

Lesson 8: Rotation patterns

Goals

- Draw and label rotations of 180 degrees of a line segment from centres of the midpoint, a point on the line segment, and a point not on the line segment.
- Generalise (orally and in writing) the outcome when rotating a line segment 180 degrees.
- Identify (orally and in writing) the translations, rotations and reflections that can build a diagram from one starting shape.

Learning Targets

- I can describe how to move one part of a shape to another using a translation, rotation or reflection.

Lesson Narrative

In this lesson, translations, rotations and reflections are applied to line segments and triangles. For line segments, students examine the impact of a 180 degree rotation. This is important preparatory work for studying parallel lines and translations, rotations and reflections, the topic of the next lesson. For triangles students look at a variety of transformations where rotations of 90 degrees and 180 degrees are again a focus. This work and the patterns that students build will be important later when they study Pythagoras' Theorem.

Throughout the lesson, students use the properties of translations, rotations and reflections (they do not change distances or angles) in order to make conclusions about the objects they are transforming.

Addressing

- Lines are taken to lines, and line segments to line segments of the same length.
- Angles are taken to angles of the same measure.

Building Towards

- Parallel lines are taken to parallel lines.

Instructional Routines

- Discussion Supports
 - Think Pair Share
-

Required Materials

Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Student Learning Goals

Let's rotate shapes in a plane.

8.1 Building a Quadrilateral

Warm Up: 5 minutes

Students rotate a copy of a right-angled isosceles triangle four times to build a quadrilateral. It turns out that the quadrilateral is a square. Students are not asked or expected to justify this but it can be addressed in the discussion. The fourth question about rotational symmetry of the quadrilateral will help students conclude that it is a square.

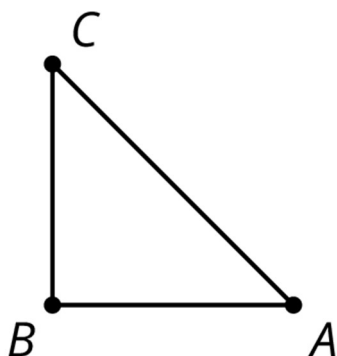
There are many more opportunities to build shapes using translations, rotations or reflections in other lessons.

Launch

Provide access to geometry toolkits, particularly tracing paper.

Student Task Statement

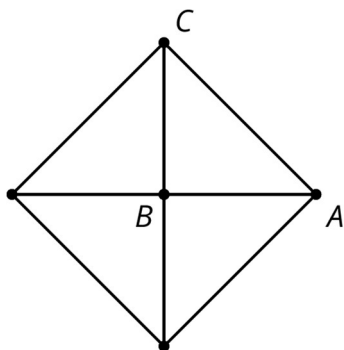
Here is a right-angled isosceles triangle:



1. Rotate triangle ABC 90 degrees clockwise around B .
2. Rotate triangle ABC 180 degrees clockwise round B .
3. Rotate triangle ABC 270 degrees clockwise around B .
4. What would it look like when you rotate the four triangles 90 degrees clockwise around B ? 180 degrees? 270 degrees clockwise?

Student Response

1-3.



4. The overall shape would look the same. These rotations just interchange the 4 triangles.

Activity Synthesis

Ask students what they notice and wonder about the quadrilateral that they have built. Likely responses include:

- It looks like a square.
- Rotating it 90 degrees clockwise or anti-clockwise interchanges the 4 copies of triangle ABC .
- Continuing the pattern of rotations, the next one will put ABC back in its original position.

Ask the students how they know the four triangles fit together without gaps or overlaps to make a quadrilateral. Here the key point is that the triangle is isosceles, so the rotations match up these sides perfectly. The four right angles make a complete 360 degrees, so the shape really is a quadrilateral. The fact that the quadrilateral is a square can be deduced from the fact that it is mapped to itself by a 90 degree rotation, but this does not need to be stressed or addressed.

8.2 Rotating a Line segment

15 minutes (there is a digital version of this activity)

The purpose of this activity is to allow students to explore special cases of rotating a line segment 180° . In general, rotating a line segment 180° produces a parallel line segment the same length as the original. This activity also treats two special cases:

- When the centre of rotation is the midpoint, the rotated line segment is the same line segment as the original, except the vertices are switched.
- When the centre of rotation is an endpoint, the line segment together with its image form a line segment twice as long as the original.

They are experimenting with a particular line segment but the conclusions that they make, especially in the last problem, are for *any* line segment.

Watch for how students explain that the 180° rotation of line segment CD in the second part of the question is parallel to CD . Some students may say that they “look parallel” while others might try to reason using the structure of the grid. Tell them that they will investigate this further in the next lesson.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Provide access to geometry toolkits. Give 3 minutes of quiet work time, followed by sharing with a partner and a whole-class discussion.

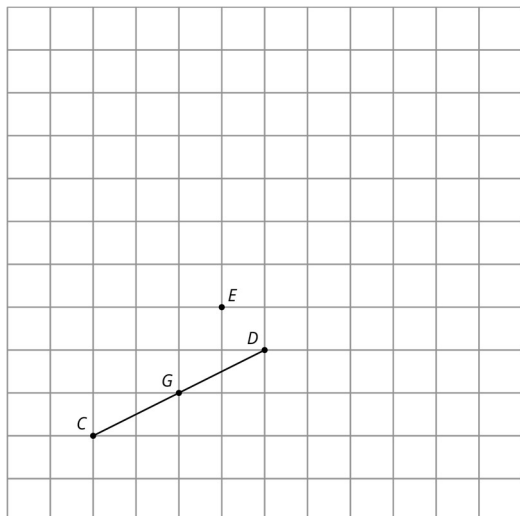
Representation: Internalise Comprehension. Begin the activity with concrete or familiar contexts. As in previous lessons, use tracing paper or digital software to rotate a line segment 180 degrees around a point. Lead the class in a think aloud considering the point of rotation to be the midpoint or endpoint of the line segment as an entry point for this activity.

Supports accessibility for: Conceptual processing; Memory

Anticipated Misconceptions

Students may be confused when rotating around the midpoint because they think the image cannot be the same line segment as the original. Assure students this can occur and highlight that point in the discussion.

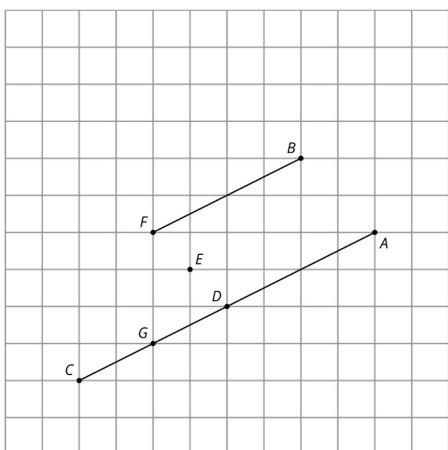
Student Task Statement



1. Rotate line segment CD 180 degrees around point D . Draw its image and label the image of C as A .
2. Rotate line segment CD 180 degrees around point E . Draw its image and label the image of C as B and the image of D as F .
3. Rotate line segment CD 180 degrees around its midpoint, G . What is the image of C ?
4. What happens when you rotate a line segment 180 degrees around a point?

Student Response

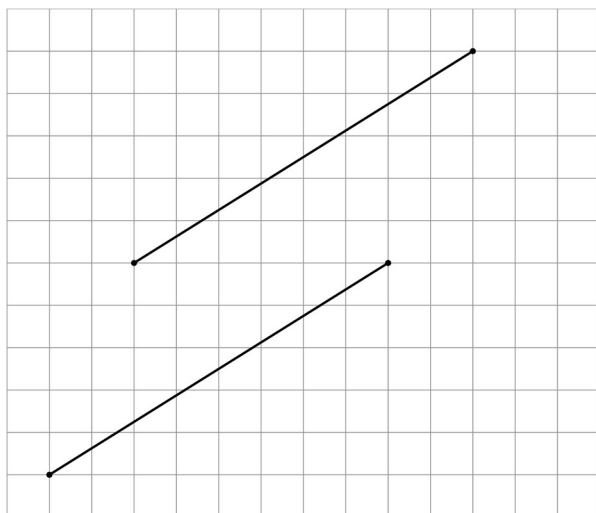
1.



2. The image of the line segment lines up with itself, but the endpoints are switched. D is now where C was and C is where D was.
3. The new line segment may change its location, but it remains the same length. The new line segment is parallel to the original line segment. When the point of rotation is

the midpoint of the line segment, then the rotated line segment is the same as the original (the endpoints swap places) and when the point of rotation is an end point of the line segment, the image connects to the original to form a line segment twice as long.

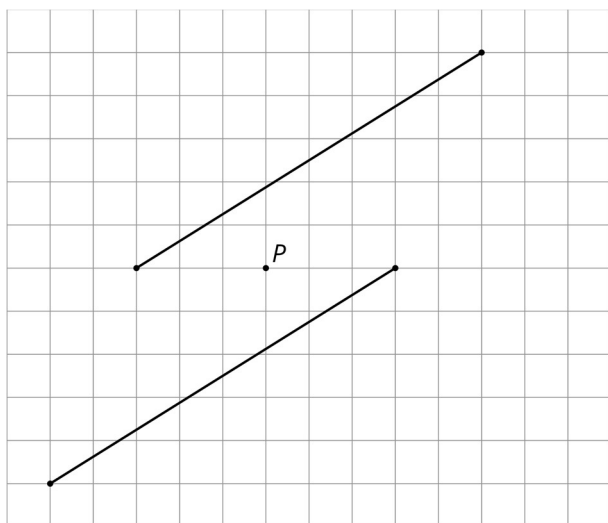
Are You Ready for More?



Here are two line segments. Is it possible to rotate one line segment to the other? If so, find the centre of such a rotation. If not, explain why not.

Student Response

Yes



Activity Synthesis

Ask students why it is not necessary to specify the direction of a 180 degree rotation (because a 180 degree clockwise rotation around point P has the same effect as a 180

degree anti-clockwise rotation around P). Invite groups to share their responses. Ask the class if they agree or disagree with each response. When there is a disagreement, have students discuss possible reasons for the differences.

Three important ideas that emerge in the discussion are:

- Rotating a line segment 180° around a point that is not on the original line segment produces a parallel line segment the same length as the original.
- When the centre of rotation is the midpoint, the rotated line segment is the same line segment as the original, except the vertices are switched.
- When the centre of rotation is an endpoint, the line segment together with its image form a line segment twice as long.

If any of the ideas above are not brought up by the students during the class discussion, be sure to make them known.

All of these ideas can be emphasised dynamically by carrying out a specified rotation in the applet and then moving the centre of rotation or an endpoint of the original line segment. Even if students are not using the digital version of the activity, you may want to display and demonstrate with the applet.

Speaking: Discussion Supports. Use this routine to support whole-class discussion when students discuss whether it is necessary to specify the direction of a 180° rotation. After a student speaks, call on students to restate and/or revoice what was shared using mathematical language (e.g., rotation, line segment, midpoint, etc.). This will provide more students with an opportunity to produce language as they explore special cases of rotating a line segment 180° .

Design Principle(s): Support sense-making; Maximise meta-awareness

8.3 A Pattern of Four Triangles

10 minutes (there is a digital version of this activity)

In this activity, students use rotations to build a pattern of triangles. In the previous lesson, students examined a right-angled triangle and a translation, rotation or reflection of the triangle. In this activity, several translations, rotations and reflections of the triangle form an interesting pattern.

Triangle ABC can be mapped to each of the three other triangles in the pattern with a single rotation. As students work on the first three questions, watch for any students who see that a single rotation can take triangle ABC to CDE . The centre for the rotation is not drawn in the diagram: it is the intersection of line segment AE and line segment CG . For students who finish early, guide them to look for a *single* transformation taking ABC to each of the other triangles.

This pattern will play an important role later when students use this shape to understand a proof of Pythagoras' Theorem.

Identify students who notice that they have already solved the first question in an earlier activity. Watch for students who think that $CAGE$ is a square and tell them that this will be addressed in a future lesson. However, encourage them to think about what they conclude about $CAGE$ now. Also watch for students who repeat the same steps to show that ABC can be mapped to each of the other three triangles.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 2–4. Provide access to geometry toolkits.

If using the digital activity, give students individual work time before allowing them to converse with a partner.

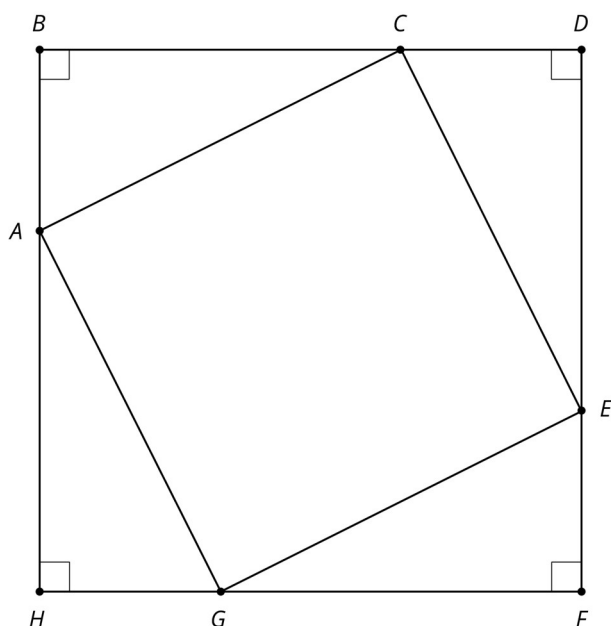
Representation: Internalise Comprehension. Demonstrate and encourage students to use colour coding and annotations to draw students' attention to the triangles in the diagram and the corresponding question. For example, highlight and isolate triangles ABC and CDE to help make connections to work in the previous activity.

Supports accessibility for: Visual-spatial processing

Anticipated Misconceptions

Some students might not recognise how this work is similar to the previous activity. For these students, ask them to step back and consider only triangles ABC and CDE , perhaps covering the bottom half of the diagram.

Student Task Statement



You can use translations, rotations or reflections of a shape to make patterns. Here is a diagram built with three different transformations of triangle ABC .

1. Describe a translation, rotation or reflection that takes triangle ABC to triangle CDE .
2. Describe a translation, rotation or reflection that takes triangle ABC to triangle EFG .
3. Describe a translation, rotation or reflection that takes triangle ABC to triangle GHA .
4. Do line segments AC , CE , EG , and GA all have the same length? Explain your reasoning.

Student Response

1. Answers vary. Sample responses:
 - Translate point B to point D , then rotate 90 degrees clockwise using D as centre.
 - Rotate anti-clockwise using C as centre until line segment CA matches up perfectly with line segment CE , then rotate 180 degrees using the midpoint of line segment CE as centre.
2. Answers vary. Sample responses:
 - Translate B to F and then rotate 180 degrees with centre F .
 - Translate so line segment AC matches up with line segment GE and then rotate 180 degrees with the midpoint of line segment GE as centre of rotation.
3. Answers vary. Sample responses:
 - Translate B to H and then rotate 90 degrees anti-clockwise with centre H .
 - Rotate with centre A so that line segment AC matches up with line segment AG and then rotate 180 degrees with the midpoint of line segment AG as centre.
4. Yes, because the size and shape of triangle ABC did not change under the transformation. Line segment AC can be matched up exactly with line segments CE , EG , and GA so the lengths of these line segments are all the same.

Activity Synthesis

Select a student previously identified who noticed how the first question relates to a previous activity to share their observation. Discuss here how previous work can be helpful in new work, since students may not be actively looking for these connections. The next questions are like the first, but the triangles have a different orientation and different transformations are needed.

Discuss translations, rotations and reflections. Focus especially on the question about lengths. A key concept in this section is the idea that lengths and angle measures are preserved under translations, rotations and reflections.

Some students may claim $CAGE$ is a square. If this comes up, leave it as an open question for now. This question will be revisited at the end of this unit, once the angle sum in a triangle is known. The last question establishes that $CAGE$ is a rhombus.

Speaking: Discussion Supports. Give students additional time to make sure that everyone in their group can explain whether the line segments AC , CE , EG , and GA all have the same lengths. Then, vary who is called on to represent the ideas of each group. This routine will prepare students for the role of group representative and to support each other to take on that role.

Design Principle(s): Optimise output (for explanation)

Lesson Synthesis

Ask students to describe the possible outcomes when a line segment AB is rotated 180 degrees.

- AB is mapped to itself, when the centre of rotation is the midpoint of the line segment
- AB is mapped to another line segment collinear with the first, when the centre of rotation is A or B (or any other point on line segment AB)
- AB is mapped to a parallel line segment, when the centre of rotation is not on line AB .

8.4 Is it a rotation?

Cool Down: 5 minutes

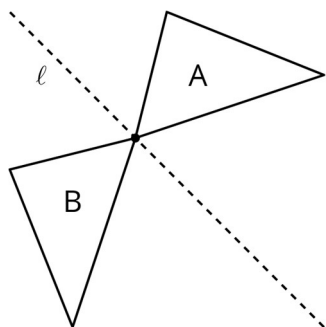
Having studied rotations in detail throughout this lesson, students look at a triangle and its image after a translation, rotation or reflection. They decide whether or not one is a rotation of the other. It turns out to be a reflection rather than a rotation. Students can use tracing paper to verify their conjectures, but at this point they should start to have an intuition for the effects of a rotation versus a reflection.

Launch

Make tracing paper available.

Student Task Statement

Here are two triangles.



Is triangle B a rotation of triangle A? Explain your reasoning.

Student Response

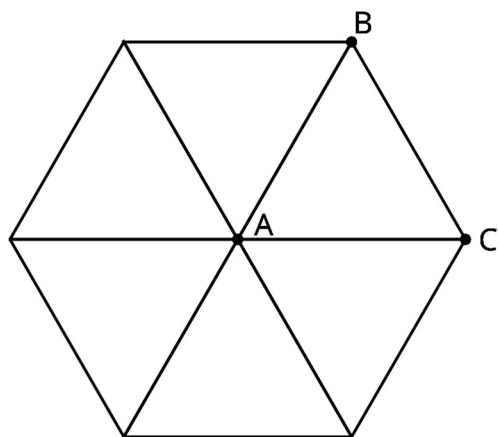
No, triangle B is a reflection of triangle A over line ℓ . A rotation can be used to match two sides of the triangles but will not match one up perfectly with the other.

Student Lesson Summary

When we apply a 180-degree rotation to a line segment, there are several possible outcomes:

- The line segment maps to itself (if the centre of rotation is the midpoint of the line segment).
- The image of the line segment overlaps with the line segment and lies on the same line (if the centre of rotation is a point on the line segment).
- The image of the line segment does not overlap with the line segment (if the centre of rotation is *not* on the line segment).

We can also build patterns by rotating a shape. For example, triangle ABC shown here has $\angle A = 60$. If we rotate triangle ABC 60 degrees, 120 degrees, 180 degrees, 240 degrees, and 300 degrees clockwise, we can build a hexagon.

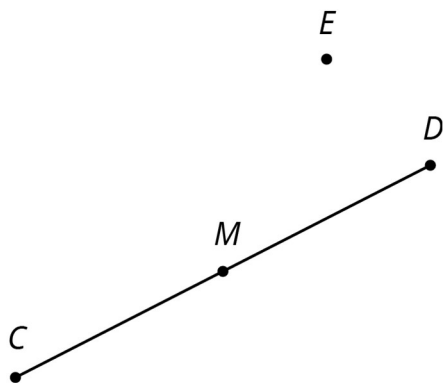


Lesson 8 Practice Problems

1. Problem 1 Statement

For the diagram shown here,

- Rotate line segment CD 180° around point D .
- Rotate line segment CD 180° around point E .
- Rotate line segment CD 180° around point M .



Solution

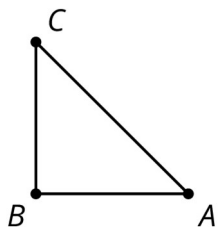
- The line segment is attached at point D and is an extension of line segment CD .
- The line segment is above point E and is parallel to line segment CD .
- The line segment is identical to line segment CD .

2. Problem 2 Statement

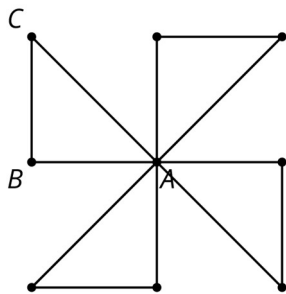
Here is an isosceles right-angled triangle:

Draw these three rotations of triangle ABC together.

- Rotate triangle ABC 90 degrees clockwise around A .
- Rotate triangle ABC 180 degrees around A .
- Rotate triangle ABC 270 degrees clockwise around A .



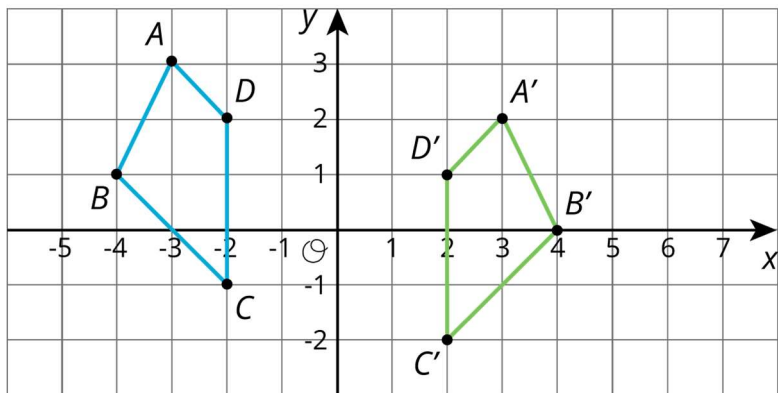
Solution



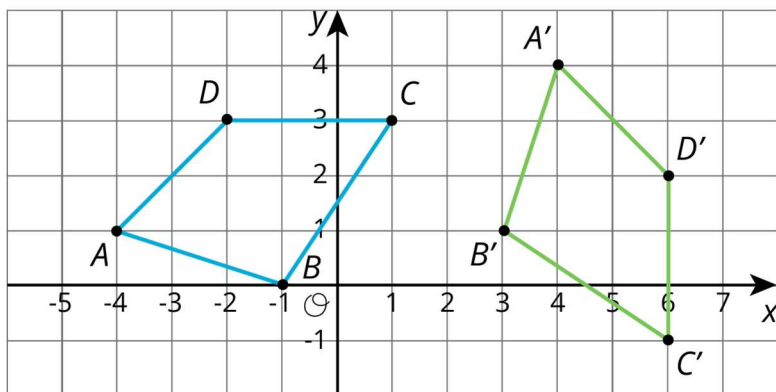
3. Problem 3 Statement

Each graph shows two polygons $ABCD$ and $A'B'C'D'$. In each case, describe a sequence of transformations that takes $ABCD$ to $A'B'C'D'$.

a.



b.

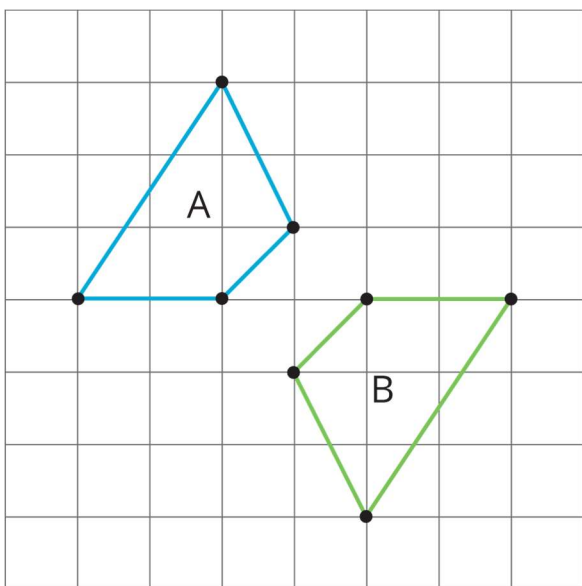


Solution

- a. Reflect $ABCD$ in the y -axis, and then translate down 1.
- b. Rotate $ABCD$ 90 degrees clockwise with centre $B = (-1,0)$, and then translate $(-1,0)$ to $(3,1)$.

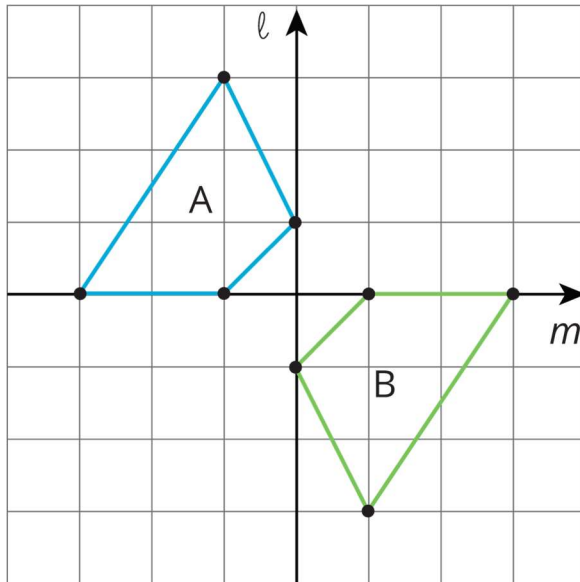
4. Problem 4 Statement

Lin says that she can map polygon A to polygon B using *only* reflections. Do you agree with Lin? Explain your reasoning.



Solution

I agree with Lin. If polygon A is reflected first over the vertical line ℓ and then in the horizontal line m , this takes polygon A to polygon B.



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