Lesson 1: Mystery bags

Goals

- Compare outcomes for different experiments, predict which experiment is more likely to produce a desired result, and justify (orally and in writing) the prediction.
- Describe (orally) how we can use the outcomes from previous experiments to help determine the relative likelihood of future events.

Learning Targets

- I can get an idea for the likelihood of an event by using results from previous experiments.

Lesson Narrative

To introduce the unit on probability, students play a game to collect data about what is inside bags and then make a decision based on the information they have collected. The process of using previous results from repeated trials to inform the likelihood of future events is one way to estimate probabilities that will be revisited later.

Bags that contain a certain number of coloured objects will be used in this unit. To reuse materials already in the classroom, coloured multi-link cubes are recommended, but any items of different colours that cannot be determined based on feeling will work. If there are not enough suitable items, equal sized pieces of paper can be coloured or have the colour written on them and used in the bags.

Addressing

- Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

Instructional Routines

- Discussion Supports
- Poll the Class

Required Materials

Paper bags
Multi-link cubes

Required Preparation

Prepare enough bags of blocks so that each group of 4 students can have one bag and all groups will have had a turn with each colour of bag after three rounds.

- Label one-third of the bags "green" and put 9 green blocks and 3 blocks of another colour into each of these bags.
- Label one-third of the bags "blue" and put 8 blue blocks and 8 blocks of another colour into each of these bags.
- Label one-third of the bags "red" and put 4 red blocks and 10 blocks of another colour into each of these bags.

Student Learning Goals

Let's make predictions based on what we know.

1.1 Going Fishing

Warm Up: 5 minutes

The purpose of this warm-up is for students to use their intuition to think about the upcoming unit on probability. In particular, students guess what type of fish might be caught after knowing the results of the previous 10 fish caught. Although no answer can be given with absolute certainty, the results being heavily skewed towards one type of fish should lead students to the idea that it is more likely to be the most commonly caught fish that will be caught again.

Instructional Routines

- Poll the Class

Launch

It may be helpful to explain that there are many types of fish that are caught while fishing for fun or sport. The two types listed in this warm-up, bluegill and yellow perch, are typically found in lakes and caught with the same type of bait. Both of these types of fish are suitable for eating after being caught or release back into the water.

Student Task Statement

Andre and his dad have been fishing for 2 hours. In that time, they have caught 9 bluegills and 1 yellow perch.

The next time Andre gets a bite, what kind of fish do you think it will be? Explain your reasoning.

Student Response

Answers vary. Sample response: I think Andre will pull out a bluegill. They have mostly caught bluegills, so it seems like the next one will probably be a bluegill, too.

Activity Synthesis

The purpose of the discussion is to show students that no single answer can be certain for this problem, but previous results can help inform the likelihood of future outcomes.

Poll the class regarding the type of fish they think will be caught next. Begin by asking for students who think the next fish caught will be a bluegill followed by students who think the next fish caught will be a yellow perch followed by students who think that another type of fish will be caught (or that they will not catch another fish). Display the results from the poll for all to see. Following the poll, ask at least one student representing each group with more than 1 vote for their reasoning. Tell students that we cannot know for certain what the next type of fish will be, but based on the results we have available, it is most likely that a bluegill will be caught next.

1.2 Playing the Block Game

30 minutes

Following the concept developed in the warm-up, students will continue to use the idea that outcomes from previous experiments can help inform the likelihood of an outcome when the experiment is repeated. In this activity, students play a game while collecting data. Bags of coloured blocks are set up with different probabilities to win and students use random chance to collect points when the colour of the block they choose matches the colour written on the bag. In the discussion, students will use the data they have collected in three rounds of the game to make a decision about which bag might be most likely to produce a winning block. As an opening activity, this is meant to motivate students into thinking about likelihood of events. As such, it is not important to resolve the questions in the discussion into perfect agreement.

Students are arranged in groups of 4 and given one of 3 different bags labelled with colours. Create enough bags so that each group will have a turn with each of the different colour bags after three rounds.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 4. Explain to students how to play the game:

- There will be 3 rounds of the game. Their group will get a different bag of blocks for each round.
- They are not allowed to look into the bag or take more than one block out of the bag at a time.
- During a round, each person in the group gets 4 turns to take a block out of the bag.

- If the block matches the colour that is written on the outside of the bag, the person scores (1 point during round 1, 2 points during round 2, and 3 points during round 3). When the block is any other colour, they do not earn any points.
- They put the block back in the bag, shake the bag to mix up the blocks, and pass the bag to the next person in between each turn.
- At the end of a round, they record everyone's points from that round and wait for a new bag of blocks.

Distribute bags of blocks. Each group gets one of these bags:

- A bag labelled "Green" that contains 9 green blocks and 3 of another colour.
- A bag labelled "Blue" that contains 8 blue blocks and 8 of another colour.
- A bag labelled "Red" that contains 4 red blocks and 10 of another colour.

Engagement: Provide Access by Recruiting Interest. Begin with a small-group or whole-class demonstration of how to play the game. Check for understanding by inviting students to rephrase directions in their own words.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Some groups may choose a bag for the bonus round based on the number of points they got in the earlier rounds. Since points are equal in the bonus round, help them see that the blocks picked out is a more important thing to consider than the points for the round.

Student Task Statement

Your teacher will give your group a bag of coloured blocks.

1. Follow these instructions to play one round of the game:
    a. Everyone in the group records the colour written on the bag in the first column of the table.
    b. Without looking in the bag, one person takes out one of the blocks and shows it to the group.
    c. If they get a block that is the same colour as the bag, they earn:
        - 1 point during round 1
        - 2 points during round 2
        - 3 points during round 3
    d. Next, they put the block back into the bag, shake the bag to mix up the blocks, and pass the bag to the next person in the group.

e. Repeat these steps until everyone in your group has had 4 turns.

2. At the end of the round, record each person's score in the table.

	What colour bag?	person 1's score	person 2's score	person 3's score	person 4's score
round 1					
round 2					
round 3					

3. Pause here so your teacher can give you a new bag of blocks for the next round.

4. Repeat the previous steps to play rounds 2 and 3 of the game.

5. After you finish playing all 3 rounds, calculate the total score for each person in your group.

Student Response

Answers vary. Sample response:

	What colour bag?	person 1's score	person 2's score	person 3's score	person 4's score
round 1	Green	3	2	3	3
round 2	Blue	8	10	8	6
round 3	Red	3	6	6	3

Are You Ready for More?

Tyler's class played the block game using purple, orange, and yellow bags of blocks.

- During round 1, Tyler's group picked 4 purple blocks and 12 blocks of other colours.
- During round 2, Tyler's group picked 11 orange blocks and 5 blocks of other colours.
- During round 3, Tyler forgot to record how many yellow blocks his group picked.

For a final round, Tyler's group can pick one block from any of the three bags. Tyler's group decides that picking from the orange bag would give them the best chance of winning, and that picking from the purple bag would give them the worst chance of winning. What results from the yellow bag could have lead Tyler's group to this conclusion? Explain your reasoning.

Student Response

Answers vary. Sample response: 8 yellow blocks and 8 other colours. The purple bag appears to have less pink than other colours, and the orange bag appears to have more orange than other colours. If the yellow bag has an equal number of yellow blocks and

other colours, it will be more likely to win than the purple bag, but less likely to win than the orange bag.

Activity Synthesis

Tell students that there is a bonus, fourth round. In this round, a block that matches the colour on the bag is worth 25 points. Each person will only have one chance to draw a bonus block. In this round, each group will get to choose which bag they would like to draw from. At least 3 people in the group must agree on the bag they will use.

Give students 5 minutes of small-group discussion to agree on which bag the group will use. Tell students that, after the bonus round, they will be asked to explain their reasoning for choosing the bag. It is ok for more than one group to select the same colour bag. Identify groups that use the results from the previous rounds to determine which bag will be more likely to draw a winning block.

Allow each group to use the bag they have selected to play the bonus round and total their points.

After all students have played the bonus round, select groups to share their reasoning for choosing which bag to use. Select previously identified students to share their reasoning as well.

Ask students, "Was one bag more likely to give you a winning cube than the others? Explain your reasoning."

Open one of each colour bag to reveal the contents. Ask students, "Based on what you see now, does this change your answer? Explain your reasoning."

The green bag should provide the best chance to win points, but it is not essential for the class to come to this understanding at this point.

Speaking, Representing: Discussion Supports. Use this routine to support whole-class discussion as groups share which bag they would choose during the bonus round. Display sentence frames for students to use as they respond to each group: "I agree because ...." or "I disagree because ...." If necessary, revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.
Design Principle(s): Support sense-making

Lesson Synthesis

Ask students, "What are some examples of times you have predicted what will happen in the future based on what you have seen happen in the past?"

Tell students that in the first half of this unit they will learn about different ways to determine how likely events are to happen. Examining previous results is one of the ways.

1.3 Jada Draws Even

Cool Down: 5 minutes

Students use a different context to continue working with the idea that outcomes from previous experiments inform the likelihood of an outcome from doing the experiment additional times. In particular, when there is more than one option available, prior outcomes can help people choose the option that is most likely to provide favourable results.

Student Task Statement

A large fish tank is filled with table tennis balls with numbers written on them. Jada chooses 10 table tennis balls from the tank and writes down their numbers.

1 3 5 1 3 2 4 1 5 3

A second tank is filled with golf balls with numbers written on them. Jada chooses 10 golf balls from the tank and writes down their numbers.

1 4 5 2 6 2 2 1 4 8

To win a prize, Jada must get a ball with an even number. Should she try to win the prize using the tank of table tennis balls or the tank of golf balls? Explain your reasoning.

Student Response

Jada should use the tank of golf balls. Explanations vary. Sample explanation: From the tank of table tennis balls, Jada only got 2 even numbers out of the 10 she chose. From the tank of golf balls, she got 7 even numbers out of the 10 she chose. There seems to be a better chance of her getting even numbered balls from the tank that has golf balls.

Student Lesson Summary

One of the main ways that humans learn is by repeating experiments and observing the results. Babies learn that dropping their cup makes it hit the floor with a loud noise by repeating this action over and over. Scientists learn about nature by observing the results of repeated experiments again and again. With enough data about the results of experiments, we can begin to predict what may happen if the experiment is repeated in the future. For example, a baseball player who has gotten a hit 33 out of 100 times at bat might be expected to get a hit about 33% of his times at bat in the future as well.

In some cases, we can predict the chances of things happening based on our knowledge of the situation. For example, a coin should land heads up about 50% of the time due to the symmetry of the coin.

In other cases, there are too many unknowns to predict the chances of things happening. For example, the chances of rain tomorrow are based on similar weather conditions we have observed in the past. In these situations, we can experiment, using past results to estimate chances.

Lesson 1 Practice Problems

Problem 1 Statement

Lin is interested in how many of her classmates watch her favourite TV show, so she starts asking around at lunch. She gets the following responses:

yes	yes	yes	no	no	no	no	no
no	no	yes	no	no	no		

If she asks one more person randomly in the cafeteria, do you think they will say "yes" or "no"? Explain your reasoning.

Solution

Answers vary. Sample response: I think they will say "no," since most people so far have said "no."

Problem 2 Statement

An engineer tests the strength of a new material by seeing how much weight it can hold before breaking. Previous tests have held these weights in pounds:

1 200	1 400	1 300	1 500	950	1 600	1 100

Do you think that this material will be able to hold more than 1 000 pounds in the next test? Explain your reasoning.

Solution

Answers vary. Sample response: Yes, 6 out of 7 tests did, and the one that didn't was pretty close.

Problem 3 Statement

A company tests two new products to make sure they last for more than a year.

- Product 1 had 950 out of 1 000 test items last for more than a year.
- Product 2 had 150 out of 200 last for more than a year.

If you had to choose one of these two products to use for more than a year, which one is more likely to last? Explain your reasoning.

Solution

Product 1, since a greater proportion of the test products lasted more than 1 year.

Problem 4 Statement

Put these numbers in order from least to greatest.

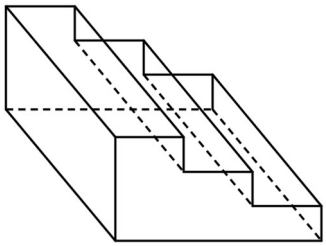
$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{5}$ 0.6 0.3

Solution

0.3, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{2}$, 0.6

Problem 5 Statement

A small staircase is made so that the horizontal piece of each step is 10 inches long and 25 inches wide. Each step is 5 inches above the previous one. What is the surface area of this staircase?



Solution

2 850 square inches