Lesson 17: Common multiples

Goals

- Comprehend (orally and in writing) the terms “multiple,” “common multiple,” and “least common multiple.”
- Explain (orally and in writing) how to calculate the least common multiple of 2 whole numbers.
- List the multiples of a number and identify common multiples for two numbers in a real-world situation.

Learning Targets

- I can explain what a common multiple is.
- I can explain what the least common multiple is.
- I can find the least common multiple of two whole numbers.

Lesson Narrative

In this lesson, students use contextual situations to learn about **common multiples** and the **least common multiples** of two whole numbers. They develop strategies for finding common multiples and least common multiples.

Addressing

- Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4(9 + 2)$.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Collect and Display
- Three Reads
- Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Materials

Multilink cubes

Required Preparation

For the first classroom activity, "The Florist's Order," provide access to two different colours of multilink cubes (100 of each colour) to students who would benefit from manipulatives. For students with visual impairment, provide access to manipulatives that are distinguished by their shape rather than by colour.

Student Learning Goals

Let's use multiples to solve problems.

17.1 Notice and Wonder: Multiples

Warm Up: 5 minutes

The purpose of this warm-up is to review factors and multiples while eliciting ideas on common factors and common multiples that will be useful in the activities of this lesson. While students may notice and wonder many things about the numbers they have circled, it is important for students to notice the multiples 4 and 6 have in common and wonder what other multiples they would have in common if the counting sequence continued.

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students 10 is a multiple of 5 because $10 = 5 \times 2$. One way you can find multiples of a number is by skip counting. For example, the multiples of 5 are 5, 10, 15, 20. . . and so on. Give students 1 minute of quiet work time to circle the multiples of 4 and 6. Give students 1 minute to discuss the things they notice and wonder with a partner, followed by a whole-class discussion.

Student Task Statement

Circle all the multiples of 4 in this list.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Circle all the multiples of 6 in this list.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

What do you notice? What do you wonder?

Student Response

Answers vary. Possible responses:

Students may notice:

- 4 and 6 have common multiples of 12, 24
- 4 has multiples not in common with 6 like 8, 16, 20...etc.
- 6 has multiples not in common with 4 like 6, 18, 30...etc.
- All of the common multiples of 4 and 6 are multiples of 12

Students may wonder:

- What other multiples would 4 and 6 have in common if we kept counting?
- Why are all of the common multiples of 4 and 6 also multiples of 12?
- Could we multiply 4 by any numbers to match all of the multiples of 6?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If there is time, ask students what other multiples 4 and 6 would have in common if the counting sequence continued. Ask them to explain their reasoning. Record and display their responses for all to see.

17.2 The Florist's Order

10 minutes

Students begin to think about common multiples and the least common multiple when finding ways to place two types of flowers into groups that contain the same number of each flower. Students find all multiples up to 100 for two different numbers. They compare these multiples to determine which ones are the same, and then they determine the least common multiple.

Look for different strategies and representations students use to describe the situation. Some students may draw pictures of groups of flowers, other students may use tables or lists, and other students may do a combination of these.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Arrange students into groups of 2. Give students 5–7 minutes of quiet work time, then 2 minutes of partner discussion. Follow with whole-class discussion.

Student Task Statement

A florist can order roses in bunches of 12 and lilies in bunches of 8. Last month she ordered the same number of roses and lilies.

1. If she ordered no more than 100 of each kind of flower, how many bunches of each could she have ordered? Find all the possible combinations.

2. What is the smallest number of bunches of roses that she could have ordered? What about the smallest number of bunches of lilies? Explain your reasoning.

Student Response

1. She could have placed 4 possible orders:
   - 2 bunches of roses and 3 bunches of lilies (24 of each kind of flower)
   - 4 bunches of roses and 6 bunches of lilies (48 of each kind of flower)
   - 6 bunches of roses and 9 bunches of lilies (72 of each kind of flower)
   - 8 bunches of roses and 12 bunches of lilies (96 of each kind of flower)
2. The smallest amount of bunches she could have ordered is 2 bunches of roses and 3 bunches of lilies (24 of each kind of flower).

Activity Synthesis

Invite students to share how they organised the different combinations of flowers, and highlight the different strategies they used. Strategies to highlight include tables, lists, multilink cubes, and other pictorial representations. Confirm that there are 4 different order combinations, and each time there are 24 more of each flower added. Discuss why the smallest number of flowers of each type is 24, and that it takes 2 bunches of roses to equal 24, and 3 bunches of lilies to equal 24. If there was a group that used multilink cubes, ask them to share what this arrangement looks like when represented with two different colours. If possible, use student responses to create a visual display of the concept of least common multiple and display it for all to see throughout the unit.

Speaking: Discussion Supports. Use this routine to amplify mathematical uses of language to communicate the multiple strategies used by different students. Provide sentence frames for students to use when they are comparing and contrasting different representations such as: “All ____ have ____ except ____.”, “What makes ____ different from the others is ____.” *Design Principle(s): Support sense-making; Cultivate conversation*

17.3 Least Common Multiple

10 minutes

In this activity, students are introduced to the terms **common multiple** and **least common multiple**.

Instructional Routines

- Collect and Display
- Think Pair Share

Launch

Arrange students in groups of 2. Ask students to discuss what they think a **common multiple** of two whole numbers is with their partner. Give 5 minutes of quiet work time followed by 2 minutes of partner discussion. Follow with whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge. Allow students to use calculators for finding greatest common factors and least common multiples to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing Conversing, Representing: Collect and Display. While pairs are working, circulate and listen to student talk about the meaning of and reason about least common multiple. Write down phrases (e.g., "the same," "the smallest,", "multiply") and representations (e.g., number lines, lists of multiples) you observe students using. Write these on a visual display, as this will help students use mathematical language as they represent least common multiples.
Design Principle(s): Support sense-making

Student Task Statement

The **least common multiple** of 6 and 8 is 24.

1. What do you think the term "least common multiple" means?
2. Find all of the **multiples** of 10 and 8 that are less than 100. Find the least common multiple of 10 and 8.
3. Find all of the multiples of 7 and 9 that are less than 100. Find the least common multiple of 7 and 9.

Student Response

1. Answers vary. Possible response: Least common multiple is the smallest multiple that numbers share.
2.
   - Multiples of 10: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
   - Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96.
   - Common multiples: 40, 80.
   - The least common multiple of 10 and 8 is 40.

3.

- Multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98.
- Multiples of 9: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99.
- Common multiples: only 63.
- The least common multiple of 7 and 9 is 63.

Are You Ready for More?

1. What is the least common multiple of 10 and 20?
2. What is the least common multiple of 4 and 24?
3. In the previous two questions, one number is a multiple of the other. What do you notice about their least common multiple? Do you think this will always happen when one number is a multiple of the other? Explain your reasoning.

Student Response

1. The multiples of 10 are 10, 20, 30, 40, . . . The multiples of 20 are 20, 40, 60, 80, . . . 20 is the least common multiple.
2. The multiples of 4 are 4, 8, 12, 16, 20, 24, . . . The multiples of 12 are 12, 24, 36, 48, 60, 72, . . . 12 is the least common multiple.
3. If one number is a multiple of the other, then it is a common multiple. For example, in the previous question with 4 and 12, we know that 12 is a multiple of 4 ($12 = 4 \times 3$) and 12 is a multiple of 12 ($12 = 12 \times 1$). If one number is a multiple of another, then it must also be the *least* common multiple, because no multiples of a number are less than the number itself. In the previous question, there are no multiples of 12 that are less than 12.

Activity Synthesis

The purpose of discussion is to clarify the process of finding common multiples and identifying the least common multiple. Ask students to discuss a way to find the least common multiple of any two numbers with a partner. Then in whole-class discussion, invite students to describe their process for finding least common multiples. Display several pairs of numbers and ask students to describe their process for finding common multiples and the least common multiple. Encourage students to use the terms “common multiple” and “least common multiple” in their responses.

17.4 Prizes on Grand Opening Day

15 minutes

In this activity, students continue to explore common multiples in context. Prizes are being given away to every 5th, 9th, and 15th customer. Students list the multiples of each number

when determining which customers get prizes and when customers get more than one prize. Customers who get more than one prize represent pairwise least common multiples (the first time this occurs for each pair). It is also true that the first customer who gets all 3 prizes represents the least common multiple of all three numbers, but this idea goes beyond the standards being addressed, and there aren't enough customers for this to happen. Students reason abstractly about common multiples and least common multiple to solve problems in context.

Monitor for students using these strategies:

- List numbers from 1 to 50 and skip count to identify common multiples.
- Analyse common multiples of pairs of numbers, rather than all three numbers at once.
- Denote multiples of different numbers with different shapes, colours, or other notations. Identify common multiples as numbers that have multiple designations.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Three Reads

Launch

Arrange students in groups of 2. Encourage students to discuss their reasoning with their partner as they work. Give students 10 minutes work time followed by a whole-class discussion.

Reading: Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., The bakery is giving away free prizes on opening day.). In the second read, ask students to describe quantities in the story (e.g., total of first 50 people get prizes, every fifth person gets a bagel, every 9th person a muffin, and every 12th person a slice of cake). In the third read, ask students to draw representations of the situation, and then brainstorm possible mathematical solution strategies to answer the questions. This helps students connect the language in the word problem and the reasoning needed to solve the problem, keeping the intended level of cognitive demand in the task.
Design Principle(s): Support sense-making

Student Task Statement

Lin's uncle is opening a bakery. On the bakery's grand opening day, he plans to give away prizes to the first 50 customers that enter the shop. Every fifth customer will get a free bagel. Every ninth customer will get a free blueberry muffin. Every 12th customer will get a free slice of carrot cake.

1. Diego is waiting in line and is the 23rd customer. He thinks that he should get farther back in line in order to get a prize. Is he right? If so, how far back should he go to get at least one prize? Explain your reasoning.

2. Jada is the 36th customer.

    a. Will she get a prize? If so, what prize will she get?

    b. Is it possible for her to get more than one prize? How do you know? Explain your reasoning.

3. How many prizes total will Lin's uncle give away? Explain your reasoning.

Student Response

1. Diego is correct that he will not get a prize because 23 is not a multiple of 5, 9, or 12, and he could get a prize if he went backward in line. If he goes back 1 spot to be the 24th customer, he will get a slice of carrot cake, because 24 is a multiple of 12. If he goes back 2 spots, he will be the 25th customer and get a bagel, because 25 is a multiple of 5. If he goes back 4 spots, he will be the 27th customer and blueberry muffin, because 27 is a multiple of 9.

2.

    a. Jada will get a muffin and a slice of carrot cake, because 36 is a multiple of 9 and of 12.

    b. Yes, it's possible. Her number in line is a common multiple of both 9 and 12, so she will get both prizes.

3. Lin's uncle will need to give away 10 bagels (to customers 5, 10, 15, 20, 25, 30, 35, 40, 45, and 50), 5 blueberry muffins (to customers 9, 18, 27, 36, and 45), and 4 slices of carrot cake (to customers 12, 24, 36, and 48). Altogether, he will give away 19 prizes ($10 + 5 + 4 = 19$).

Activity Synthesis

For each unique strategy, select one group to display their work for all to see and explain their reasoning. Sequence their presentations in the order they are presented in the Activity Narrative. If one or more of these strategies isn't brought up by students, demonstrate them. Encourage students to use the terms "common multiple" and "least common multiple."

Lesson Synthesis

In this lesson, students learned what a common multiple of two whole numbers is as well as the least common multiple. They also developed strategies for how to find the least common multiple of two whole numbers. Discuss:

- "What are some situations when finding least common multiple is helpful?" (When forming the smallest amount of equal groups, or when events both first happen at the same time.)
- "Explain what least common multiple means." (It is the smallest multiple that numbers share.)
- "How can we can determine least common multiple?" (List multiples of each number until you find the first one that is common to both lists.)

17.5 In Your Own Words Again

Cool Down: 5 minutes

Student Task Statement

1. What is the least common multiple of 6 and 9? Show your reasoning.
2. In your own words, what is the least common multiple of two whole numbers? How can you find it?

Student Response

1. The least common multiple of 6 and 9 is 18. The first few multiples of 6 are 6, 12, 18, 24, 30, 36. The first few multiples of 9 are 9, 18, 27, 36. The number 18 is the first to appear on both lists.
2. Answers vary. Possible response: The least common multiple of two numbers is the smallest multiple that the numbers share. You can find least common multiple by listing the multiples of each number until you find one that is common to both lists. The first multiple that is common to both lists is the least common multiple.

Student Lesson Summary

A multiple of a whole number is a product of that number with another whole number. For example, 20 is a multiple of 4 because $20 = 5 \times 4$.

A **common multiple** for two whole numbers is a number that is a multiple of both numbers. For example, 20 is a multiple of 2 and a multiple of 5, so 20 is a common multiple of 2 and 5.

The **least common multiple** (sometimes written as LCM) of two whole numbers is the smallest multiple they have in common. For example, 30 is the least common multiple of 6 and 10.

One way to find the least common multiple of two numbers is to list multiples of each in order until we find the smallest multiple they have in common. Let's find the least common multiple for 4 and 10. First, we list some multiples of each number.

- Multiples of 4: 4, 8, 12, 16, **20**, 24, 28, 32, 36, **40**, 44 . . .

- Multiples of 10: 10, **20**, 30, **40**, 50, . . .

20 and 40 are both common multiples of 4 and 10 (as are 60, 80, . . .), but 20 is the smallest number that is on *both* lists, so 20 is the least common multiple.

Glossary

- common multiple
- least common multiple

Lesson 17 Practice Problems

1. **Problem 1 Statement**

    a. A green light blinks every 4 seconds and a yellow light blinks every 5 seconds. When will both lights blink at the same time?

    b. A red light blinks every 12 seconds and a blue light blinks every 9 seconds. When will both lights blink at the same time?

    c. Explain how to determine when 2 lights blink together.

    Solution

    a. 20, 40, 60, 80, 100. . . seconds, because these are common multiples of 4 and 5.

    b. 36, 72, 108 . . . seconds, because these are common multiples of 12 and 9.

    c. Answers vary. Sample response: They blink together every common multiple.

2. **Problem 2 Statement**

    a. List all multiples of 10 up to 100.

    b. List all multiples of 15 up to 100.

    c. What is the least common multiple of 10 and 15?

    Solution

    a. 10, 20, 30, 40, 50, 60, 70, 80, 90, 100

    b. 15, 30, 45, 60, 75, 90

    c. 30

3. **Problem 3 Statement**

    Cups are sold in packages of 8. Napkins are sold in packages of 12.

a. What is the fewest number of packages of cups and the fewest number of packages of napkins that can be purchased so there will be the same number of cups as napkins?

b. How many sets of cups and napkins will there be?

Solution

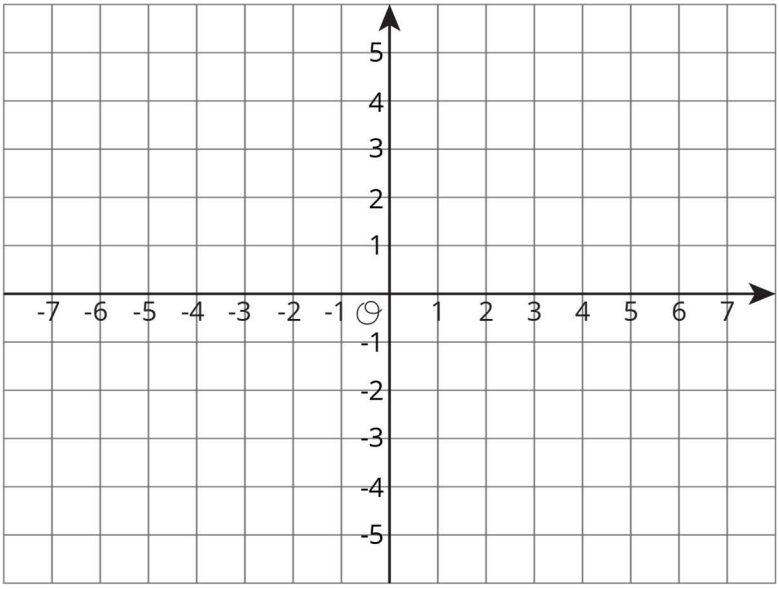
a. 3 packages of cups ($3 \times 8 = 24$) and 2 packages of napkins ($2 \times 12 = 24$)

b. 24 sets

4. **Problem 4 Statement**

a. Plot and connect these points to form a polygon.

(-5,3), (3,3), (1,-2), (-3,-2)



b. Find the lengths of the two horizontal sides of the polygon.

Solution

a. A trapezium is plotted.

b. The longer horizontal side is 8 units long, and the shorter horizontal side is 4 units long.

5. **Problem 5 Statement**

Rectangle ABCD is drawn on a coordinate grid. A = (-6,9) and B = (5,9). What could be the locations of points C and D?

Solution

Answers vary. Sample responses: C could be (-6,-9) and D could be (5,-9), or C could be (-6,18), and D could be (5,18).

6. **Problem 6 Statement**

A school wants to raise £2 500 to support its music programme.

a. If it has met 20% of its goal so far, how much money has it raised?

b. If it raises 175% of its goal, how much money will the music programme receive? Show your reasoning.

Solution

a. £500 ($2\,500 \times 0.2 = 500$)

b. £4 375 ($2\,500 \times 1.75 = 4\,375$)