Lesson 7: Building polygons (Part 2)

Goals

- Explain (in writing) how to use circles to locate the point where the sides of a triangle with known side lengths should meet.
- Use manipulatives to justify when it is not possible to make a triangle with three given side lengths.
- Use manipulatives to show that there is a minimum and maximum length the third side of a triangle could be, given the other two side lengths.

Learning Targets

- I can reason about a shape with an unknown angle.
- I can show whether or not 3 side lengths will make a triangle.

Lesson Narrative

In this lesson, students experiment with constructing triangles given 2 or 3 side lengths. They start by working with cardboard strips and metal fasteners, as in the previous lesson. They discover that there are some combinations of lengths that do not make a triangle. Then students move toward using a ruler and compass, seeing that it recreates the functionality of the cardboard strips and metal fasteners more efficiently. The purpose of this transition is to help students move toward a mental understanding that does not depend on physical objects, helping them work toward the understanding that in a triangle the sum of any two sides must be greater than the other side.

Addressing

- Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

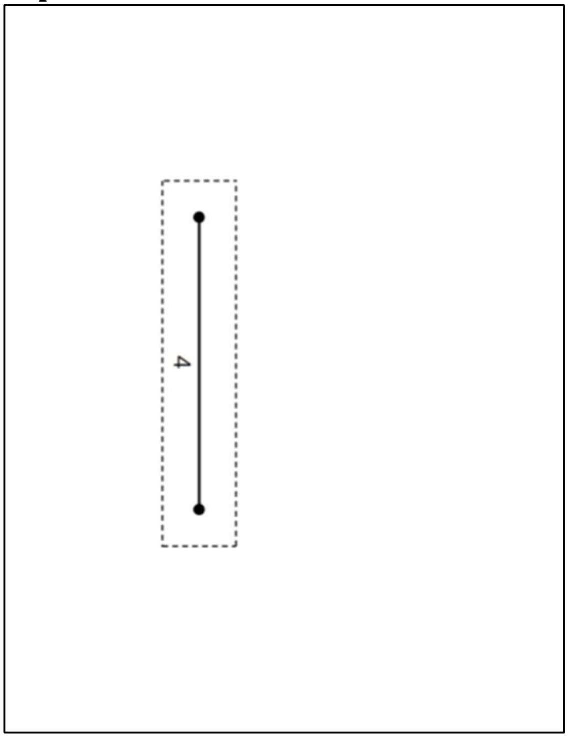
Instructional Routines

- Compare and Connect
- Discussion Supports
- Think Pair Share

Required Materials

Compasses

Copies of blackline master



Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Metal paper fasteners

Required Preparation

Print the Swinging the Sides Around blackline master. Prepare 1 copy for every 2 students.

Students will also need the cardboard strips and metal paper fasteners from the previous lesson, as well as access to geometry toolkits and compasses.

Note: If using the digital version of every activity, these supplies will not be needed.

Student Learning Goals

Let’s build more triangles.

7.1 Where Is Lin?

Warm Up: 5 minutes (there is a digital version of this activity)

The purpose of this warm-up is to remind students that when you have a fixed starting point, all the possible endpoints for a line segment of a given length form a circle (centred around the starting point). The context of finding Lin's position in the playground helps make the geometric relationships more concrete for students. Since there are many possible distances between Lin and the swings (but not infinitely many), this activity serves as an introduction to formalising rules about what lengths can and cannot be used to form a triangle.

Monitor for students who come up with different locations for Lin, as well as students who recognise that there are many possible locations, to share during the whole-class discussion.

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. If necessary, remind students of the directions north, south, east, and west and their relative position on a map. Provide access to geometry toolkits. Give students 2 minutes of quiet work time, followed by a partner and whole-class discussion.

Students with access to the digital materials can explore the applet.

Representation: Develop Language and Symbols. Display or provide charts with symbols and meanings. Display a chart of a compass showing the directions north, south, east and west. *Supports accessibility for: Conceptual processing; Memory*

Anticipated Misconceptions

Some students might assume that the swings, the slide, and Lin are all on a straight line, and that she must be 8 metres away. Ask these students if the problem tells us which direction Lin is from the slide.

Some students may confuse the type of compass discussed in the Launch and the type of compass discussed in the Activity Synthesis. Consider displaying a sample object or image of each of them and explain that the same name refers to two different tools.

Student Task Statement

At a park, the slide is 5 metres east of the swings. Lin is standing 3 metres away from the slide.

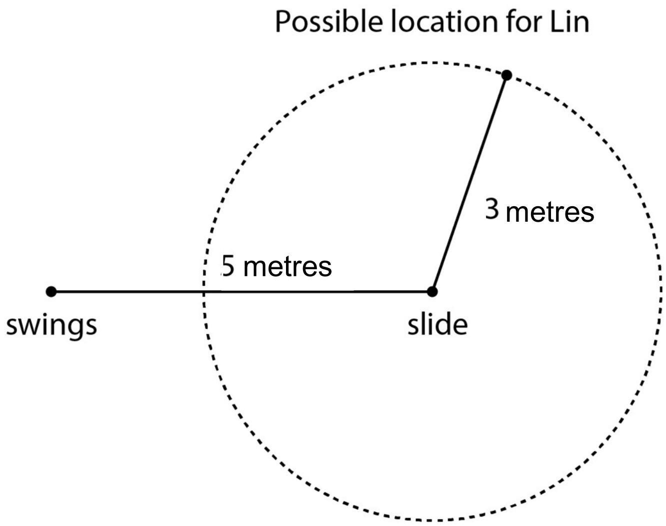
1. Draw a diagram of the situation including a place where Lin could be.

2. How far away from the swings is Lin in your diagram?

3. Where are some other places Lin could be?

Student Response

1. Answers vary. See diagram.



2. There is no way to know for sure, because we don't know what direction Lin is from the slide. She could be anywhere between 2 and 8 metres away from the swings.

3. Lin could be at any position along a circle that is centred on the slide and has a radius of 3 metres.

Activity Synthesis

First, have students compare answers and share their reasoning with a partner until they reach an agreement.

Next, ask selected students to share their diagrams of where Lin is located. Discuss the following questions with the whole class:

- "Do we know for sure where Lin is?" (No, because we don't know what direction she is from the swings.)
- "What shape is made by all the possible locations where Lin could be?" (a circle)
- "What is the closest Lin could be to the swings?" (2 m)
- "What is the farthest Lin could be away from the swings?" (8 m)

Consider using the applet at https://ggbm.at/qkHk6TpJ to show all the locations where Lin could be. Based on their work with drawing circles in a previous unit, some students may suggest that a compass could be used to draw all the possible locations where Lin could be. Consider having a student demonstrate how this could be done. If not mentioned by students, it is *not* necessary for the teacher to bring it up at this point.

7.2 How Long Is the Third Side?

15 minutes (there is a digital version of this activity)

The purpose of this activity is for students to experience that the sum of the lengths of the two shorter sides of a triangle must be greater than the length of the longest side. Students continue working with the cardboard strips and fasteners from the previous lesson to see how many different triangles they can build given two of the three side lengths. In the Activity Synthesis, the possible triangles are arranged in a way that helps students see the unknown angle between two known side lengths as a hinge. This prepares students for using compasses to draw triangles with given side lengths. They also continue to work at recognising when two triangles are identical copies that are oriented differently.

As students work, monitor for those who:

- find different lengths for the third side of the triangle
- use precise language to describe how the two side lengths can move in relation to each other
- make a connection to the circle of Lin's possible positions from the previous activity

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 4. Distribute *two* sets of strips and fasteners (from the previous lesson) to each group. Give students 7–10 minutes of group work time, followed by a whole-class discussion.

If using the digital lesson, students will be familiar with this applet from the previous lesson.

Anticipated Misconceptions

Some students may think that there are more than 5 possible triangles they can build, because they don't realise that some of the triangles they have listed are identical copies of other triangles on their list, with the side lengths written in a different order.

Some students may think that the third side of the triangle cannot be 4 or 5 inches, because then the triangle would have two sides of that length instead of the one asked for in the question. Explain that the triangle is acceptable as long as *at least* one side is 5 inches long and *at least* one side is 4 inches.

Some students may think that 8 is the longest the third side can be and 2 would be the shortest (if they were given a strip of that length), because they don't realise that there could be fractional side lengths.

Student Task Statement

Your teacher will give you some strips of different lengths and fasteners you can use to attach the corners.

1. Build as many different triangles as you can that have one side length of 5 inches and one of 4 inches. Record the side lengths of each triangle you build.

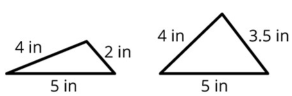
2. Are there any other lengths that could be used for the third side of the triangle but weren't in your set?

3. Are there any lengths that were in your set but could not be used as the third side of the triangle?

Student Response

1. There are 5 possible triangles, with the third side measuring 3 inches, 4 inches, 5 inches, 6 inches, or 8 inches.

2. Yes. The third side could be 2 inches, 7 inches, or any fractional length between 1 and 9 inches. Sample responses:



3. Yes. We could not use the 9 inch side to make a triangle, because it is a straight line when we connect it.

Are You Ready for More?

Assuming you had access to strips of any length, and you used the 9-inch and 5-inch strips as the first two sides, complete the sentences:

1. The third side can't be ____ inches or longer.

2. The third side can't be ____ inches or shorter.

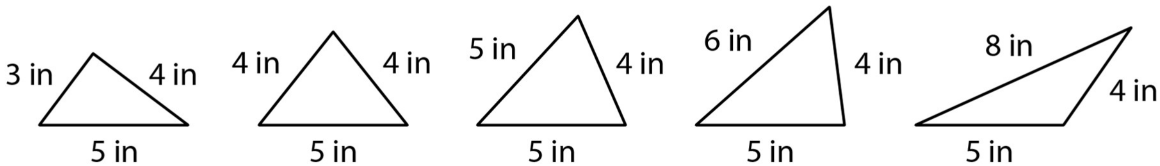
Student Response

The longest the third side could be is shorter than 14 inches (such as 13 or 13.9).

The shortest the third side could be is longer than 4 inches (such as 5 or 4.1).

Activity Synthesis

Select previously identified groups to share a triangle they created. Establish whether each new triangle shared is the same as a triangle previously shared or is a different triangle. Collect one example of each possible triangle and display them for all to see, in order of increasing side length for the third side. Continue until students agree all possible triangles are displayed.



To help students generalise about all the possible triangles that could be built with sides 4 inches and 5 inches, ask questions like the following:

- “What do you notice about the triangles?”
- “Why was it impossible to use the 9 inch side to create a triangle?”
- “What is the longest the third side of the triangle could be?” (more than 8, but less than 9, e.g., 8.5, 8.75, 8.9)
- “What is the shortest the third side of the triangle could be?” (less than 2, but more than 1, e.g., 1.5, 1.25, 1.1)
- “What happens when the third side is 1 inch or 9 inches?” (You get a straight line instead of a triangle.)

Display a 5 inch strip fastened to a 4 inch strip for all to see. Demonstrate rotating the 4-inch strip around 180° to line up with each of the displayed triangles, as well as to show the idea that the third side could have a fractional side length. Invite students to share how this relates to the previous activity about Lin’s distance from the swings. (If we hold one strip fixed, then all the possible locations where the other strip could end form a circle.)

Speaking: Discussion Supports. Use this routine to support students as they generalise about the length of the third side of the triangle. Provide sentence frames such as: “The third side of a triangle will always be ___ because…” This will help students use mathematical language to generalise that the sum of the lengths of the two shorter sides of a triangle must be greater than the length of the longest side.
Design Principle(s): Optimise output (for generalisation)

7.3 Swinging the Sides Around

15 minutes (there is a digital version of this activity)

The purpose of this activity is to relate the process for *building* a triangle given 3 side lengths (using cardboard strips and metal fasteners) to the process for *drawing* a triangle given 3 side lengths (using a compass). Students use the cardboard strips as an informal compass for drawing all the possible locations where the given line segments could end. They are reminded of their work with circles in a previous unit: that a circle is the set of all the points that are equally distant from a centre point and that a compass is a useful tool, not just for drawing circles, but also for transferring lengths in general. This prepares them for using a compass to draw triangles in future lessons.

In this activity, students also consider what their drawing would look like if the two shorter sides were too short to make a triangle with the third given side length.

Left-handed students may find it easier to start by drawing the 3-inch circle on the left side of the 4-inch line segment.

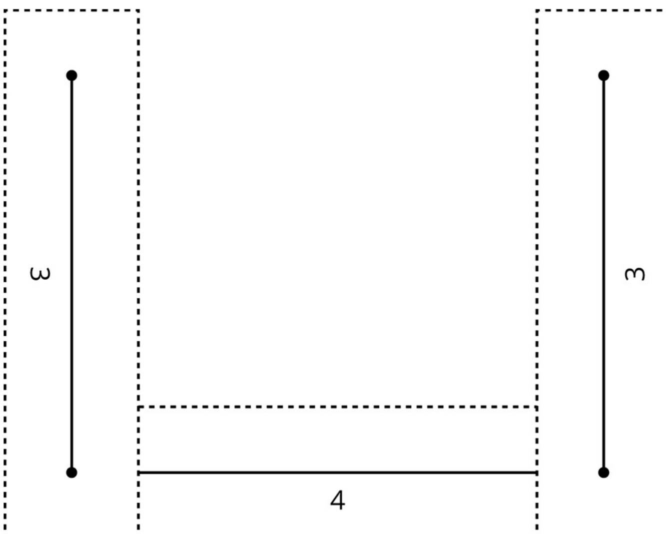
Instructional Routines

- Compare and Connect

Launch

Arrange students in groups of 2. Distribute one copy of the blackline master to each group. Make sure each group has one complete set of strips and fasteners from the previous activity. Provide access to geometry toolkits and compasses.

Tell students to take one 4-inch piece and two 3-inch pieces and connect them so that the 4-inch piece is in between the 3-inch pieces as seen in the image. If necessary, display the image for all to see. Students should not connect the 3-inch pieces to each other.



Explain to students that the sheet distributed to them is the 4-inch line segment that is mentioned in the task statement and they will be drawing on that sheet.

If using the digital version of the activity, students will be using the Show trace feature to see the path of the point. It is accessed by right-clicking on the point.

Action and Expression: Internalise Executive Functions. Begin with a small-group or whole-class demonstration of how to use the cardboard strips as an informal compass for drawing all the possible locations where the given line segments could end.
Supports accessibility for: Memory; Conceptual processing

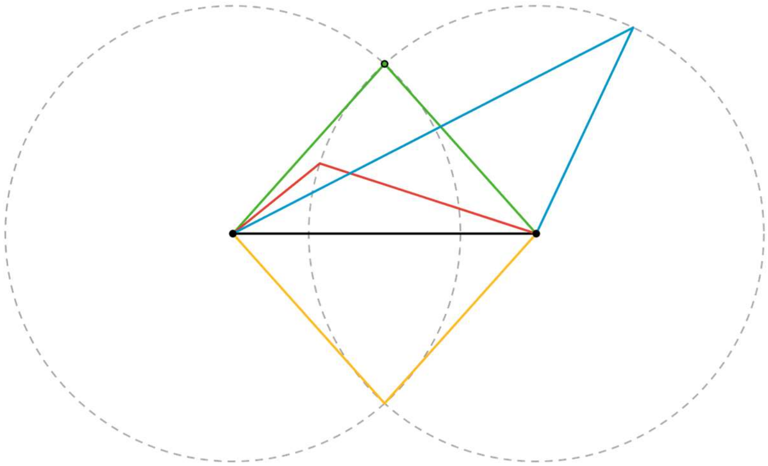
Student Task Statement

We'll explore a method for drawing a triangle that has three specific side lengths. Your teacher will give you a piece of paper showing a 4-inch line segment as well as some instructions for which strips to use and how to connect them.

1. Follow these instructions to mark the possible endpoints of one side:
    a. Put your 4-inch strip directly on top of the 4-inch line segment on the piece of paper. Hold it in place.
    b. For now, ignore the 3-inch strip on the left side. Rotate it so that it is out of the way.
    c. In the 3-inch strip on the *right* side, put the tip of your pencil in the hole on the end that is not connected to anything. Use the pencil to move the strip around its hinge, drawing all the places where a 3-inch side could end.
    d. Remove the connected strips from your paper.
2. What shape have you drawn while moving the 3-inch strip around? Why? Which tool in your geometry toolkit can do something similar?
3. Use your drawing to create two unique triangles, each with a base of length 4 inches and a side of length 3 inches. Use a different colour to draw each triangle.
4. Reposition the strips on the paper so that the 4-inch strip is on top of the 4-inch line segment again. In the 3-inch strip on the *left* side, put the tip of your pencil in the hole on the end that is not connected to anything. Use the pencil to move the strip around its hinge, drawing all the places where another 3-inch side could end.
5. Using a third colour, draw a point where the two marks intersect. Using this third colour, draw a triangle with side lengths of 4 inches, 3 inches, and 3 inches.

Student Response

1. Students draw the circle shown on the right side of the diagram.
2. A circle is all the points that are the same distance away from a centre point. A compass can be used to draw this.
3. Answers vary. Possible response: Students draw the small red triangle and the large blue triangle shown in the diagram.
4. Students draw the circle shown on the left side of the diagram.
5. Answers vary. Possible response: Students draw the green or yellow triangle shown in the diagram.



Activity Synthesis

Display a 4-inch strip connected to two 3-inch strips, positioned parallel to each other as pictured in the Launch. To help students connect the process of building with cardboard strips to drawing on paper, ask questions like:

- "If you want to *build* a triangle with these side lengths, how do you know at what angle to position the cardboard strips?" (Turn the sides until their unattached endpoints are touching.)
- "If you want to *draw* a triangle with these side length, how can you know at what angle to draw the sides?" (Find the point where both circles intersect.)
- "We have seen with the cardboard strips that an unknown angle works like a hinge. How is that represented in your drawing?" (with a circle centred on the endpoint of one line segment and a radius the length of the other line segment)

Select students to share their drawings with the class. To reinforce the patterns that students noticed in the previous activity, consider asking questions like these:

- "How many different triangles could we draw when we had only traced a circle on one side? Why?" (Lots of different triangles, because we were only using two of the given side lengths.)
- "What is the longest the third side could have been? And the shortest?" (Less the 7 inches; More than 1 inch)
- "How many different triangles could we draw once we had traced a circle on each side?" (It looked like there were 2 different triangles, but they are identical copies, so there's really only 1 unique triangle.)

Speaking, Listening, Representing: Compare and Connect. Use this routine to help students compare their processes for building and drawing their triangles. Ask students "What is the same and what is different?" about the approaches. Draw students' attention to the connections between building and drawing such as: opening the hinge between the cardboard strips and drawing the circle using the compass. These exchanges strengthen students' language use and reasoning from concrete to representational approaches. *Design Principle(s): Maximise meta-awareness*

Lesson Synthesis

- When you are given side lengths and asked to draw a triangle, how can you get started? (Hold one length fixed and swing the other around in a circle.)
- If you draw one side of the triangle with circles (of the correct radius for the other two side lengths) on each end, what does it look like when it is impossible to make a triangle? (The two circles do not intersect, or they intersect at a point on the first line segment.)
- If you draw one side of the triangle with circles on each end, and the circles *do* cross, they will cross twice. Why do we say there's only one possible triangle instead of two? (The two triangles are identical copies.)

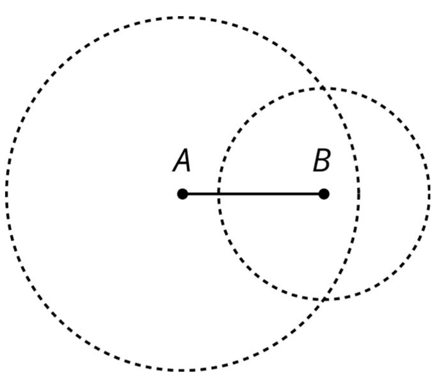
7.4 Finishing Elena's Triangles

Cool Down: 5 minutes

Student Task Statement

Elena is trying to draw a triangle with side lengths 4 inches, 3 inches, and 5 inches.

- She uses her ruler to draw a 4 inch line segment AB.
- She uses her compass to draw a circle around point B with radius 3 inches
- She draws another circle, around point A with radius 5 inches.

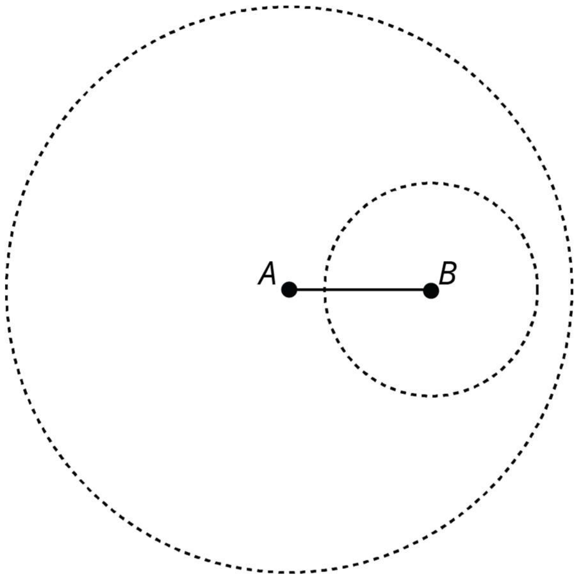


1. What should Elena do next? Explain and show how she can finish drawing the triangle.

Now Elena is trying to draw a triangle with side lengths 4 inches, 3 inches, and 8 inches.

- She uses her ruler to draw a 4 inch line segment AB.

- She uses her compass to draw a circle around point B with radius 3 inches.
- She draws another circle, around point A with radius 8 inches.



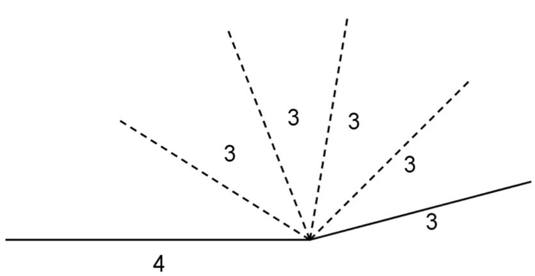
2. Explain what Elena's drawing means.

Student Response

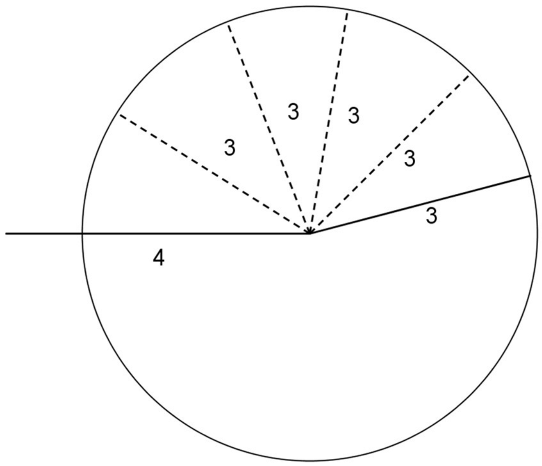
1. Elena should put a point where the two circles intersect and draw line segments connecting that point to points A and B to finish her triangle.
2. Elena's drawing means that there is no way to draw a triangle with these side lengths. The circles do not intersect, because the side lengths of 3 inches and 4 inches are too short to make a triangle with the third side of 8 inches.

Student Lesson Summary

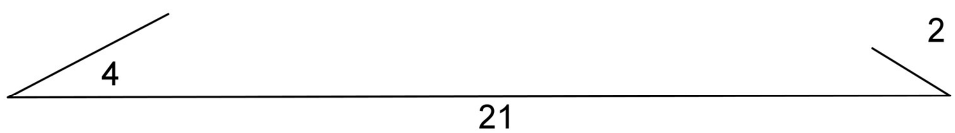
If we want to build a polygon with two given side lengths that share a vertex, we can think of them as being connected by a hinge that can be opened or closed:



All of the possible positions of the endpoint of the moving side form a circle:

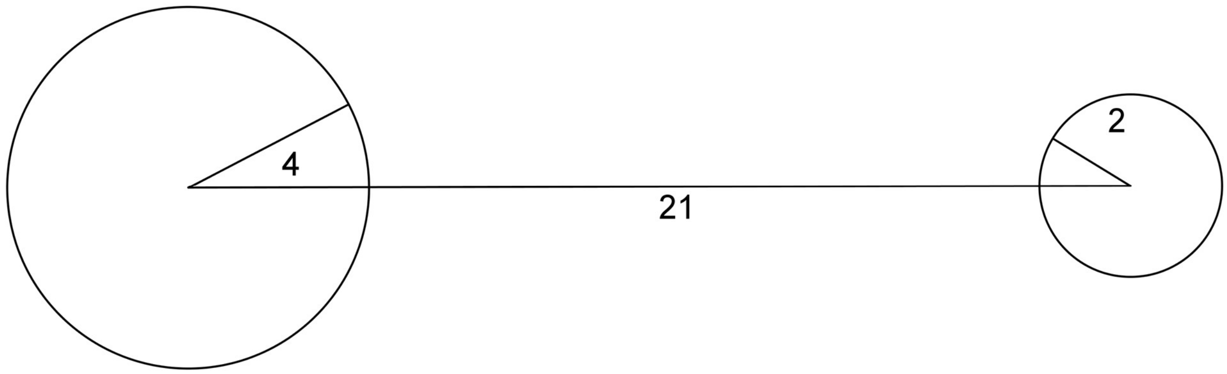


You may have noticed that sometimes it is not possible to build a polygon given a set of lengths. For example, if we have one really, really long line segment and a bunch of short line segments, we may not be able to connect them all up. Here's what happens if you try to make a triangle with side lengths 21, 4, and 2:



The short sides don't seem like they can meet up because they are too far away from each other.

If we draw circles of radius 4 and 2 on the endpoints of the side of length 21 to represent positions for the shorter sides, we can see that there are no places for the short sides that would allow them to meet up and form a triangle.



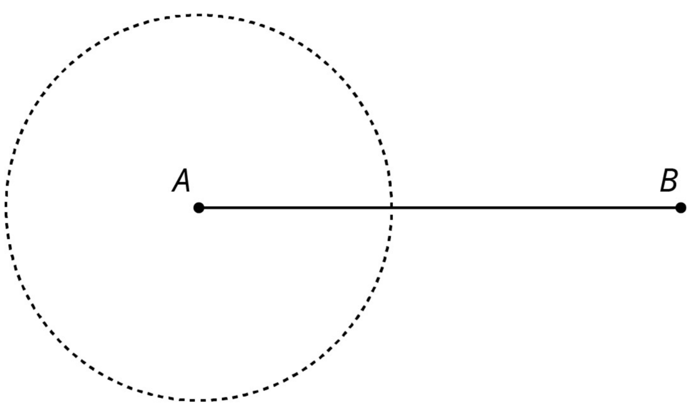
In general, the longest side length must be less than the sum of the other two side lengths. If not, we can't make a triangle!

If we *can* make a triangle with three given side lengths, it turns out that the sizes of the corresponding angles will *always* be the same. For example, if two triangles have side lengths 3, 4, and 5, they will have the same corresponding angles.

Lesson 7 Practice Problems

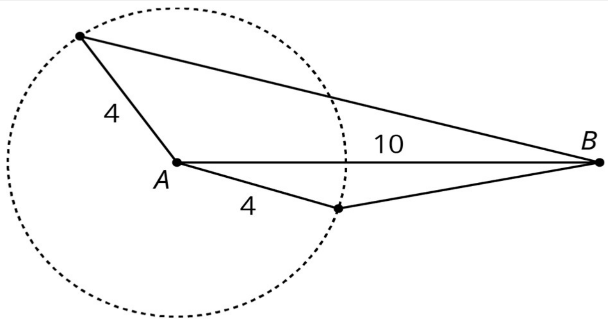
1. **Problem 1 Statement**

   In the diagram, the length of line segment AB is 10 units and the radius of the circle centred at A is 4 units. Use this to create two unique triangles, each with a side of length 10 and a side of length 4. Label the sides that have length 10 and 4.



   Solution

   Answers vary. Possible response:



2. **Problem 2 Statement**

   Select **all** the sets of three side lengths that will make a triangle.

   a. 3, 4, 8

   b. 7, 6, 12

   c. 5, 11, 13

   d. 4, 6, 12

   e. 4, 6, 10

   Solution ["B", "C"]

3. **Problem 3 Statement**

   Based on signal strength, a person knows their lost phone is exactly 47 feet from the nearest mobile phone tower. The person is currently standing 23 feet from the same mobile phone tower. What is the closest the phone could be to the person? What is the furthest their phone could be from them?

   Solution

   24 feet, 70 feet

4. **Problem 4 Statement**

   Each row contains the sizes in degrees of two complementary angles. Complete the table.

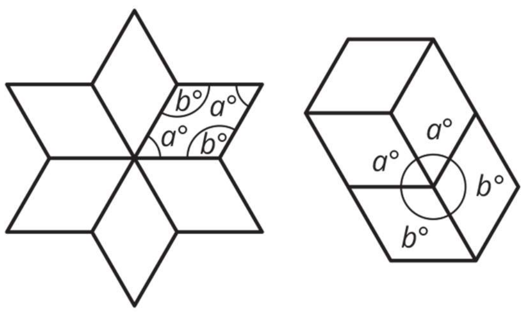
size of an angle	size of its complement
80°	
25°	
54°	
x	

Solution

size of an angle	size of its complement
80°	10°
25°	65°
54°	36°
x	$90 - x$

5. **Problem 5 Statement**

Here are two patterns made using identical rhombuses. Without using a protractor, determine the value of a and b. Explain or show your reasoning.



Solution

$a = 60$ because 6 a's make 360. $b = 120$ because 2 a's and 2 b's make 360.

6. **Problem 6 Statement**

Mai's family is traveling in a car at a constant speed of 65 miles per hour.

a. At that speed, how long will it take them to travel 200 miles?

b. How far do they travel in 25 minutes?

Solution

a. $3\frac{5}{65}$ hours or about 3 hours and 4.6 minutes ($200 \div 65$ hours)

b. $65 \times \frac{25}{60}$ miles or about 27.1 miles