13.7 Ejercicios

- Hallar un vector unitario normal a la superficie en el punto dado. Normalizar el vector gradiente $\nabla f(x,y,z)$.

⑦ $x^2+y^2+z^2=6$, $(1,1,2)$.

$F_x = 2x$ | $F_y = 2y$ | $F_z = 2z$

$F_x(1,1,2) = 2(1) = 2$ | $F_y(1,1,2) = 2(1) = 2$ | $F_z(1,1,2) = 2(2) = 4$

$$\nabla F = 2x\,i + 2y\,j + 2z\,k$$

$$\boxed{\nabla F(1,1,2) = 2\,i + 2\,j + 4k}$$

$$\nabla F(1,1,2) = \langle 2,2,4 \rangle$$

$$\|\nabla F(1,1,2)\| = \sqrt{2^2+2^2+4} = \sqrt{24} = \pm 2\sqrt{6}$$

$$\frac{\nabla F}{\|\nabla F\|} = \frac{\langle 2,2,4 \rangle}{\sqrt{24}} = \frac{2}{2\sqrt{6}}\,i + \frac{2}{2\sqrt{6}}\,j + \frac{4}{2\sqrt{6}}\,k$$

$$= \boxed{\frac{1}{\sqrt{6}}\,i + \frac{1}{\sqrt{6}}\,j + \frac{2}{\sqrt{6}}\,k}$$

- Hallar una ecuación del plano tangente a la superficie en el punto dado.

(17) $z = x^2 + y^2 + 3$, $(2, 1, 8)$

$x^2 + y^2 + 3 - z = 0$

$F_x = 2x$	$F_y = 2y$	$F_z = -1$
$F_x(2,1) = 2(2) = 4$	$F_y(2,1) = 2(1) = 2$	$F_y = -1$

$F(x, y, z) = x^2 + y^2 + 3 - z$

$4(x-2) + 2(y-1) + 1(z-8) = 0$

$4x - 8 + 2y - 2 - z + 8 = 0$

$\boxed{4x + 2y - z = 2}$

(27) $x^2 + 4y^2 + z^2 = 36$, $(2, -2, 4)$.

$F_x = 2x$	$F_y = 8y$	$F_z = 2z$
$F_x(2,-2,4) = 2(2) = 4$	$F_y(2,-2,4) = 8(-2) = -16$	$F_z(2,-2,4) = 2(4) = 8$

$F(x, y, z) = x^2 + 4y^2 + z^2 - 36$

$4(x-2) - 16(y+2) + 8(z-4) = 0$

$4x - 8 - 16y - 32 + 8z - 32 = 0$

$\frac{4x}{4} - \frac{16y}{4} + \frac{8z}{4} - \frac{72}{4} = 0$

$x - 4y + 2z - 18 = 0$

$\boxed{x - 4y + 2z = 18}$

- Hallar la ecuación del plano tangente y hallar ecuaciones simétricas para la recta normal a la superficie en el punto dado.

(35) $z = x^2 - y^2$, $(3, 2, 5)$

$F(x, y, z) = x^2 - y^2 - z$

$F_x = 2x$ | $F_y = -2y$ | $F_z = -1$

$F(3,2,5) = 2(3) = 6$ | $F(3,2,5) = -2(2) = -4$ | $F_z = -1$

$6(x-3) - 4(y-2) - 1(z-5) = 0$

$6x - 18 - 4y + 8 - z + 5 = 0$

$6x - 4y - z = 18 - 13$

$\boxed{6x - 4y - z = 5}$

Números directores: $6, -4, -1$

$$\boxed{\frac{x-3}{6} = \frac{y-2}{-4} = \frac{z-5}{5}}$$

(37) $xyz = 10$, $(1, 2, 5)$

$F(x, y, z) = xyz - 10$

$F_x = yz$ | $F_y = xz$ | $F_z = xy$

$F_x(1,2,5) = (2)(5) = 10$ | $F(1,2,5) = (1)(5) = 5$ | $F(1,2,5) = (1)(2) = 2$

$10(x-1) + 5(x-2) + 2(z-5) = 0$

$10x - 10 + 5x - 10 + 2z - 10 = \boxed{10x + 5x + 2z = 30}$

$$\boxed{\frac{x-1}{10} = \frac{x-2}{5} = \frac{z-5}{2}}$$