on a line that passes through (0,0). They match tables and graphs of given situations and articulate their reasons for each match.

Building On

- Graph points on the coordinate grid to solve real-world and mathematical problems.
- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate grid. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Addressing

- Recognise and represent proportional relationships between quantities.
- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate grid and observing whether the graph is a straight line through the origin.

Building Towards

- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate grid and observing whether the graph is a straight line through the origin.

Instructional Routines

- Stronger and Clearer Each Time
- Discussion Supports
- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the blackline master
Rulers

Required Preparation

Prepare the Matching Tables and Graphs activity by printing one copy for each group of 2 students and cutting them up ahead of time. Prepare a few copies of an answer key and place them in envelopes for students to access to check their work when they finish.

Student Learning Goals

Let's see how graphs of proportional relationships differ from graphs of other relationships.

10.1 Notice These Points

Warm Up: 5 minutes (there is a digital version of this activity)

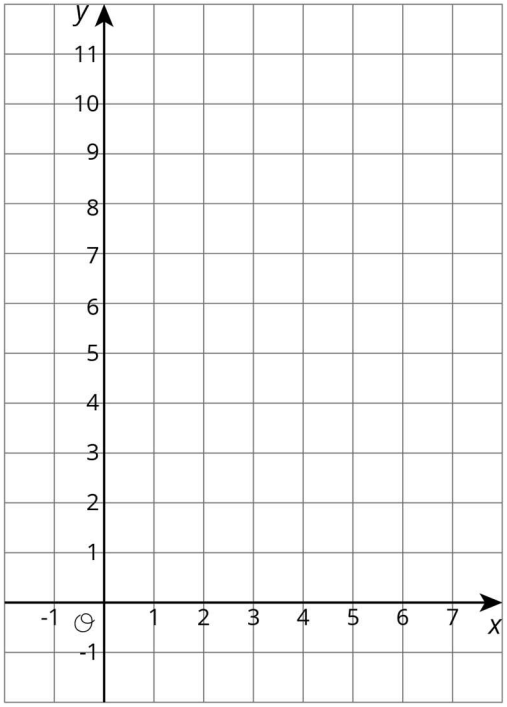
This warm-up prepares students for graphing proportional relationships in the coordinate grid. They practice graphing coordinate points and notice that all points lie on a straight line.

Launch

Give students 3 minutes quiet work time followed by a whole-class discussion.

Student Task Statement

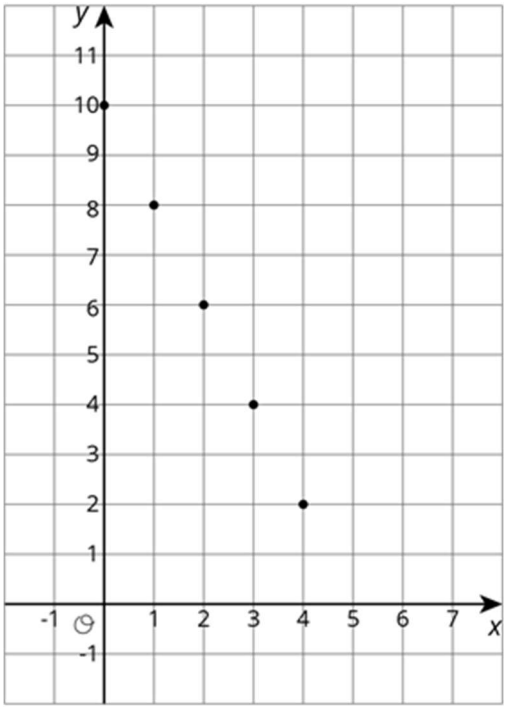
1. Plot the points (0,10), (1,8), (2,6), (3,4), (4,2).



2. What do you notice about the graph?

Student Response

1.



2. Answers vary. Sample responses:

   - The points line up so that they could all be connected with a single line.
   - The line goes down when reading left to right.
   - Every time the x-coordinate goes up 1, the y-coordinate goes down 2.

Activity Synthesis

Invite students to share their observations. Ask if other students agree. If some students do not agree that the points lie on a straight line, ask which points break the pattern and give students a chance to self-correct their work.

10.2 T-shirts for Sale

10 minutes (there is a digital version of this activity)

This introductory activity asks students to plot points using tables of values that represent scenarios familiar from previous lessons. This activity is intended as a review of the coordinate grid, its axes, and plotting ordered pairs.

Instructional Routines

- Discussion Supports

- Think Pair Share

Launch

Arrange students in groups of 2. Provide access to rulers. Give students 5 minutes of quiet work time followed by students discussing responses with a partner, followed by whole-class discussion.

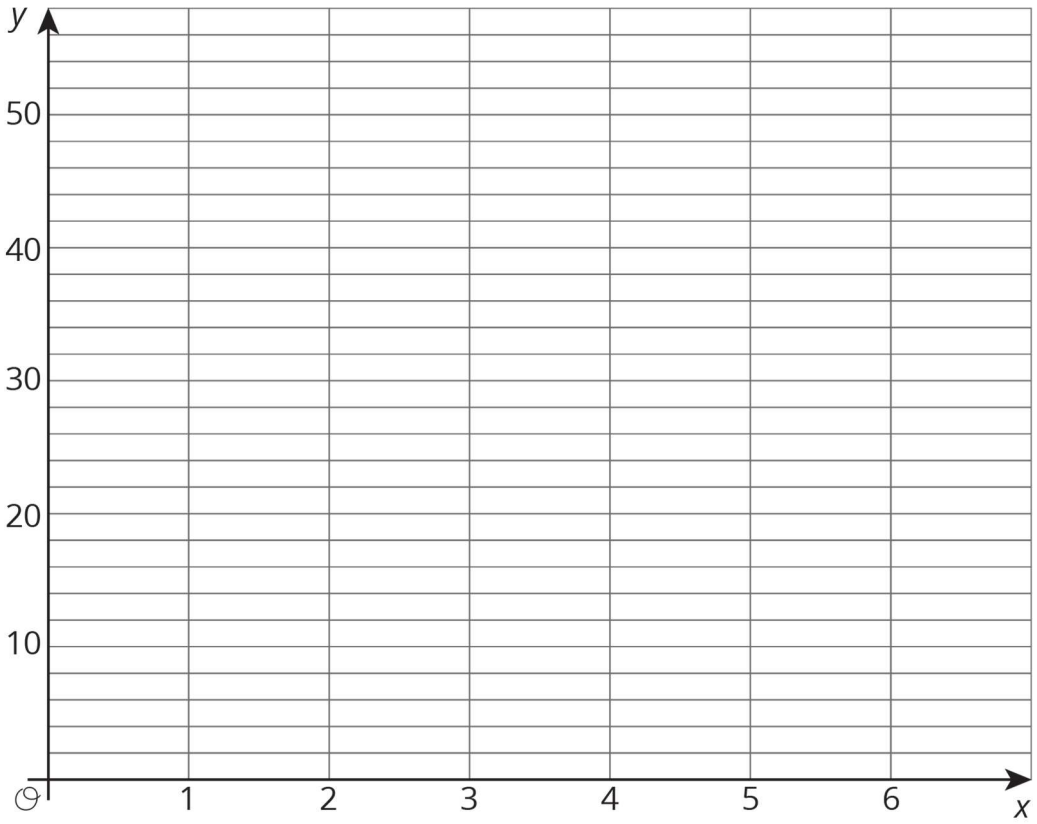
The second question reviews earlier work, but it is important that students understand how to plot points in the coordinate grid. Display the graph for all to see. Show students how to plot the pair from the first row in the table in the coordinate grid. Ask students to plot the remaining pairs and check with nearby students as they work. Be on the lookout for students plotting coordinates in the wrong order.

Student Task Statement

Some T-shirts cost £8 each.

x	y
1	8
2	16
3	24
4	32
5	40
6	48

1. Use the table to answer these questions.
    a. What does x represent?
    b. What does y represent?
    c. Is there a proportional relationship between x and y?
2. Plot the pairs in the table on the **coordinate grid**.



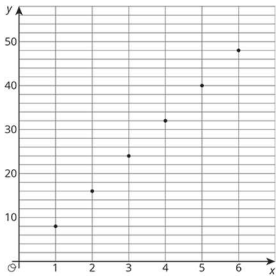
3. What do you notice about the graph?

Student Response

a. x is the number of T-shirts

b. y is the total cost of those T-shirts

c. x is proportional to y. Students may identify 8 as the constant of proportionality.

1. 

2. Students may notice that the points lie on a line.

Activity Synthesis

Discuss the first question if students had trouble with it.

Ask students to share their observations about the plotted points. Ask, "Could we buy 0 shirts? 7 T-shirts? 10 T-shirts? Can we buy half of a T-shirt?" Note that the graph consists of discrete points because only whole numbers of T-shirts make sense in this context; however, people often connect discrete points with a line to make the relationship more clear, even when the in-between values don't make sense.

Ask the students, "Suppose instead of price per shirt, this graph displayed the cost of cherries that are £8 per pound. Given that context, how should we change the graph?" Weights need not have integer values, so the graph is not restricted to discrete points. If you haven't done so already, draw the ray starting at (0,0) that passes through the points.

Speaking: Discussion Supports. As students describe their observations about the proportional relationship represented in the graph, revoice student ideas to demonstrate mathematical language use. Press for details in students' explanations by requesting that students challenge an idea, elaborate on an idea, or give an example. Show central concepts multi-modally by using different types of sensory inputs: acting out scenarios or inviting students to do so, using gestures, and talking about the context of selling T-shirts or cherries. This will help students to produce and make sense of the language needed to communicate their own ideas.
Design Principle(s): Support sense-making, Optimise output (for explanation)

10.3 Matching Tables and Graphs

25 minutes

Students work in pairs to match tables to graphs and to practice articulating their reasoning. This task is intended to foster understanding of correspondences between tables and graphs.

Students sort the graphs and justify their sorting schemes. Then, they compare the way they sorted their graphs with a different group. The purpose of this activity is to illustrate the idea that the graph of a proportional relationships is a line through the **origin**. Students will not have the tools for a formal explanation until later in KS3.

Demonstrate how the matching activity works and how to have mathematical dialogue about the decisions being made (see the instructions in the task statement). When students finish the activity, they use an answer key to check their answers. If adjustments need to be made, students discuss any errors they made.

While students are working to match graphs to written descriptions and tables, circulate and ask them to justify their choices.

Instructional Routines

Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Place copies of answer keys in envelopes.

Ask students to observe similarities and differences in the graphs and to create a rationale for sorting them. Provide access to sticky notes. If necessary, specify how the groups will trade places after they finish sorting their graphs into categories.

Demonstrate how to set up and conduct the matching activity. Choose a student to act as your partner. Mix up the cards and place them face-up. Point out that the cards contain either tables or graphs. Select one of each style of card and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree, e.g. by explaining your mathematical thinking, asking clarifying questions, etc.

Give each group cut-up slips for matching. Tell students to check their matches after they complete the activity using the answer keys.

Representation: Internalise Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual processing; Organisation Speaking and listening: Stronger and Clearer Each Time. Use this with successive pair shares to give students a structured opportunity to revise and refine their response to “Which of the relationships are proportional?” Ask each student to meet with 2–3 other partners in a row for feedback. Display prompts for feedback that students can use to help their partner strengthen and clarify their ideas. For example, "Your explanation tells me . . .", "Can you say more about why you . . . ?", and "A detail (or word) you could add is _____, because . . . ." Give students with 3–4 minutes to revise their initial draft based on feedback from their peers.
Design Principle(s): Optimise output (for justification)

Anticipated Misconceptions

If students struggle to get started making any matches, ask questions like “How would we expect this row in the table to look on the graph?” Or, “See this point on the graph? What corresponds to it in the table?”

A common misunderstanding is to assume that if the points lie on a line, then the graph represents a proportional relationship. Ask questions about the table to assist students in realising the error.

Student Task Statement

Your teacher will give you papers showing tables and graphs.

1. Examine the graphs closely. What is the same and what is different about the graphs?
2. Sort the graphs into categories of your choosing. Label each category. Be prepared to explain why you sorted the graphs the way you did.
3. Take turns with a partner to match a table with a graph.
    a. For each match you find, explain to your partner how you know it is a match.

b. For each match your partner finds, listen carefully to their explanation. If you disagree, work to reach an agreement.

Pause here so your teacher can review your work.

4. Trade places with another group. How are their categories the same as your group's categories? How are they different?

5. Return to your original place. Discuss any changes you may wish to make to your categories based on what the other group did.

6. Which of the relationships are proportional?

7. What have you noticed about the graphs of proportional relationships? Do you think this will hold true for *all* graphs of proportional relationships?

Student Response

1. Answers vary. Sample response: All of the graphs have points that can be connected by a single, straight line. Some of the graphs will go through $(0,0)$, but others will not.

2. Answers vary. Many students will sort the graphs into proportional and nonproportional. Some students may add more categories (i.e., not straight, straight but not proportional, proportional).

3. Here are the correct matches: 1H, 2B, 3G, 4D, 5A, 6E, 7F, 8I, 9C, 10J.

4. Answers vary.

5. No response required.

6. The proportional relationships are 2B, 4D, 7F, 8I, and 9C

7. Answers vary. Possible responses: All points on a graph of a proportional relationship lie on a line. All such lines pass through $(0,0)$. The constant of proportionality can be seen in the graphs as the y-coordinate when x is 1.

Are You Ready for More?

1. All the graphs in this activity show points where both coordinates are positive. Would it make sense for any of them to have one or more coordinates that are negative?

2. The equation of a proportional relationship is of the form $y = kx$, where k is a positive number, and the graph is a line through $(0,0)$. What would the graph look like if k were a negative number?

Student Response

1. The temperature graph could have negative coordinates because temperatures can be negative.

2. The line would still be through the origin, but it would slant downward from left to right.

1. When you buy two shirts, you get the second one at half-price.

x	y
1	10
2	15
3	25
4	30
5	40
6	45

2. These t-shirts cost £8 each.

x	y
1	8
2	16
4	32
5	40
7	56
8	64

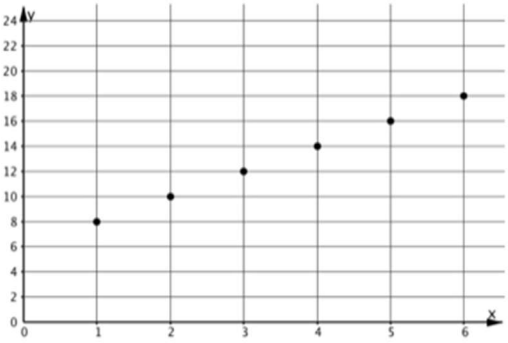
3. In the science lab there is a chart to help students convert temperatures from Celsius to Fahrenheit.

x	y
0	32
10	50
20	68
30	86
40	104
50	122

4. She is planning on serving $\frac{1}{3}$ cup of rice per person.

x	y
1	$\frac{1}{3}$
2	$\frac{2}{3}$
3	1
4	$1\frac{1}{3}$
5	$1\frac{2}{3}$
6	2

A.



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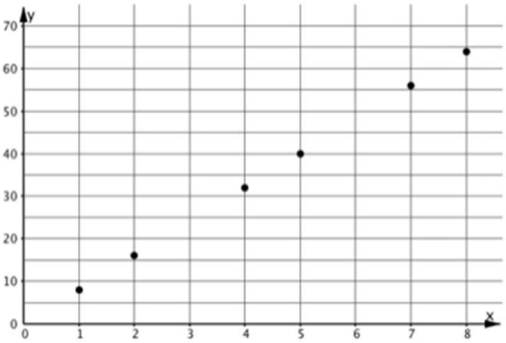

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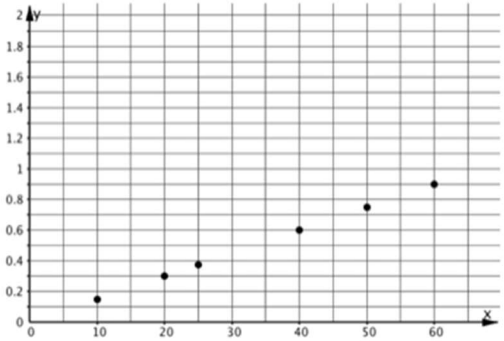

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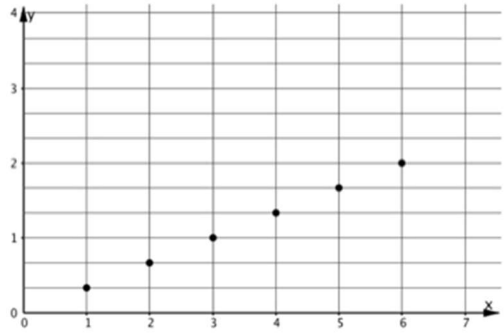


5. Entrance to a state park costs £6.00 per vehicle, plus £2.00 per person in the vehicle. One vehicle can seat 6 people.

x	*y*
1	8
2	10
3	12
4	14
5	16
6	18

6. He measures the time that has elapsed after each lap he runs.

x	*y*
1	2
2	4
3	7
4	9
5	11
6	15

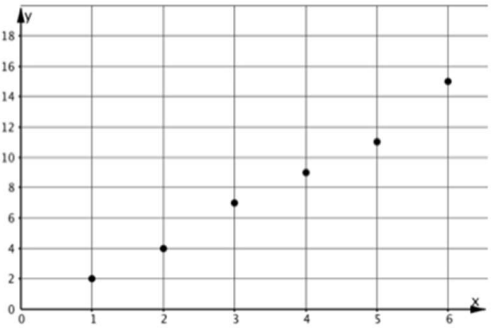
7. A recipe uses 2 tablespoons of honey for every 8 cups of flour.

x	*y*
2	8
3	12
6	24
7	28
10	40
12	48

8. She is filling her fish tank with water. The chart shows the gallons of water after so many minutes.

x	*y*
1	1.6
2	3.2
3	4.8
4	6.4
5	8.0
6	9.6

E.



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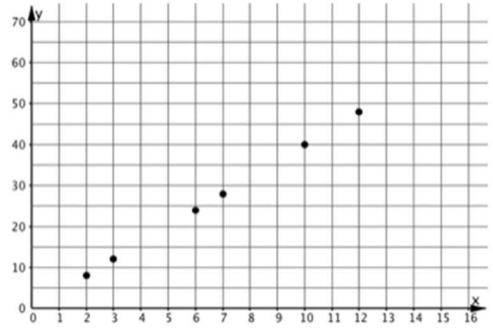

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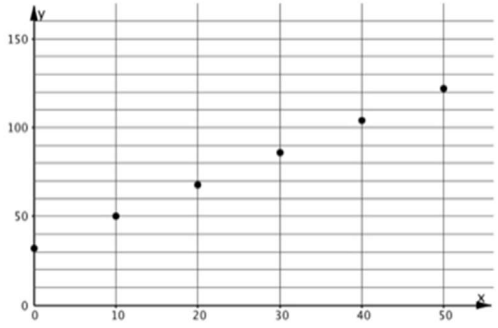

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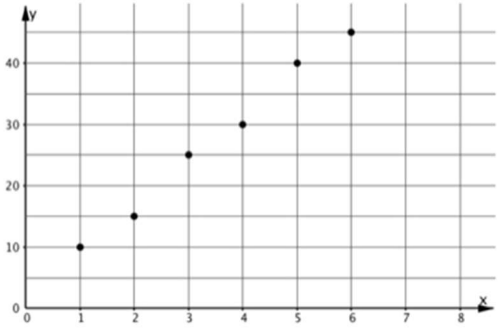
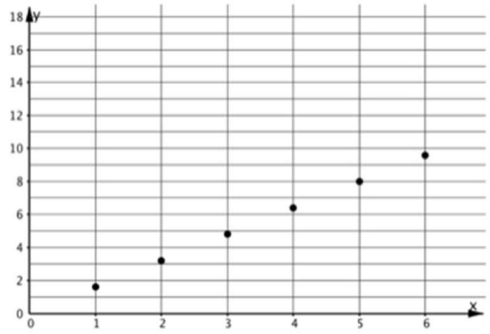


9. Ten empty aluminum cans weigh 0.15 kg.

x	y
10	0.15
20	0.30
25	0.375
40	0.60
50	0.75
60	0.90

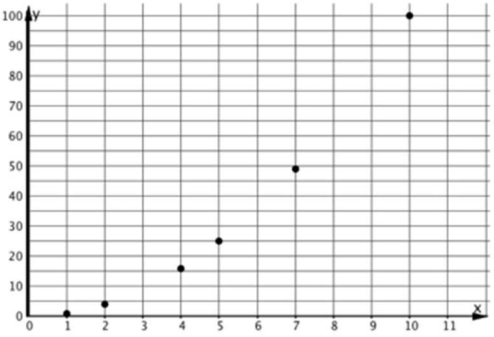
I.



10. The area of a square is the square of the side length.

x	y
1	1
2	4
4	16
5	25
7	49
10	100

J.



Activity Synthesis

Ask students how they determined which relationships were proportional. Invite volunteers to share the proportional relationships to ensure common understanding. Address any discrepancies through questioning: "How do you know the relationship is proportional? What have you learned about proportional relationships that applies here?"

Select students to share what they noticed about the characteristics of graphs of proportional relationships. Some observations might conclude:

- Points whose coordinates satisfy the relationship lie on a line.
- The line passes through the point (0,0).

This would be a good place to either introduce the term **origin** to refer to the point (0,0) (or to remind students of it, if they have encountered it before).

If time permits, discuss which written descriptions of proportional relationships would warrant "connecting the dots." In other words, which proportional relationships are best represented with dots, and which are best represented with an unbroken line? Of the cards which describe a proportional relationship, it makes sense to draw an unbroken line for 7 and 8. The rest should use dots that are not connected. None include negative values without some assumptions (e.g., scenarios that involve owing money for card 2). Students should realise that even when the graph of a proportional relationship is represented by unconnected points, they lie on a line through the origin.

Lesson Synthesis

At the end of the lesson, make sure that students know that the graph of a proportional relationship lies on a line through the **origin**. (They will be able to explain why this is true later in KS3.) Students should understand that the context sometimes restricts which points on the line should be included in the graph.

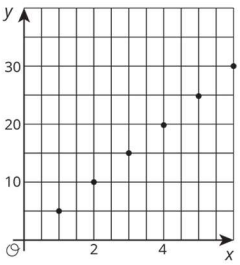
10.4 Which Are Not Proportional

Cool Down: 5 minutes

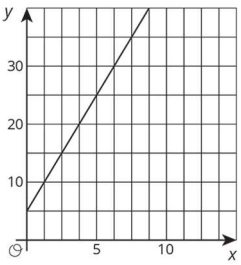
Student Task Statement

Which graphs cannot represent a proportional relationship? Select **all** that apply. Explain how you know.

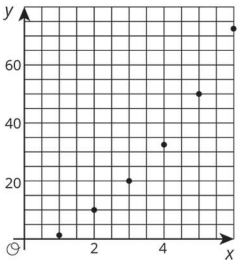
A



B



C



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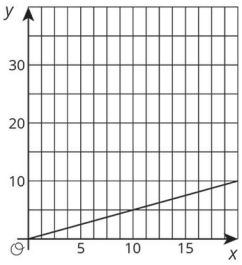
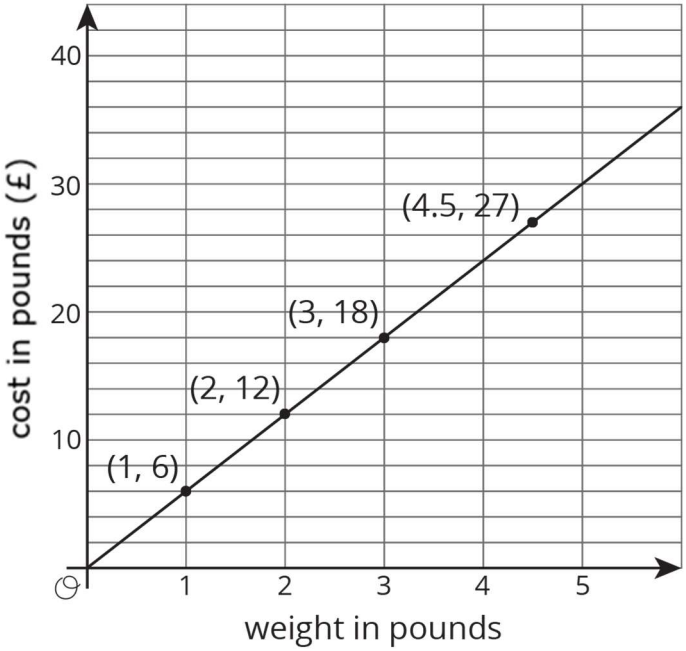


Student Response

B and C. Since graph B does not go through the origin, it cannot be a proportional relationship. Since the points in graph C cannot be connected by a single, straight line, it cannot be a proportional relationship.

Student Lesson Summary

One way to represent a proportional relationship is with a graph. Here is a graph that represents different amounts that fit the situation, “Blueberries cost £6 per pound.”



Different points on the graph tell us, for example, that 2 pounds of blueberries cost £12, and 4.5 pounds of blueberries cost £27.

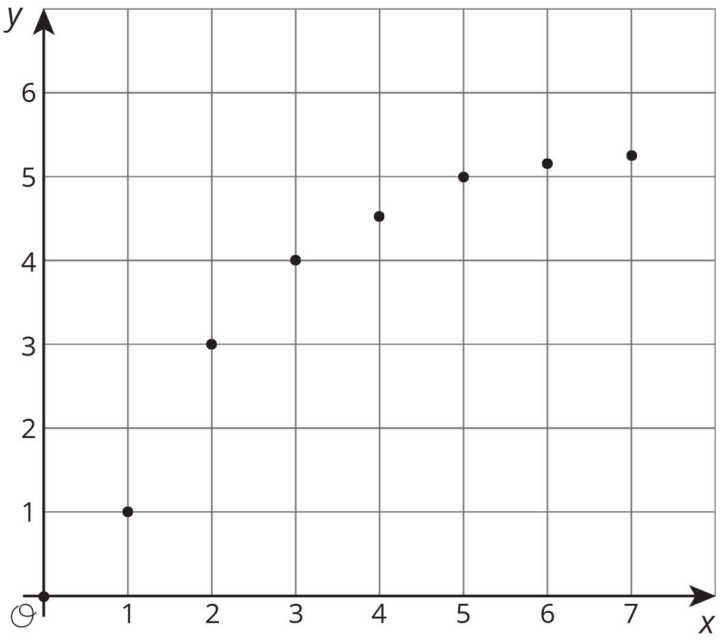
Sometimes it makes sense to connect the points with a line, and sometimes it doesn't. We could buy, for example, 4.5 pounds of blueberries or 1.875 pounds of blueberries, so all the points in between the whole numbers make sense in the situation, so any point on the line is meaningful.

If the graph represented the cost for different *numbers of sandwiches* (instead of pounds of blueberries), it might not make sense to connect the points with a line, because it is often not possible to buy 4.5 sandwiches or 1.875 sandwiches. Even if only points make sense in the situation, though, sometimes we connect them with a line anyway to make the relationship easier to see.

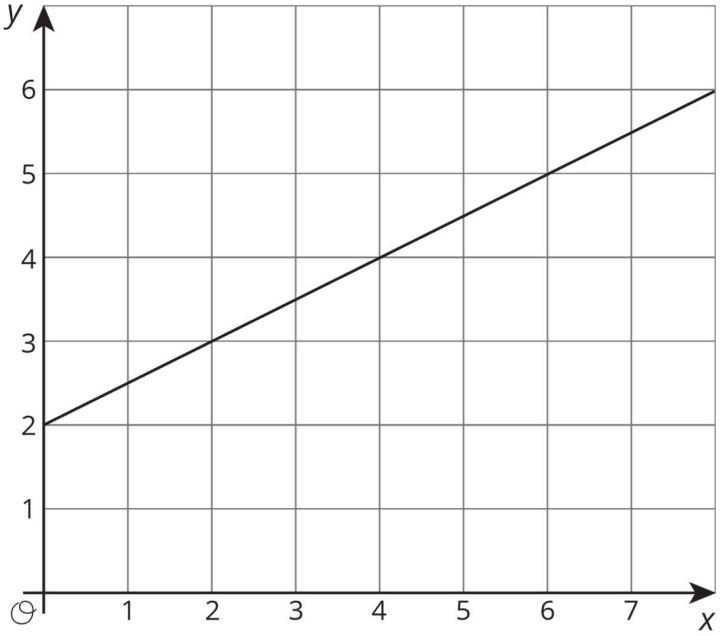
Graphs that represent proportional relationships all have a few things in common:

- Points that satisfy the relationship lie on a straight line.
- The line that they lie on passes through the **origin**, (0,0).

Here are some graphs that do *not* represent proportional relationships:



These points do not lie on a line.



This is a line, but it doesn't go through the origin.

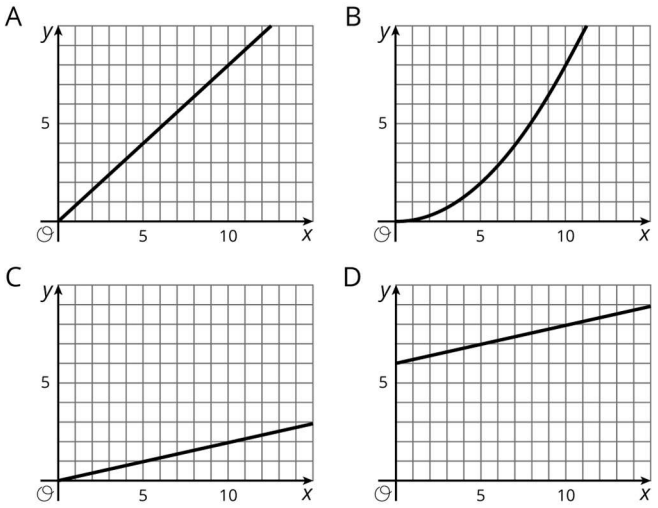
Glossary

- coordinate grid
- origin

Lesson 10 Practice Problems

1. **Problem 1 Statement**

   Which graphs could represent a proportional relationship?



   a. A

   b. B

   c. C

   d. D

   Solution ["A", "C"]

2. **Problem 2 Statement**

   A lemonade recipe calls for $\frac{1}{4}$ cup of lemon juice for every cup of water.

   a. Use the table to answer these questions.

      i. What does x represent?

      ii. What does y represent?

      iii. Is there a proportional relationship between x and y?

   b. Plot the pairs in the table in a coordinate grid.

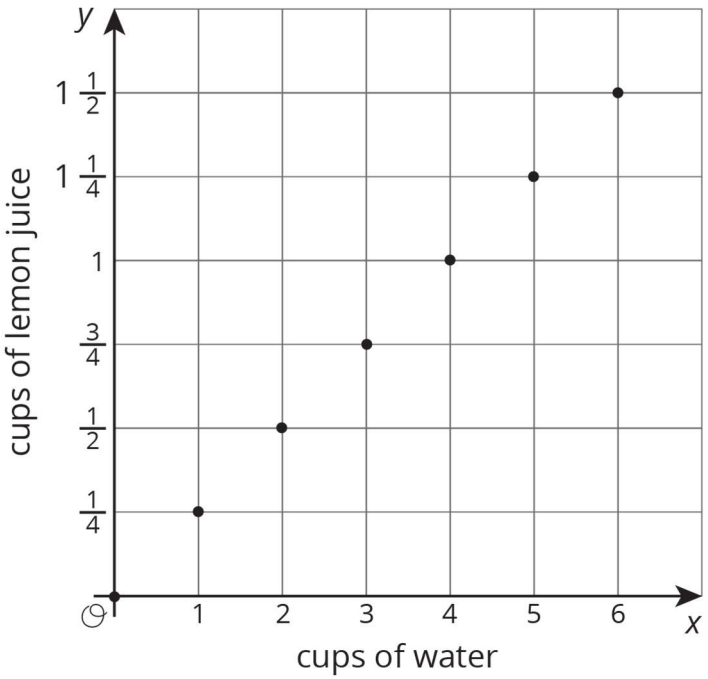
x	y
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

Solution

i. x represents the cups of water

ii. y represents the cups of lemon juice

iii. Yes



3. **Problem 3 Statement**

Select **all** the pieces of information that would tell you x and y have a proportional relationship. Let y represent the distance in metres between a rock and a turtle's current position and x represent the time in minutes the turtle has been moving.

a. $y = 3x$

b. After 4 minutes, the turtle has walked 12 feet away from the rock.

c. The turtle walks for a bit, then stops for a minute before walking again.

d. The turtle walks away from the rock at a constant rate.

Solution ["A", "D"]

4. **Problem 4 Statement**

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

a. The sizes you can print a photo.

width of photo (inches)	height of photo (inches)
2	3
4	6
5	7
8	10

b. The distance from which a lighthouse is visible.

height of a lighthouse (feet)	distance it can be seen (miles)
20	6
45	9
70	11
95	13

Solution

a. Not proportional since the ratios of width to height are not all equivalent.

b. Not proportional since the ratios of height to distance are not all equivalent.