Grades 7-8 (A), 9-12 (A)

Duration: 30-45 min

Tools: one Logifaces Set / 3-5 students

Individual / Pair work

Keywords: Logical thinking, Cases, Nets

511 - Logifaces Nets

MATHS / NET

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

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DESCRIPTION

In Levels 1-4 students consider the given net of the block 222 and find the number of nets of this type only. Two nets are different if they cannot be transformed into each other by reflection or rotation. (This can be told to students as "you can not distinguish the nets after cutting them out of the paper").

diagram 1

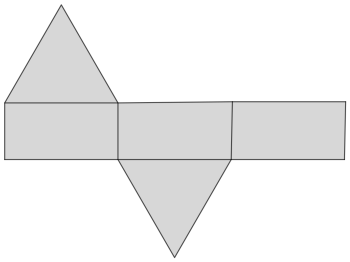
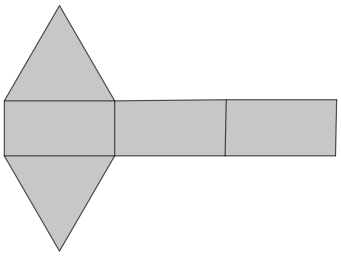


diagram 2



They work with the following nets and blocks:

LEVEL 1 diagram 1 and blocks 123 or 132

LEVEL 2 diagram 1 and blocks 133, 122, 233, 113, 112 or 223

LEVEL 3 diagram 2 and blocks 123 or 132

LEVEL 4 diagram 2 and blocks 133, 122, 233, 113, 112 or 223

In Levels 5 and 6 students consider any type of nets and find the number of all different nets of the given block.

They work with the following blocks:

LEVEL 5 blocks 123 or 132

LEVEL 6 blocks 133, 122, 233, 113, 112 or 223

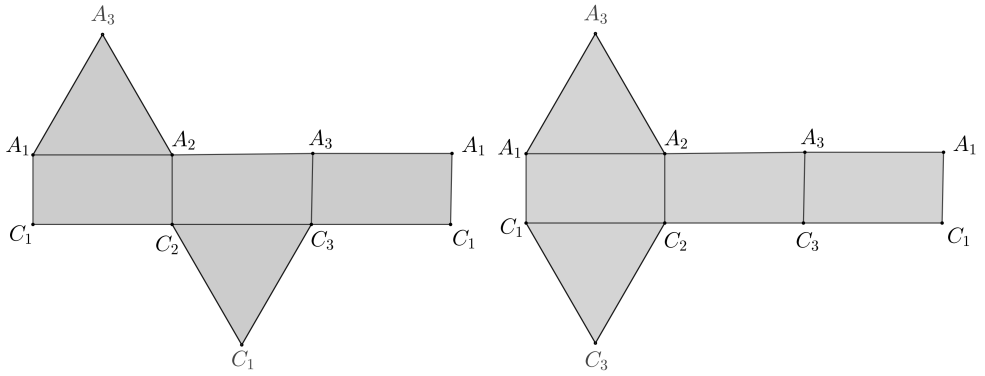
SOLUTIONS / EXAMPLES

It may be useful to emphasise what we mean by equivalent nets.
Note on the symmetries of the solutions: The nets of blocks 123 and 132 are exactly the same because of the equivalence of the nets that can be transformed into each other by reflections.
The symmetries of block 133 imply less or equal number of nets than in the case of block 123.

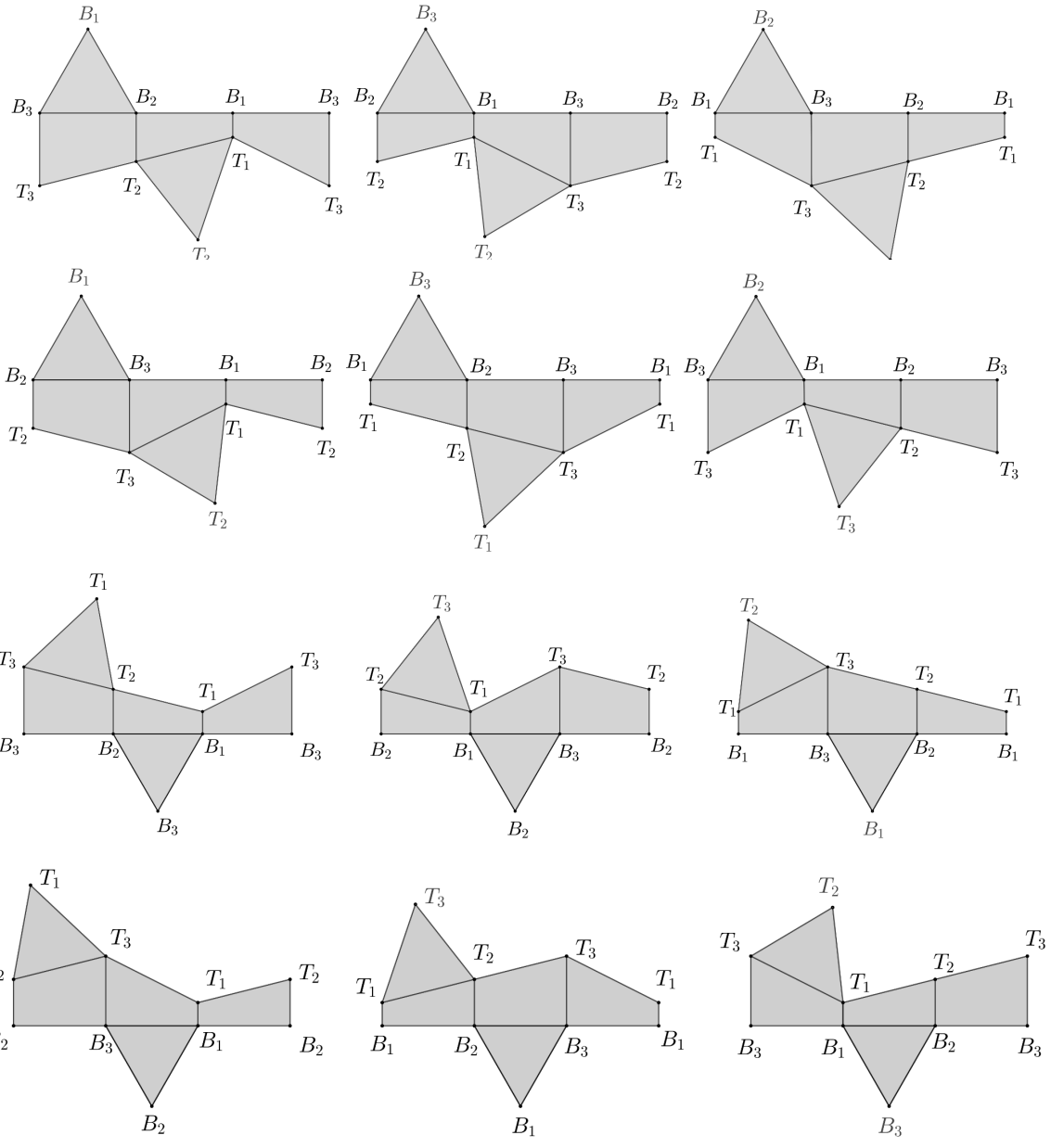
In the solution the vertices of the given net type representing the same vertex are marked with the same letter in the figure of the net type. The letters B_i and T_i stand for the base and top vertices of the block of an edge of height i, respectively.

The solutions of Levels 1-4 are given both by a combinatorial argument and by the list of the solutions.

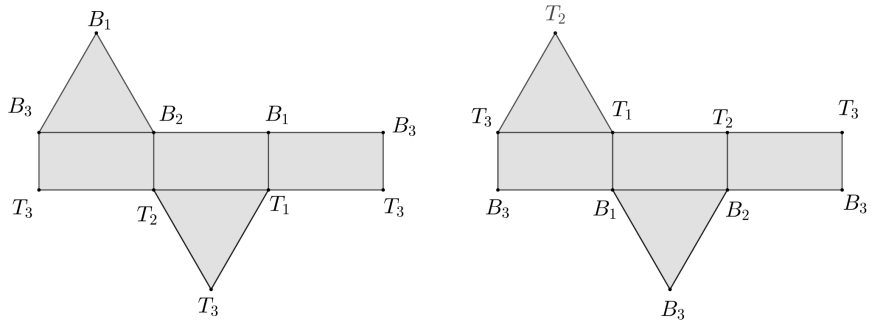
LEVELS 1-2 Each vertex on the figure is distinguishable, hence the number of nets can be counted by a simple combinatorial argument.



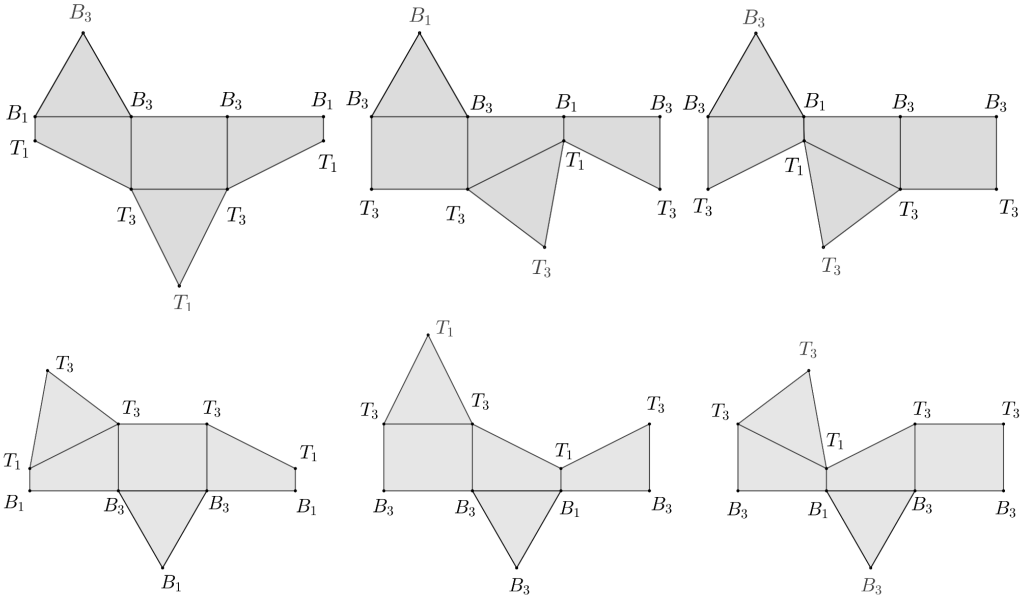
LEVEL 1 The vertex B_1 can be chosen among the 6 vertices A_1, A_2, A_3, C_1, C_2, C_3. The vertex B_2 must be a neighbouring vertex with the same letter (one vertex of the same triangle), so there are 2 choices for this vertex given the choice for B_1. All other vertices are determined by B_1 and B_2, hence there are $6 \times 2 = 12$ different nets of the given types for the blocks 123 or 132. The 12 different nets are listed below:



ASSISTANCE FOR THE STUDENTS The different nets are maybe too difficult to draw. An easier way to list the nets is to write the vertices in the diagram of the given net of the block 222. In this way, the diagram is the same (which is easy to draw), and only the letters change. For example, the first and the last net of the solution of the Level 1 exercise are drawn below in this easier way.

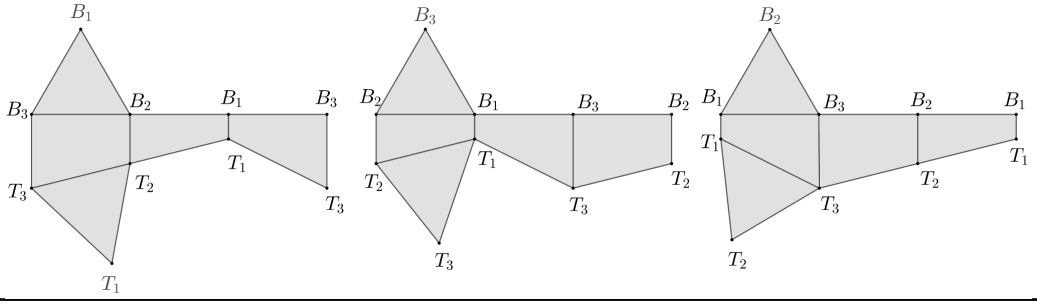


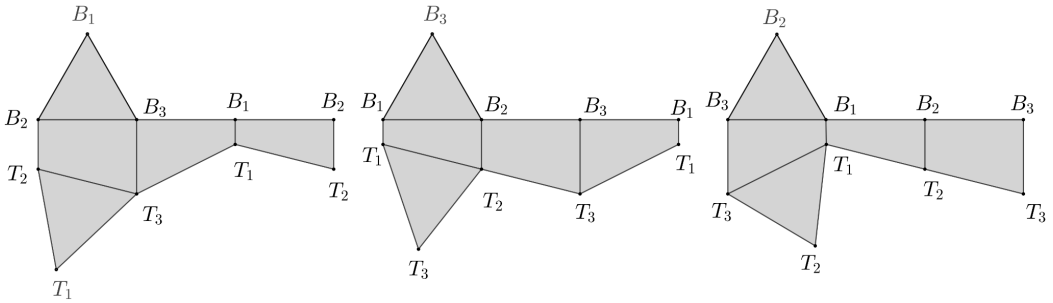
LEVEL 2 The solution is given for block 133. For the other blocks it is similar, and the number of nets is the same. The vertex B_1 can be chosen among the 6 vertices A_1, A_2, A_3, C_1, C_2, C_3, and this choice determines all other vertices, hence there are 6 different nets of the given type for block 133. The 6 different nets are listed below:



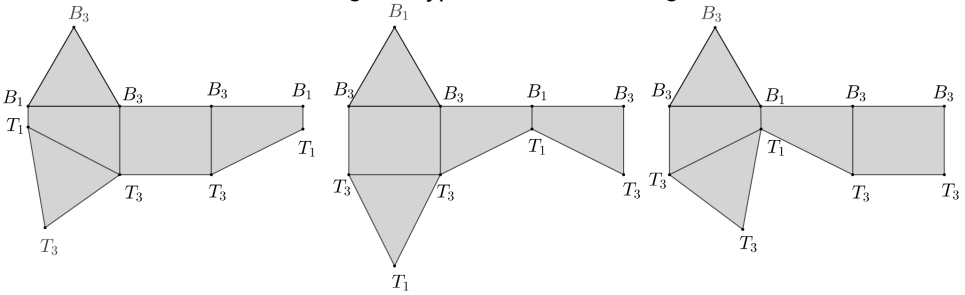
LEVELS 3-4 The vertices of the net type are not distinguishable, because the perpendicular line through the midpoint of the segment A_1C_1 is an axis of symmetry of the diagram. Hence every choice of the vertices gives the same net as the choice symmetrical to this axis. It follows that if the number of nets of this type is counted by the same combinatorial argument as in the solutions of Levels 1-2, then every choice has a symmetrical pair that gives the same net. It follows that the number of nets is half of the numbers in Levels 1 and 2: the number of nets in Level 3 is $12/2 = 6$, and in Level 4 is $6/2 = 3$.

LEVEL 3 Block 123 has 6 nets of the given type, as seen in the figure below.





LEVEL 4 Block 133 has 3 nets with the given type, as seen in the figure below.

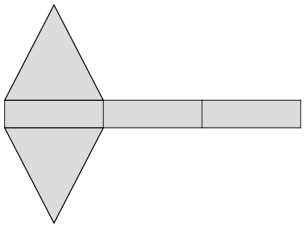


GUIDELINES FOR STUDENTS By repeating the method used in Levels 1-4 to all nets of block 222 (see exercise 510 - Net of Prism), all possible nets can be found. The combinatorial argument using the symmetries must be performed carefully when the axis of symmetry goes through some vertices.

LEVEL 5 The block 123 has 69 nets, see the number of nets for each net type in the tabular below.
In some of the net types, the upper vertex of the edge of length 1 can be any vertex of the triangles (6 cases) and the upper vertex of the edge of length 2 can be any neighbour of it (2 cases). This determines the positions of the remaining vertices, hence there are 12 nets. By symmetry, in some net types two such vertex labelings give equivalent nets, this gives the net types of 6 cases, and in the net type with the most symmetries this gives only 3 cases.

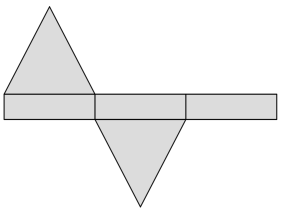
LEVEL 6 The block 133 has 39 nets, see the number of nets for each net type in the table below.
In some of the net types, the upper vertex of the edge of length 1 can be any vertex of the triangles (6 cases), this determines the positions of the remaining vertices, hence there are 6 nets. By symmetry, in some net types two such vertex labelings give equivalent nets, this gives the net types of 4, 3 or 2 cases.

Net type



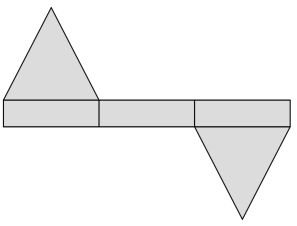
Number of nets of block 123: 6
Number of nets of block 133: 3

Net type



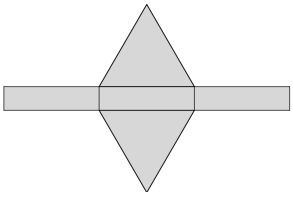
Number of nets of block 123: 12
Number of nets of block 133: 6

Net type



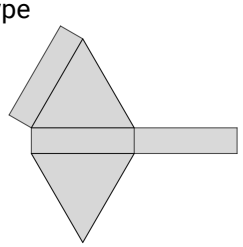
Number of nets of block 123: 6
Number of nets of block 133: 3

Net type



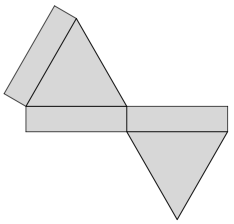
Number of nets of block 123: 3
Number of nets of block 133: 2

Net type

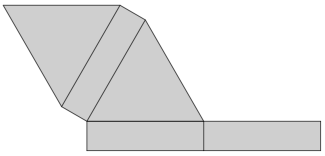
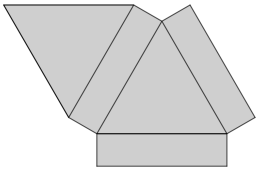
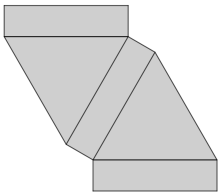


Number of nets of block 123: 12
Number of nets of block 133: 6

Net type



Number of nets of block 123: 12
Number of nets of block 133: 6

Net type	Net type	Net type
		
Number of nets of block 123: 12 Number of nets of block 133: 6	Number of nets of block 123: 6 Number of nets of block 133: 4	Number of nets of block 123: 6 Number of nets of block 133: 3

PRIOR KNOWLEDGE

Net of a polyhedron

RECOMMENDATIONS / COMMENTS

This exercise can be used to develop spatial vision and systematic thinking.
Exercise 510 - Net of Prism is recommended before this exercise.
This exercise is suitable for differentiation. Different blocks can be given to different students or pairs. The Level 5 and 6 exercises are challenging questions.