Lesson 6: Absolute values of numbers

Goals

- Compare rational numbers and their absolute values, and explain (orally and in writing) the reasoning.
- Comprehend the phrase “absolute value” and the symbol || to refer to a number’s distance from zero on the number line.
- Interpret rational numbers and their absolute values in the context of height above sea level or temperature.

Learning Targets

- I can explain what the absolute value of a number is.
- I can find the absolute values of rational numbers.
- I can recognise and use the notation for absolute value.

Lesson Narrative

In the past several lessons, students have reasoned about the structure of rational numbers by plotting them on a number line and noting their relative positions and distances from zero. They learned that opposite numbers have the same distance from zero. Students now formalise the concept of a number’s magnitude with the term **absolute value**. They learn that the absolute value of a number is its distance from zero, which means that opposite numbers have the same absolute value. Students reason abstractly about the familiar contexts of temperature and height above sea level using the concept and notation of absolute value.

Building On

- Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognise that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, =, or $<$, and justify the conclusions, e.g., by using a visual fraction model.
- Understand the place value system.

Addressing

- Understand ordering and absolute value of rational numbers.
- Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 pounds, write $|-30| = 30$ to describe the size of the debt in pounds.

- Distinguish comparisons of absolute value from statements about order. For example, recognise that an account balance less than -30 pounds represents a debt greater than 30 pounds.

Building Towards

- Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 pounds, write $|-30| = 30$ to describe the size of the debt in pounds.

Instructional Routines

- Stronger and Clearer Each Time
- Compare and Connect
- Discussion Supports
- Number Talk

Student Learning Goals

Let’s explore distances from zero more closely.

6.1 Number Talk: Closer to Zero

Warm Up: 5 minutes

The purpose of this Number Talk is to elicit strategies and understandings students have about the distance from 0 on the number line. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to think about distance from 0 for various rational numbers. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Reveal one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the talk. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

For each pair of expressions, decide mentally which one has a value that is closer to 0.

$\frac{9}{11}$ or $\frac{15}{11}$

$\frac{1}{5}$ or $\frac{1}{9}$

1.25 or $\frac{5}{4}$

0.01 or 0.001

Student Response

- $\frac{9}{11}$. Sample explanation: $\frac{9}{11}$ is positive and less than 1, whereas $\frac{15}{11}$ is greater than 1, so $\frac{9}{11}$ is closer to 0.
- $\frac{1}{9}$. Sample explanation: Ninths are smaller than fifths, so $\frac{1}{9}$ is closer to zero.
- They are equal, so equally close to zero.
- 0.001 is 10 times closer to zero than 0.01.

Activity Synthesis

Ask students to share their reasoning for each problem. Record and display their explanations for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same answer but would explain it differently?”
- “Did anyone reason about the problem in a different way?”
- “Does anyone want to add on to _____’s reasoning?”
- “Do you agree or disagree? Why?

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

6.2 Jumping Flea

15 minutes (there is a digital version of this activity)

The purpose of this task is to help students understand the **absolute value** of a number as its distance from 0 on the number line. The context is not realistic, but helps students visualise relationships on the number line in a more concrete way. Students have used the concept of absolute value informally in previous lessons, but this is where the term is formally introduced and used precisely.

Instructional Routines

- Stronger and Clearer Each Time

Launch

Allow students 10 minutes quiet work time followed by whole-class discussion.

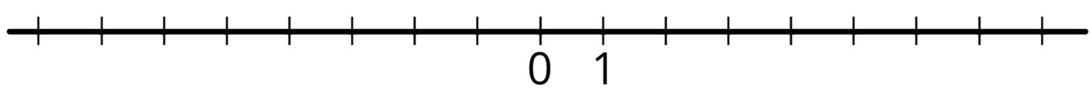
Students using the digital materials, will use an applet to visualise the bug jumping. Students can pick a starting point for the bug, choose the direction it jumps, and then check where it lands.

Anticipated Misconceptions

Students may confuse absolute value with opposites, thinking that absolute value changes the sign of the number. Remind students that absolute value represents the distance of the number from 0 without worrying about the sign or direction. Use some concrete examples: in a board game, 3 moves forward and 3 moves backward both involve 3 jumps but you land in different places; traveling 5 miles east or 5 miles west would put the same number of miles on your car's odometer, but you end up in different places depending on the direction; earning £10 and spending £10 both involve the amount £10 but in one case you gain it and in the other you lose it. Absolute value is just the amount involved, not including the sign.

Student Task Statement

1. A flea is jumping around on a number line.



    a. If the flea starts at 1 and jumps 4 units to the right, where does it end up? How far away from 0 is this?

    b. If the flea starts at 1 and jumps 4 units to the left, where does it end up? How far away from 0 is this?

    c. If the flea starts at 0 and jumps 3 units away, where might it land?

    d. If the flea jumps 7 units and lands at 0, where could it have started?

    e. The **absolute value** of a number is the distance it is from 0. The flea is currently to the left of 0 and the absolute value of its location is 4. Where on the number line is it?

f. If the flea is to the left of 0 and the absolute value of its location is 5, where on the number line is it?

g. If the flea is to the right of 0 and the absolute value of its location is 2.5, where on the number line is it?

2. We use the notation |-2| to say "the absolute value of -2," which means "the distance of -2 from 0 on the number line."

   a. What does |-7| mean and what is its value?

   b. What does |1.8| mean and what is its value?

Student Response

1.

   a. Counting 4 spaces to the right gets to 5; this is 5 units from 0.

   b. Counting 4 spaces to the left gets to -3; this is 3 units from 0.

   c. Counting 3 spaces to the right gets to 3, or counting 3 spaces to the left gets to -3.

   d. Counting 7 spaces to the right of zero gets to 7, or counting 7 spaces to the left of 0 gets to -7.

   e. 4 units to the left of 0 is -4.

   f. 5 units to the left of 0 is -5.

   g. 2.5 units to the right of 0 is 2.5.

2.

   a. |-7| means the absolute value of -7 or the distance of -7 from 0. Its value is 7.

   b. |1.8| means the absolute value of 1.8 or the distance of 1.8 from 0. Its value is 1.8.

Activity Synthesis

Define the **absolute value** of a number as its distance from 0. Ask students to contrast |-8| and |8| to come to the conclusion that they have the same value but represent the distance between two distinct points and zero. Also ask for situations where they think the absolute value might be useful (example: your car's odometer tracks the miles you drove, but if you make a round trip—the same distance in two opposite directions—the difference between where you started and where you ended is zero).

To help clear up misconceptions related to opposites and absolute values, ask:

- "What is the difference between a number's opposite and a number's absolute value?" (Opposite is another number on the number line whose distance from zero is the same; absolute value is a number that describes that distance.)
- "Does finding a number's absolute value always mean changing the sign?" (No, absolute value represents the distance from zero. If the number is positive, the number and its absolute value are the same. If the number is negative, the distance is represented by the number without its negative sign.)
- "If n is any number that can be positive or negative, what is the sign of the absolute value of n?"

Representation: Internalise Comprehension. Use colour and annotations to illustrate student thinking. As students share and explain their reasoning, scribe their thinking on a display of each problem so it is visible for all students. Use colour and annotations to illustrate the difference between absolute value and opposites.
Supports accessibility for: Visual-spatial processing; Conceptual processing Writing, Speaking, Listening: Stronger and Clearer Each Time. To support students to recognise the similarities and differences between "absolute value" and "opposite," ask students to respond to the prompt "How is a number's absolute value the same and different than its opposite?" Encourage students to include diagrams with their explanations. Ask each student to meet with 2–3 other partners for feedback. Provide student with prompts for feedback "Can you show on your diagram . . ." or "Why are the values the same for this number?" Students can borrow ideas and language from each partner to strengthen their final response. This will help student to refine their understanding of absolute value and improve their ability to discuss rational numbers.
Design Principle(s): Support sense-making, Cultivate conversation

6.3 Absolute Height Above Sea Level and Temperature

15 minutes

The purpose of this task is for students to develop their understanding of the $|x|$ notation in familiar contexts. They should build their understanding that $|x|$ represents the distance from zero to x and that $|-x|$ and $|x|$ are equal.

Instructional Routines

- Compare and Connect

Launch

Allow students 10 minutes quiet work time followed by whole-class discussion.

Student Task Statement

1. A part of the city of New Orleans is 6 feet below sea level. We can use "-6 feet" to describe its height above sea level, and "|-6| feet" to describe its vertical distance from sea level. In the context of height above sea level, what would each of the following numbers describe?

a. 25 feet

b. |25| feet

c. -8 feet

d. |-8| feet

2. The height of a city is different from sea level by 10 feet. Name the two heights above sea level that the city could have.

3. We write “-5°C” to describe a temperature that is 5 degrees Celsius below freezing point and “5°C” for a temperature that is 5 degrees above freezing. In this context, what do each of the following numbers describe?

    a. 1°C

    b. -4°C

    c. |12|°C

    d. |-7|°C

4.

    a. Which temperature is colder: -6°C or 3°C?

    b. Which temperature is closer to freezing temperature: -6°C or 3°C?

    c. Which temperature has a smaller absolute value? Explain how you know.

Student Response

1.

    a. 25 feet above sea level.

    b. The distance in feet between a point 25 feet above sea level and sea level.

    c. 8 feet below sea level.

    d. The distance in feet between a point 8 feet below sea level and sea level.

2. -10 feet or 10 feet.

3.

    a. 1 degree Celsius above freezing point.

    b. 4 degrees Celsius below freezing point.

    c. The distance in degrees between freezing point and 12 degrees Celsius.

d. The distance in degrees between freezing point and -7 degrees Celsius.

4.

a. -6°C

b. 3°C

c. 3°C, because it has a closer distance to 0 than does -6°C.

Are You Ready for More?

At a certain time, the difference between the temperature in New York City and in Boston was 7 degrees Celsius. The difference between the temperature in Boston and in Chicago was also 7 degrees Celsius. Was the temperature in New York City the same as the temperature in Chicago? Explain your answer.

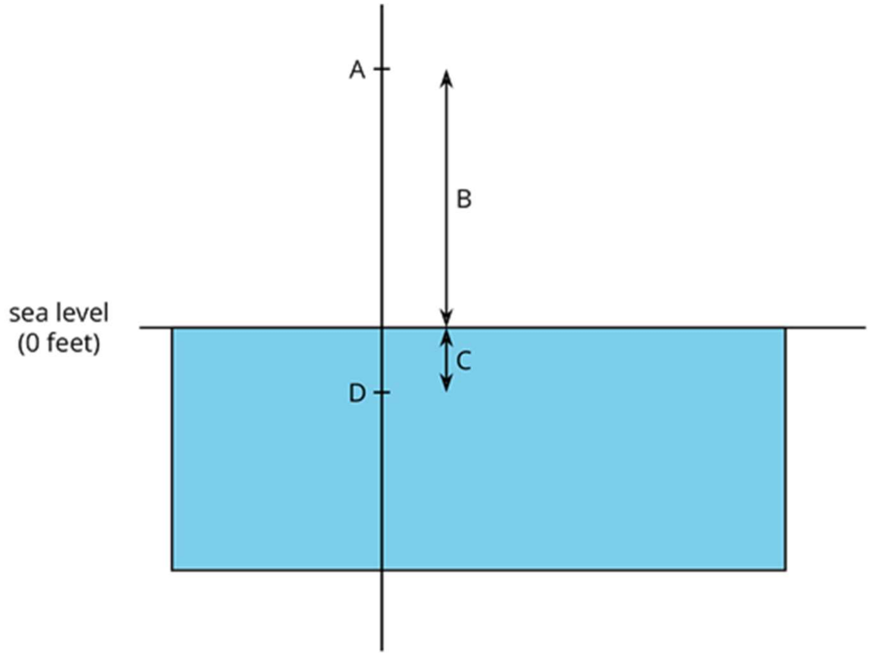
Student Response

Answers vary. Sample response: It is not possible to say for sure. There are the two temperatures that differ from the temperature in Boston by 7 degrees. The temperature in New York City could be either 7 degrees above or below the temperature in Boston. The temperature in Chicago could also be either 7 degrees above or below the temperature in Boston. So the temperatures in New York City and Chicago could be equal, or they could differ by 14 degrees.

Activity Synthesis

To summarise students' work, consider displaying the following diagram and these four expressions. Give students a minute to study the diagram and match each letter on the diagram to an appropriate expression.

- -4 feet
- |15| feet
- |-4| feet
- 15 feet



Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: absolute value of a number, height above sea level, sea level, less than, greater than.
Supports accessibility for: Conceptual processing; Language; Memory Representing, Speaking: Compare and Connect. To help students connect the numerical values with the diagram in the activity synthesis, provide students phrases that also connect to the situation (e.g., "4 feet below sea level," "15 feet from sea level," "4 feet from sea level," and "15 feet above sea level"). Emphasise the language needed to say what the numerical values mean and how those values connect to the diagram and the phrases.
Design Principle(s): Maximise meta-awareness

Lesson Synthesis

In this lesson, students learned the definition of absolute value and its relationship to rational numbers. Ask students to do the following:

- Order the numbers 1, 3, and 6 from least to greatest. (1, 3, 6.)
- Order |1|, |3|, and |6| from least to greatest. (|1|, |3|, |6|.)
- Order the numbers -1, -3, and -6 from least to greatest. (-6, -3, -1.)
- Order |-1|, |-3|, and |-6| from least to greatest. (|-1|, |-3|, |-6|.)
- What do you notice about the order of numbers after taking absolute value? Explain why this happens.

Consider asking students to sketch a number line if they get stuck. Students should see that the order remained the same for the positive numbers but reversed for the negative numbers. They should be able to explain that as numbers move to the left on the number line, their absolute value gets larger because they are further from 0. This realisation should help solidify the thinking that has been building for the past several lessons about the ordering and magnitude of rational numbers.

6.4 Greater, Less, the Same

Cool Down: 5 minutes

Student Task Statement

1. Write a number that has the same value as each expression:
    a. |5|
    b. |-12.9|
2. Write a number that has a value less than |4.7|
3. Write a number that has a value greater than |-2.6|

Student Response

1.
    a. 5 or |-5|
    b. 12.9 or |12.9|
2. Answers vary. Sample responses: 4.5, |-4.5|, or -10
3. Answers vary. Sample responses: 2.7 or |-2.7|

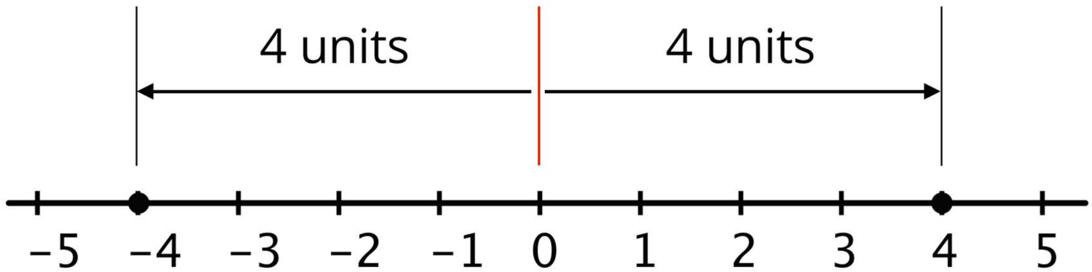
Student Lesson Summary

We compare numbers by comparing their positions on the number line: the one farther to the right is greater; the one farther to the left is less.

Sometimes we wish to compare which one is closer to or farther from 0. For example, we may want to know how far away the temperature is from the freezing point of 0°C, regardless of whether it is above or below freezing.

The **absolute value** of a number tells us its distance from 0.

The absolute value of -4 is 4, because -4 is 4 units to the left of 0. The absolute value of 4 is also 4, because 4 is 4 units to the right of 0. Opposites always have the same absolute value because they both have the same distance from 0.



The distance from 0 to itself is 0, so the absolute value of 0 is 0. Zero is the *only* number whose distance to 0 is 0. For all other absolute values, there are always two numbers—one positive and one negative—that have that distance from 0.

To say “the absolute value of 4,” we write: $|4|$

To say that “the absolute value of -8 is 8,” we write: $|-8| = 8$

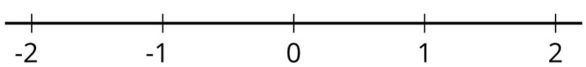
Glossary

- absolute value

Lesson 6 Practice Problems

1. **Problem 1 Statement**

   On the number line, plot and label all numbers with an absolute value of $\frac{3}{2}$.



   Solution

   Points $\frac{3}{2}$ and $\frac{-3}{2}$, $1\frac{1}{2}$ and $-1\frac{1}{2}$, or 1.5 and -1.5 should be plotted.

2. **Problem 2 Statement**

   The temperature at dawn is 6°C away from 0. Select **all** the temperatures that are possible.

   a. -12°C

   b. -6°C

   c. 0°C

   d. 6°C

   e. 12°C

   Solution ["B", "D"]

3. **Problem 3 Statement**

Put these numbers in order, from least to greatest.

|-2.7|

0

1.3

|-1|

2

Solution

0 |-1| 1.3 2 |-2.7|

4. **Problem 4 Statement**

Lin's family needs to travel 325 miles to reach her grandmother's house.

a. At 26 miles, what percentage of the trip's distance have they completed?

b. How far have they travelled when they have completed 72% of the trip's distance?

c. At 377 miles, what percentage of the trip's distance have they completed?

Solution

a. 8% of the trip, because $26 \div 325 = 0.08$.

b. 234 miles, because $0.72 \times 325 = 234$.

c. 116% of the trip, because $377 \div 325 = 1.16$.

5. **Problem 5 Statement**

Elena donates some money to charity whenever she earns money as a babysitter. The table shows how much money, d, she donates for different amounts of money, m, that she earns.

d	4.44	1.80	3.12	3.60	2.16
m	37	15	26	30	18

a. What percent of her income does Elena donate to charity? Explain or show your work.

b. Which quantity, m or d, would be the better choice for the dependent variable in an equation describing the relationship between m and d? Explain your reasoning.

c. Use your choice from the second question to write an equation that relates m and d.

Solution

a. Elena donates 12% of her income to charity. Sample reasoning: We want to know what percent of 30 is 3.6, so we can write $30p = 3.6$. To solve this, divide 3.6 by 30, which is 0.12. So 12% of 30 is 3.6.

b. Answers vary. Sample response: Since the amount of the donation depends on how much money she earns, d would be better as the dependent variable. If she wants to donate a certain amount and needs to figure out how much she needs to earn to achieve that donation, then m would be better as the dependent variable.

c. $d = 0.12m$ or equivalent. $m = \frac{100}{12}d$ or equivalent

6. **Problem 6 Statement**

How many times larger is the first number in the pair than the second?

a. 3^4 is ____ times larger than 3^3.

b. 5^3 is ____ times larger than 5^2.

c. 7^{10} is ____ times larger than 7^8.

d. 17^6 is ____ times larger than 17^4.

e. 5^{10} is ____ times larger than 5^4.

Solution

a. 3

b. 5

c. 7^2 or 49

d. 17^2 or 289

e. 5^6 or 15 625