Resolução 14.5

1) Calcule a integral iterada

1) $\int_{-1}^{1}\int_{0}^{2}\int_{0}^{1} (x^2+y^2+z^2)\, dx\, dy\, dz$

$\int_{0}^{1} (x^2+y^2+z^2)\, dx = \frac{x^3}{3} + y^2x + z^2x \Big|_{0}^{1} =$

$\Rightarrow \left(\frac{1^3}{3} + y^2 + z^2\right) - 0 = \frac{1}{3} + y^2 + z^2$ //

$\int_{0}^{2} \frac{1}{3} + y^2 + z^2\, dy = \frac{1}{3}y + \frac{y^3}{3} + z^2 y \Big|_{0}^{2}$

$\Rightarrow \left(\frac{1.2}{3} + \frac{2^3}{3} + z^2.2\right) - 0 = \frac{2}{3} + \frac{8}{3} + 2z^2 = \frac{10}{3} + 2z^2$

$\int_{-1}^{1} \frac{10}{3} + 2z^2\, dz = \int_{-1}^{1} \frac{10}{3}\, dz + 2\int_{-1}^{1} z^2\, dz$

$\frac{10z}{3}\Big|_{-1}^{1} = \frac{10}{3} + \frac{10}{3} = \frac{20}{3}$ // $+ \; 2 . \frac{z^3}{3}\Big|_{-1}^{1}$

$2 . \left[\frac{1}{3} + \frac{1}{3}\right] = \frac{4}{3} = \frac{20}{3} + \frac{4}{3} = \frac{24}{3} = 8$ //

5) $\int_0^3 \int_0^{\sqrt{9-z^2}} \int_0^x xy \, dy \, dx \, dz$

$$\int_0^x xy \, dy = x\frac{y^2}{2}\Big|_0^x = x.\frac{x^2}{2} - 0 = \frac{x^3}{2}$$

$$\int_0^{\sqrt{9-z^2}} \frac{x^3}{2} dx = \frac{x^4}{2.4}\Big|_0^{\sqrt{9-z^2}} = \frac{(\sqrt{9-z^2})^4}{8} - 0$$

$$\frac{((9-z^2)^{1/2})^4}{8} = \frac{(9-z^2)^2}{8}$$

$$\int_0^3 \frac{(9-z^2)^2}{8} dz = \frac{1}{8}\int_0^3 z^4 - 18z^2 - 81 \, dz$$

$$= \frac{1}{8}\left(\int_0^3 z^4 dz - \int_0^3 -18z^2 dz - \int_0^3 81 \, dz\right)$$

$$\int_0^3 z^4 dz = \frac{z^5}{5}\Big|_0^3 = \frac{3^5}{5} - 0 = \frac{243}{5}$$

$$\int_0^3 18z^2 dz = 18\frac{z^3}{3}\Big|_0^3 = 18.\frac{3^3}{3} = 162$$

$$\int_0^3 81 \, dz = 81z\Big|_0^3 = 81.3 = 243$$

$$\frac{1}{8}\left(243 - 162 + \frac{243}{5}\right) = \boxed{\frac{81}{5}}$$

credeal

9) $\iiint_G xy\,\text{sen}(yz)\,dv$, onde G é a caixa retangular definida pelas desigualdades $0 \leq x \leq \pi$, $0 \leq y \leq 1$, $0 \leq z \leq \frac{\pi}{6}$.

$dz\,dy\,dx$

$$\int_0^{\pi}\int_0^{1}\int_0^{\pi/6} xy\,\text{sen}(yz)\,dz\,dy\,dx$$

$$\int_0^{\pi/6} xy\,\text{sen}(yz)\,dz = xy\int_0^{\pi/6} \text{sen}(\underbrace{yz}_{u})\,dz$$

$u = yz$

$du = y\,dz$

$\frac{du}{y} = dz$

$$x\int_0^{\pi/6} \text{sen}(u)\,du$$

$$x(-\cos(u))\Big|_0^{\pi/6} =$$

$$-\cos(yz)x\Big|_0^{\pi/6} = -\cos\left(y\cdot\frac{\pi}{6}\right)x + \cos(y\cdot 0)x$$

$$= -\cos\left(y\frac{\pi}{6}\right)x + \cos(0)\cdot x$$

$$= -\cos\left(y\frac{\pi}{6}\right)x + x \;//$$

$$\int_0^1 -\cos\left(\frac{y\pi}{6}\right)x + x\;dy = -\int_0^1 \cos\left(\frac{y\pi}{6}\right)x\,dy + \int x\,dy$$

$$-x\int_0^1 \cos\left(\underbrace{\frac{y\pi}{6}}_{u}\right)dy + \int x\,dy$$

$u = \frac{y\pi}{6}$ $\qquad \frac{6du}{\pi} = dy$

$du = \frac{\pi}{6}dy$

$$-x\int_0^1 \cos(u)\,\frac{6du}{\pi}$$

$$-x\cdot\frac{6}{\pi}\int_0^1 \cos(u)\,du$$

$$-x\frac{6}{\pi}\sin(u) + C$$