Lesson 6: Area of parallelograms

Goals

- Apply the formula for area of a parallelogram to find the area, the length of the base, or the height, and explain (orally and in writing) the solution method.
- Choose which measurements to use for calculating the area of a parallelogram when more than one base or height measurement is given, and explain (orally and in writing) the choice.

Learning Targets

- I can use the area formula to find the area of any parallelogram.

Lesson Narrative

This lesson allows students to practise using the formula for the area of parallelograms, and to choose the measurements to use as a base and a corresponding height. Through repeated reasoning, they see that some measurements are more helpful than others. For example, if a parallelogram on a grid has a vertical side or horizontal side, both the base and height can be more easily determined if the vertical or horizontal side is used as a base.

Along the way, students see that parallelograms with the same base and the same height have the same area because the products of those two numbers are equal, even if the parallelograms look very different. This gives us a way to use given dimensions to find others.

Building On

- Represent and solve problems involving multiplication and division.

Addressing

- Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no brackets to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.
- Find the area of right-angled triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Discussion Supports

Required Materials

Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Student Learning Goals

Let's practise finding the area of parallelograms.

6.1 Missing Dots

Warm Up: 5 minutes

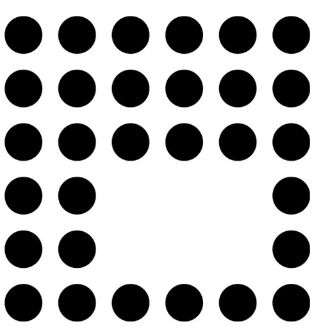
In this warm-up, students determine the number of dots in an image without counting and explain how they arrive at that answer. The activity also gives students a chance use decomposition and structure to quantify something, in a setting that is slightly different than what they have seen in this unit. To arrive at the total number of dots, students need to visualise and articulate how the dots can be decomposed, and use what they know about arrays, multiplication, and area to arrive at the number of interest. To encourage students to refer to the image in their explanation, ask students how they *saw* the dots instead of how they *found* the number of dots.

As in an earlier warm-up, consider establishing a small, discreet hand signal that students can display to indicate that they have an answer they can support with reasoning. This signal could be a thumbs-up, a certain number of fingers that tells the number of responses they have, or another subtle signal. This is a quick way to see if the students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class.

Launch

Give students 1–2 minutes of quiet think time and ask them to give a signal showing how many solutions they have. Encourage students who have found one way of seeing the dots to think about another way while they wait.

Student Task Statement



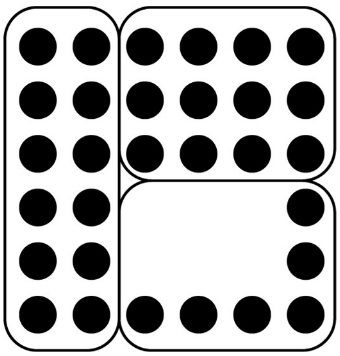
How many dots are in the image?

How do you see them?

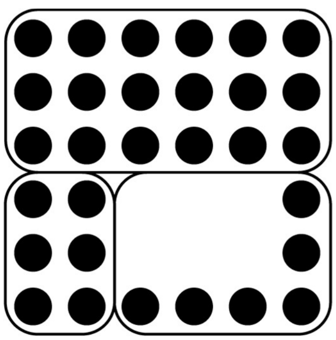
Student Response

30 dots. Strategies vary. Sample strategies:

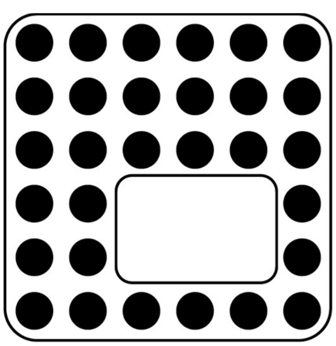
- Decomposing the image into parts, then multiplying and adding.
  - $(2 \times 6) + (3 \times 4) + 6 = 30$



  - $(3 \times 6) + (3 \times 2) + 6 = 30$



- Multiply to find the dots in the entire array and subtract the missing array of dots.
  - $(6 \times 6) - (2 \times 3) = 30$



Activity Synthesis

Ask students to share how many dots they saw and how they saw them. Record and display student explanations for all to see. To involve more students in the conversation, consider asking some of the following questions:

- "Who can restate the way ___ saw the dots in different words?"
- "Did anyone see the dots the same way but would explain it differently?"
- "Does anyone want to add an observation to the way ____ saw the dots?"
- "Do you agree or disagree? Why?"

6.2 More Areas of Parallelograms

25 minutes (there is a digital version of this activity)

This activity allows students to practise finding and reasoning about the area of various parallelograms—on and off a grid. Students need to make sense of the measurements and relationships in the given shapes, identify an appropriate pair of base-height measurements to use, and recognise that two parallelograms with the same base-height measurements (or with different base-height measurements but the same product) have the same area.

As they work individually, notice how students determine base-height pairs to use. As they work in groups, listen to their discussions and identify those who can clearly explain how they found the area of each of the parallelograms.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 4. Give each student access to their geometry toolkits and 5 minutes of quiet time to find the areas of the parallelograms in the first question. Then, assign each student one parallelogram (A, B, C or D). Ask each student to explain to the group, one at a time, how they found the area of the assigned parallelogram. After each student shares, check for agreement or disagreement from the rest of the group. Discuss any disagreement and come to a consensus on the correct answer before moving to the next parallelogram.

Afterwards, give students another 5–7 minutes of quiet work time to complete the rest of the activity.

For classrooms using the digital activity, arrange students in groups of 2. Ask each student to explain to their partner how they found the area of each parallelogram. When using the second applet, each student should each find one pair of quadrilaterals with equal area.

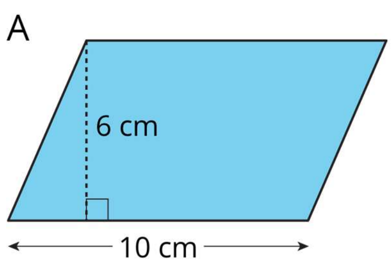
Representation: Internalise Comprehension. Use colour and annotations to illustrate student thinking. As students describe how they calculated the area of each parallelogram, use colour and annotations to scribe their thinking on a display. Ask students how they knew which measurements to use, and label each base and height accordingly.
Supports accessibility for: Visual-spatial processing; Conceptual processing

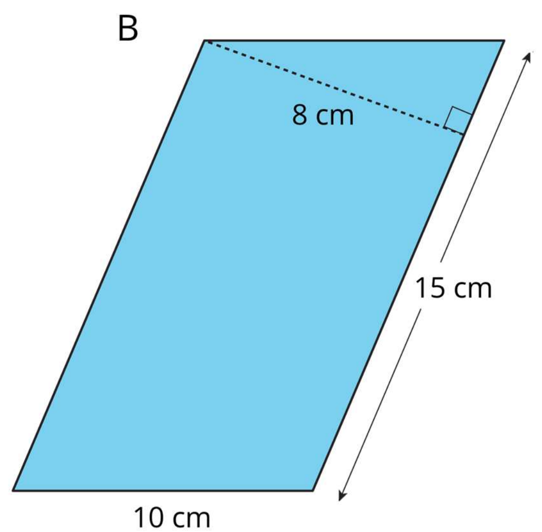
Anticipated Misconceptions

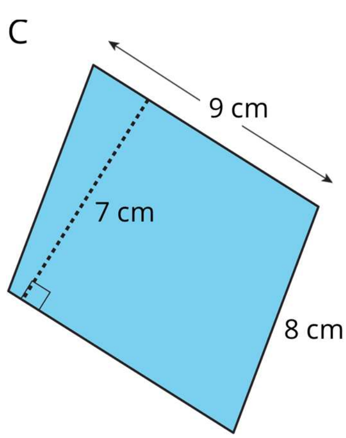
Some students may continue to use visual reasoning strategies (decomposition, rearranging, enclosing, and subtracting) to find the area of parallelograms. This is fine at this stage, but to help them gradually transition toward abstract reasoning, encourage them to try solving one problem both ways—using visual reasoning and their generalisation about bases and heights from an earlier lesson. They can start with one method and use the other to check their work.

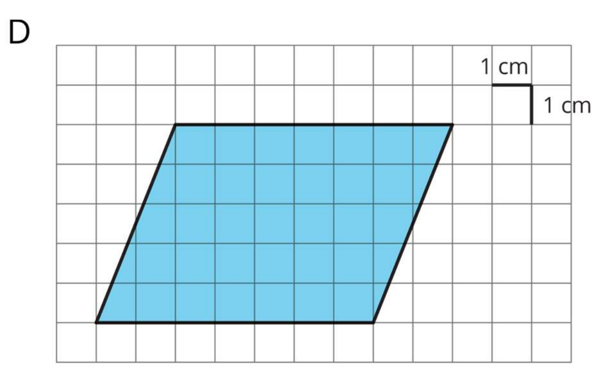
Student Task Statement

1. Find the area of each parallelogram. Show your reasoning.









2. In parallelogram B, what is the corresponding height for the base that is 10 cm long? Explain or show your reasoning.

3. Two different parallelograms P and Q both have an area of 20 square units. Neither of the parallelograms are rectangles.

   On the grid, draw two parallelograms that could be P and Q.

Student Response

1. A: $10 \times 6 = 60$ square centimetres
   B: $15 \times 8 = 120$ square centimetres
   C: $9 \times 7 = 63$ square centimetres
   D: $7 \times 5 = 35$ square centimetres

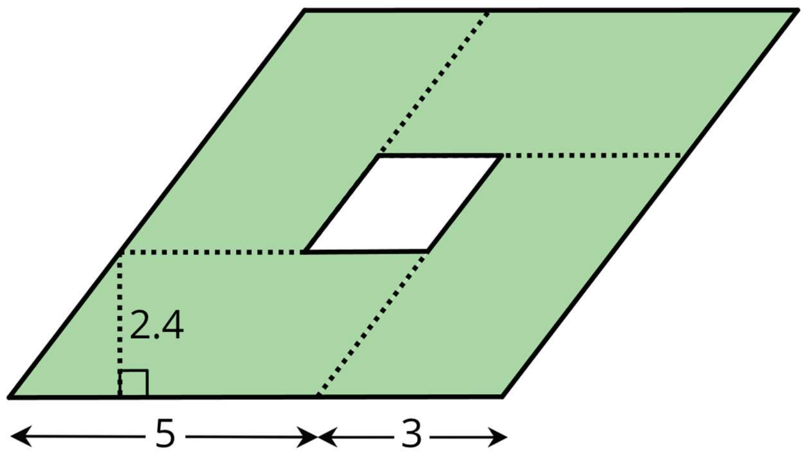
2. 12 centimetres. Sample reasoning: We found the area of the parallelogram to be 120 square centimetres. If the side that is 10 centimetres is the base, then $10 \times h$ must equal 120, so the height must be $120 \div 10$ or 12 centimetres.

3. Answers vary. Sample responses:

   - One parallelogram has a base of 10 units and a height of 2 units; another one has a base that is 4 units and a height that is 5 units.
   - One parallelogram has a base of 5 units and a height of 4 units; another one has a base that is 4 units and a height that is 5 units.
   - Two parallelograms with equal base and equal height but with different orientations, or with the pair of parallel bases positioned differently.

Are You Ready for More?

Here is a parallelogram composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches.



What is the area of the unshaded parallelogram in the middle? Explain or show your reasoning.

Student Response

3.2 square inches. Reasonings vary. Sample responses:

- The area of one shaded parallelogram is 12 square inches, because one base is 5 inches and its corresponding height is 2.4 inches $(5 \times (2.4) = 12)$. This means the corresponding height for the side that is 3 inches is 4 inches $(3 \times 4 = 12)$. The height of the small parallelogram is the difference between 4 inches and 2.4 inches, which is 1.6 inches. The horizontal side of the the unshaded parallelogram, which can be a base, is 2 inches $(5 - 3 = 2)$. The area of the unshaded parallelogram is therefore $2 \times (1.6)$ or 3.2 square inches.

- The base of the overall parallelogram is 8 inches $(5 + 3 = 8)$. Its height is 6.4 inches $(4 + 2.4 = 6.4)$. Its area is therefore $8 \times (6.4)$ or 51.2 square inches. The area of the four shaded parallelograms is 4×12 or 48 square inches. The area of the unshaded region is therefore $51.2 - 48$ or 3.2 square inches.

Activity Synthesis

Use whole-class discussion to draw out three important points:

1. We need base and height information to help us calculate the area of a parallelogram, so we generally look for the length of one side and the length of a perpendicular segment that connects the base to the opposite side. Other measurements may not be as useful.

2. A parallelogram generally has two pairs of base and height. Both pairs produce the same area (it's the same parallelogram), so the product of pair of numbers should equal the product of the other pair.

3. Two parallelograms with different pairs of base and height can have the same area, as long as their products are equal. So a 3-by-6 rectangle and a parallelogram with base 1 and height 18 will have the same area because $3 \times 6 = 1 \times 18$.

To highlight the first point, ask how students decided which measurements to use when calculating area.

- "When multiple measurements are shown, how did you know which of the measurements would help you find area?"
- "Which pieces of information in parallelograms B and C were not needed? Why not?"

To highlight the second point, select 1–2 previously identified students to share how they went about finding the missing height in the second question. Emphasise that the product 8×15 and that of 10 and the unknown h must be equal because both give us the area of the same parallelogram.

To highlight the last point, invite a few students to share their pair of parallelograms with equal area and an explanation of how they know the areas are equal. If not made explicit in students' explanations, stress that the base-height pairs must have the same product.

Speaking: Discussion Supports. Use this routine to support whole-class discussion when students explain how they created two parallelograms with equal area. For each explanation that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. If students are not able to restate, they should ask for clarification. Call students' attention to any words or phrases that helped to clarify the original statement, such as area, product, base, or height. This provides more students with an opportunity to produce language as they interpret the reasoning of others.
Design Principle(s): Support sense-making; Maximise meta-awareness

Lesson Synthesis

We used the formula for area to practise finding the area of various parallelograms.

- "When a parallelogram is on a grid, how do we know which side to choose for a base? Can we use any side?" (It is helpful to use a horizontal or a vertical side as a base; it would be easier to tell the length of that side and of its corresponding height.)
- "Off a grid, how do we know which measurements can help us find the area of a parallelogram?" (We need the length of one side of the parallelogram and of a perpendicular segment that connects that side to the opposite side.)

- "Do parallelograms that have the same area always look the same?" (No.) "Can you show an example?"
- "Do parallelograms that have the same base and height always look the same?" (No.) "Can you show an example?"
- "How can we draw two different parallelograms with the same area?" (We can find any two pairs of base-height lengths that have the same product. We can also use the same pair of numbers by drawing the parallelograms differently.)

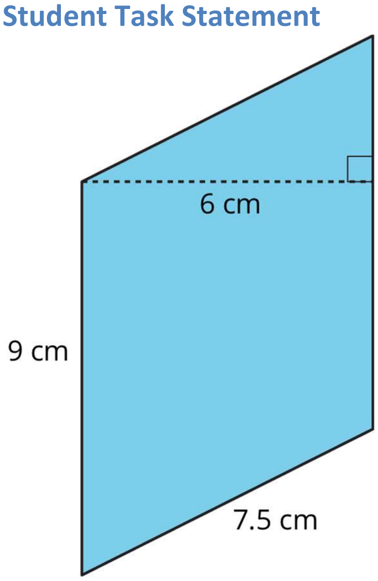
6.3 One More Parallelogram

Cool Down: 5 minutes

Launch

Access to geometry toolkits.

Student Task Statement



1. Find the area of the parallelogram. Explain or show your reasoning.
2. Was there a length measurement you did not use to find the area? If so, explain why it was not used.

Student Response

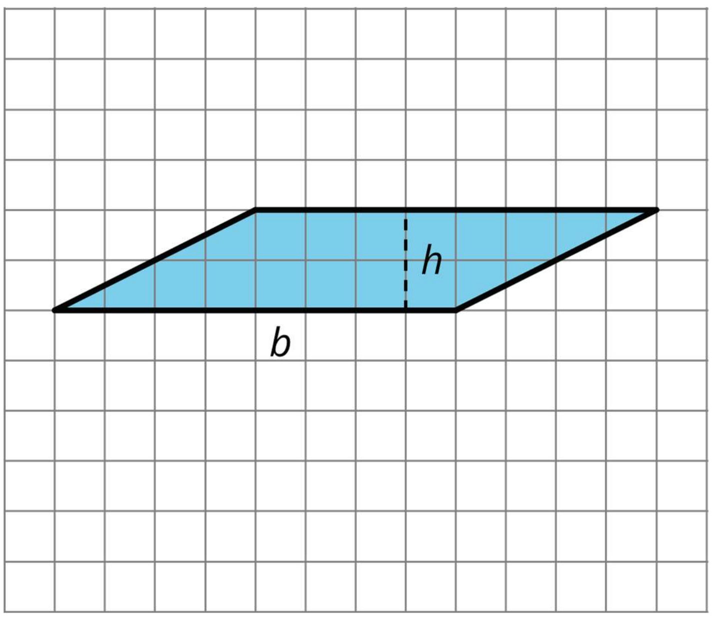
1. 54 cm^2. A base is 9 cm and its corresponding height is 6 cm. $9 \times 6 = 54$.
2. The 7.5 cm length was not used. Explanations vary. Sample explanations:
    - If the side that is 7.5 cm was used to find area, we would need the length of a perpendicular segment between that side and the opposite side as its corresponding height. We don't have that information.

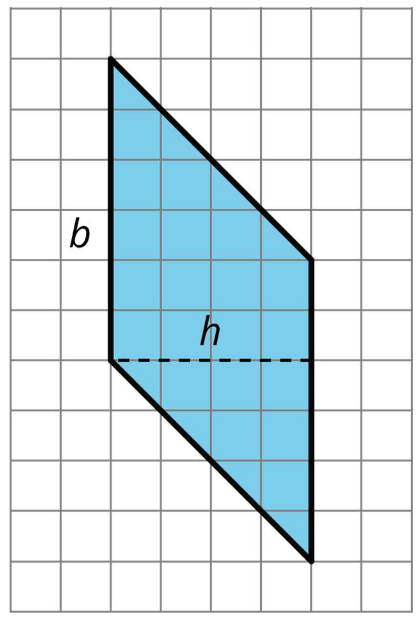
- The parallelogram can be decomposed and rearranged into a rectangle by cutting it along the horizontal line and moving the right-angled triangle to the bottom side. Doing this means the side that is 7.5 cm is no longer relevant. The rectangle is 6 cm by 9 cm; we can use those side lengths to find area.

Student Lesson Summary

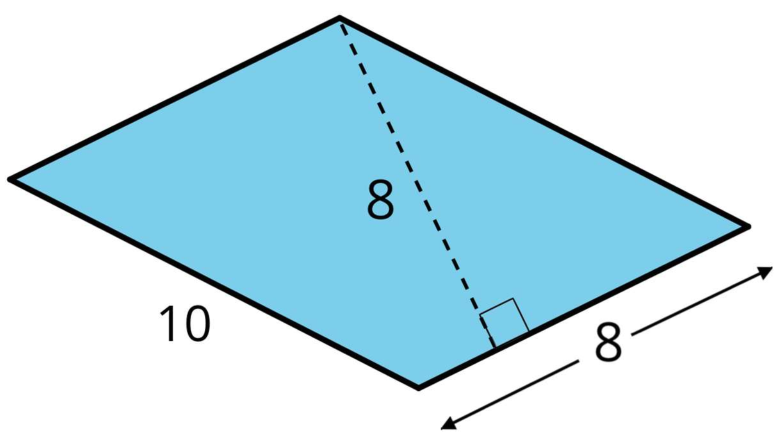
Any pair of base and corresponding height can help us find the area of a parallelogram, but some base-height pairs are more easily identified than others.

When a parallelogram is drawn on a grid and has *horizontal* sides, we can use a horizontal side as the base. When it has *vertical* sides, we can use a vertical side as the base. The grid can help us find (or estimate) the lengths of the base and of the corresponding height.



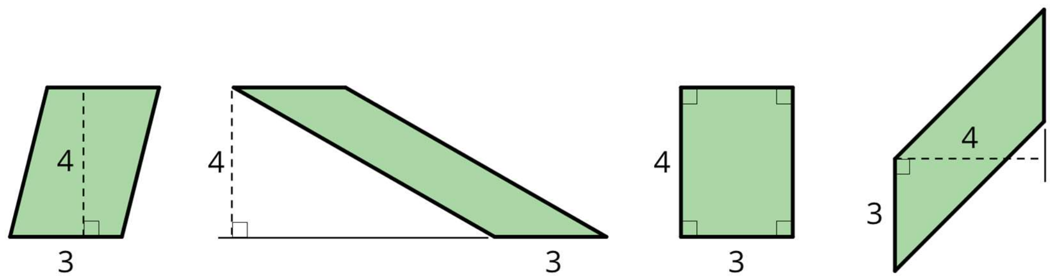


When a parallelogram is *not* drawn on a grid, we can still find its area if a base and a corresponding height are known.



In this parallelogram, the corresponding height for the side that is 10 units long is not given, but the height for the side that is 8 units long is given. This base-height pair can help us find the area.

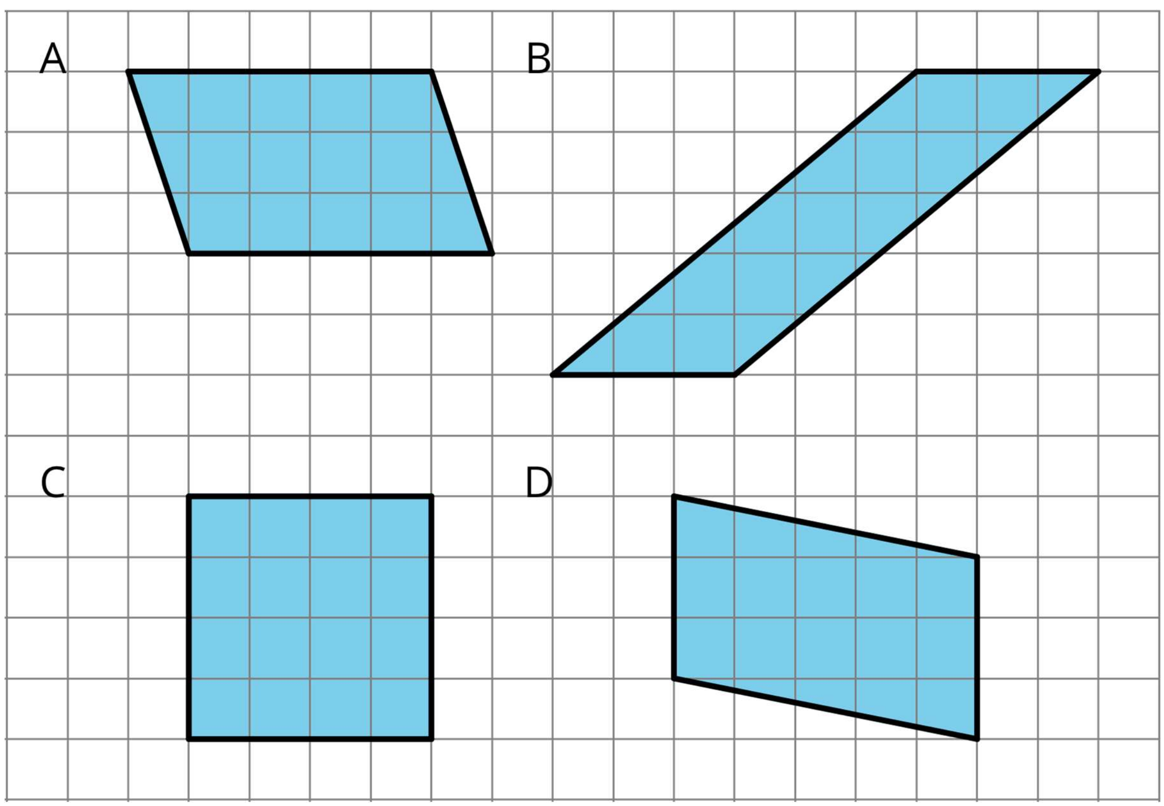
Regardless of their shape, parallelograms that have the same base and the same height will have the same area; the product of the base and height will be equal. Here are some parallelograms with the same pair of base-height measurements.



Lesson 6 Practice Problems

1. **Problem 1 Statement**

   Which three of these parallelograms have the same area as each other?



   a. A

   b. B

c. C

d. D

Solution ["A", "B", "D"]

2. **Problem 2 Statement**

Which pair of base and height produces the greatest area? All measurements are in centimetres.

a. $b = 4, h = 3.5$

b. $b = 0.8, h = 20$

c. $b = 6, h = 2.25$

d. $b = 10, h = 1.4$

Solution B

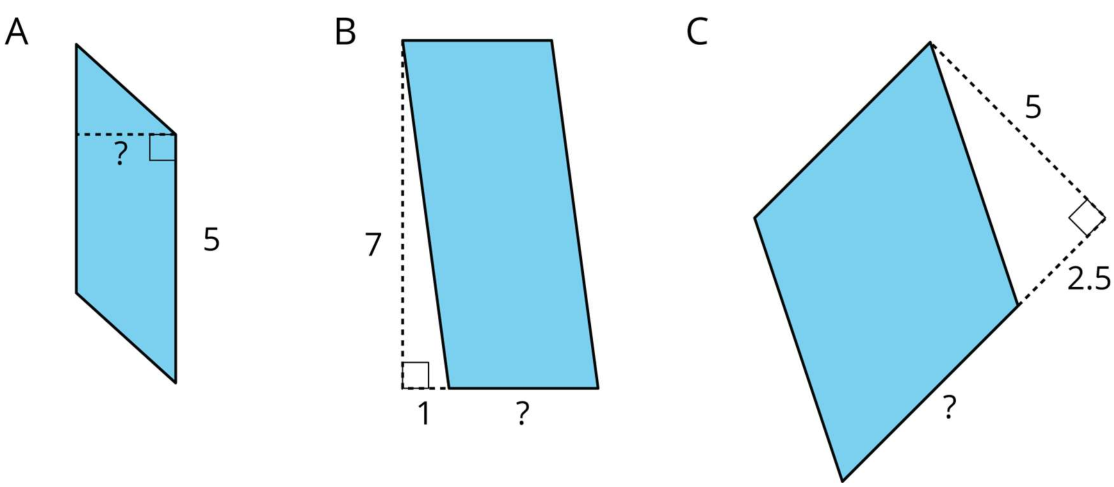
3. **Problem 3 Statement**

Here are the areas of three parallelograms. Use them to find the missing length (labelled with a "?") on each parallelogram.

A: 10 square units

B: 21 square units

C: 25 square units



Solution

A: 2 units

B: 3 units

C: 5 units

4. **Problem 4 Statement**

The Dockland Building in Hamburg, Germany is shaped like a parallelogram.

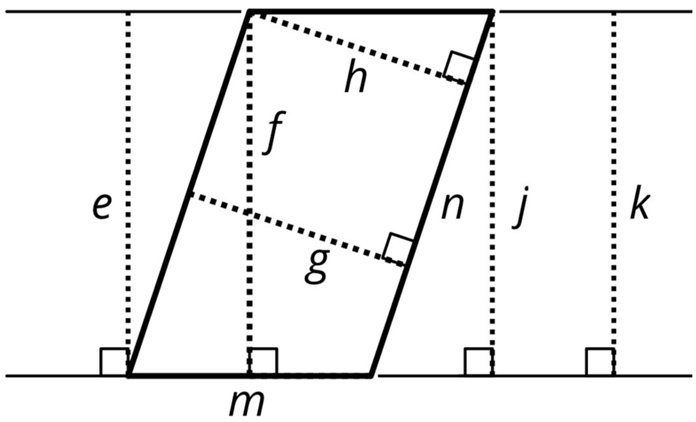
If the length of the building is 86 metres and its height is 55 metres, what is the area of this face of the building?

Solution

4 730 square metres ($86 \times 55 = 4\,730$).

5. **Problem 5 Statement**

Select **all** segments that could represent a corresponding height if the side m is the base.



A. e

B. f

C. g

D. h

E. j

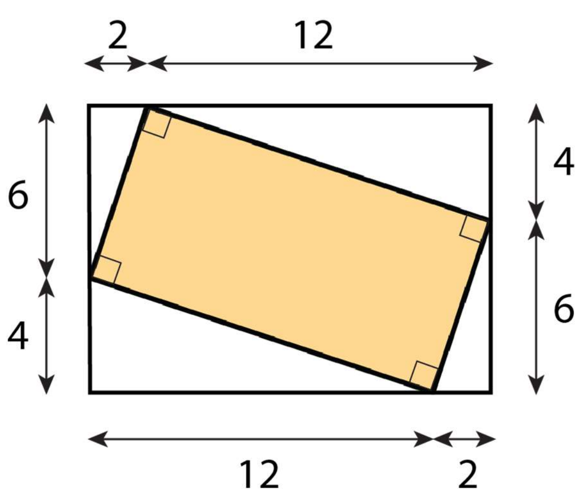
F. k

G. n

Solution ["A", "B", "E", "F"]

6. **Problem 6 Statement**

Find the area of the shaded region. All measurements are in centimetres. Show your reasoning.



Solution

80 square centimetres. Sample reasoning: The area of the large rectangle is 140 square centimetres, because $14 \times 10 = 140$. The areas of the small, unshaded right-angled triangles are each 6 square centimetres, because $6 \times 2 \div 2 = 6$. The areas of the larger, unshaded right-angled triangles are each 24 square centimetres, because $4 \times 12 \div 2 = 24$. Subtracting the areas of the four unshaded right-angled triangles from the area of the large rectangle yields 80: $140 - 6 - 6 - 24 - 24 = 80$.