Lesson 5: Solving any linear equation

Goals

- Calculate a value that is a solution to a linear equation in one variable, and explain (orally) the steps used to solve.
- Create an expression to represent a number puzzle, and justify (orally) that it is equivalent to another expression.
- Justify (orally) that each step used in solving a linear equation maintains equality.

Learning Targets

- I can solve an equation where the variable appears on both sides.

Lesson Narrative

The purpose of this lesson is to move towards a general method for solving linear equations. In the warm-up, students solve equations mentally, including equations with negative coefficients, prompting a discussion of multiplying or dividing each side of an equation by a negative number. In the first activity, students encounter several different structures of equations, and take turns suggesting moves for solving them. Then they apply their growing fluency in solving equations to constructing number puzzles of the sort they encountered in the first lesson in this unit.

Students explain their reasoning for choosing a particular move while solving equations and critiquing the choice of their partner.

Addressing

- Analyse and solve linear equations and pairs of simultaneous linear equations.
- Solve linear equations in one variable.

Instructional Routines

- Algebra Talk
- Clarify, Critique, Correct
- Compare and Connect
- Discussion Supports

Required Materials

Pre-printed cards, cut from copies of the blackline master

Trading moves 1 $-6x - 7 = 4x - 2$	Trading moves 2 $\frac{1}{2}(7x - 6) = 6x - 10$
Trading moves 3 $\frac{1}{2}x + 7 = x + 13$	Trading moves 4 $2(x + 7) = 4x + 14$

Required Preparation

Print and cut up the Trading Steps blackline master for the matching activity. Prepare one set of cards for every 2 students.

Student Learning Goals

Let's solve linear equations.

5.1 Equation Talk

Warm Up: 5 minutes

The purpose of this warm-up is to elicit students' strategies for solving an equation for the value of x. The negative integers and location of x in each equation were purposeful to spark a discussion about operations of integers and a negative coefficient of a variable.

Instructional Routines

- Algebra Talk

Launch

Tell students to close their books or devices and that they are going to see some equations they are to solve mentally. Display each problem one at a time for all to see. Give students 30 seconds of quiet think time for each equation. Ask students to share their strategies for finding the value of x. Record and display their responses for all to see.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Solve each equation mentally.

$5 - x = 8$

$-1 = x - 2$

$-3x = 9$

$-10 = -5x$

Student Response

- $x = -3$
- $x = 1$
- $x = -3$
- $x = 2$

Activity Synthesis

Some students may reason about the value of x using logic. For example, in $-3x = 9$, the x must be -3 since $-3 \times -3 = 9$. Other students may reason about the value of x

by changing the value of each side of the equation equally by, for example, dividing each side of -3$x = 9$ by -3 to get the result $x =$ -3. Both of these strategies should be highlighted during the discussion where possible.

To involve more students in the conversation, consider asking as the students share their ideas:

- "Can you explain why you chose your strategy?"
- "Can anyone restate ___'s reasoning in a different way?"
- "Did anyone reason about the problem the same way but would explain it differently?"
- "Did anyone reason about the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

5.2 Trading Moves

20 minutes

The goal of this activity is for students to build fluency solving equations with variables on each side. Students describe each step in their solution process to a partner and justify how each of their changes maintains the equality of the two expressions.

Look for groups solving problems in different, but efficient, ways. For example, one group may multiply through by the $\frac{1}{2}$ on the left side in problem 2 while another may multiply each side of the equation by 2 in order to re-write the equation with less factors on each side.

Instructional Routines

- Clarify, Critique, Correct
- Discussion Supports

Launch

Arrange students in groups of 2. Instruct the class that they will receive 4 cards with problems on them and that their goal is to create a solution to the problems.

For the first two cards they draw, students will alternate solving by stating to their partner the step they plan to do to each side of the equation and why before writing down the step and passing the card. For the final two problem cards, each partner picks one and writes out its solution individually before trading to check each other's work.

To help students understand how they are expected to solve the first two problems, demonstrate the trading process with a student volunteer and a sample equation. Emphasise that the "why" justification should include how their step maintains the equality of the equation. Use *Clarify, Critique, Correct* by reminding students to push each other to explain how their step guarantees that the equation is still balanced as they are working. For example, a student might say they are combining two terms on one side of the equation, which maintains the equality as the value of that side does not change, only the appearance.

Distribute 4 slips from the blackline master to each group. Give time for groups to complete the problems, leaving at least 5 minutes for a whole-class discussion. If any groups finish early, make sure they have checked their solutions and then challenge them to try and find a new solution to one of the problems that uses less steps than their first solution. Conclude with a whole-class discussion.

If time is a concern, give each group 2 cards rather than all 4 and have them only doing the trading steps portion of the activity, but make sure that all 4 cards are distributed throughout the class. Give 6–7 minutes for groups to complete their problems. Make sure each problem is discussed in a final whole-group discussion. Alternatively, extend the activity by selecting more problems for students to solve with their partners.

Student Task Statement

Your teacher will give you 4 cards, each with an equation.

1. With your partner, select a card and choose who will take the first turn.
2. During your turn, decide what the next move to solve the equation should be, explain your choice to your partner, and then write it down once you both agree. Switch roles for the next move. This continues until the equation is solved.
3. Choose a second equation to solve in the same way, trading the card back and forth after each move.
4. For the last two equations, choose one each to solve and then trade with your partner when you finish to check one another's work.

Student Response

Answers vary. The solutions to the given equations are as follows:

1. $-6x - 7 = 4x - 2$ ($x = -0.5$)
2. $\frac{1}{2}(7x - 6) = 6x - 10$ ($x = 2.8$)

3. $\frac{1}{2}x + 7 = x + 13$ (x = -12)

4. $2(x + 7) = 4x + 14$ ($x = 0$)

Sample response: Students may decide the next step is to factor out a 2 from the right hand side of this equation $2(x + 7) = -4x + 14$. So the next line would be $2(x + 7) = 2(-2x + 7)$.

Activity Synthesis

Depending on your observations as students worked, you may wish to begin the discussion with a few common errors and ask students to explain why they are errors. For example, write out a solution to problem 2 where the second line has $3.5x - 6$ instead of $3.5x - 3$ and then ask students to find the error.

The goal of this discussion is for the class to see different, successful ways of solving the same equation. Record and display the student thinking that emerges during the discussion to help the class follow what is being said. To highlight some of the differences in solution paths, ask:

- "Did your partner ever make a move different than the one you expected them to? Describe it."
- "For problem 4, could you start by halving the value of each side? Why might you want to do this?"
- "What's an arithmetic error you made but then caught when you checked your work?"

Representation: Internalise Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed the initial set, or create an additional set of cards with equations that are more accessible.
Supports accessibility for: Conceptual processing Speaking: Discussion Supports. Use this routine to support students in producing statements about common errors in problem solving. Use the example offered, (the second line has $3.5x - 6$ instead of $3.5x - 3$ for problem 2), and provide sentence frames to support the discussion. For example, "The error this student made was… and I believe this happened because…." and "A different solution path could be…." Restate or revoice student language to demonstrate use of correct mathematical language to describe each move (e.g., "multiply through by the $\frac{1}{2}$," "combine like terms," etc.), and include mathematical reasoning (e.g., "...because this maintains the equality"). Clarify explanations that detail differences in problem-solving strategies rather than errors, to help students see differences in solution paths. This will help students describe differences in solution paths and justify each step.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

5.3 A Puzzling Puzzle

10 minutes

In this activity, students investigate a number puzzle. After the puzzle is demonstrated, students are tasked with figuring out how it works and encouraged to create an algebraic representation of the puzzle. The goal of this activity is to build student fluency working with equations with complex structure. This activity also looks ahead to the future work on functions where students will revisit some of these ideas and learn the language of inputs and outputs. More immediately, this activity points to the study of equations that do not have a single answer, which students will learn about in more depth later in this unit.

While students work, identify those using expressions with different structures for their representation of the number puzzle to share during the whole-class discussion. For example, some students may use $\frac{(3x-7)\times 2-22}{6}$ while others write $\frac{1}{6}((3x-7)\times 2-22)$ and connecting these together with the outcome of the number puzzle, namely $x-6$, is the focus of the whole-class discussion.

Instructional Routines

- Compare and Connect

Launch

Tell students to close their books or devices and choose a number (but not share the number with anyone else). Tell them they will perform a sequence of operations on their number and then tell you their final answer. Say each step of Tyler's number puzzle, giving students time to calculate their new number after each step. Select 5–6 students to share their final number, and after each, tell them their original number as quickly as you can.

Pause here and ask students if they can tell how you are able to figure out their number so fast. If no students notice that each number you say is always 6 more than the number given at the end of the steps, you may wish to record and display the pairs of numbers for all students to see or call on more students so that everyone can hear more pairs of numbers.

Once the class agrees that you are able to figure out their original numbers by adding 6 to their final number, tell them that the number puzzle is really Tyler's and that their task is to figure out how it works. Tell students to open their books or devices.

Give 3–4 minutes quiet work time for students to write their explanations, followed by a whole-class discussion.

Student Task Statement

Tyler says he invented a number puzzle. He asks Clare to pick a number, and then asks her to do the following:

- Triple the number

- Subtract 7
- Double the result
- Subtract 22
- Divide by 6

Clare says she now has a -3. Tyler says her original number must have been a 3. How did Tyler know that? Explain or show your reasoning. Be prepared to share your reasoning with the class.

Student Response

Answers vary. Possible solution: Following Tyler's instructions for a number x results in the expression $\frac{1}{6}((3x-7)\times 2-22)$. Simplified, this expression is just $x-6$. Tyler knows that to figure out Clare's original number, 3, he only needs to solve the equation $x-6=$ -3.

Activity Synthesis

Select previously identified students to share their representations with the class. Record and display the different ways Tyler's number puzzle can be written as an expression. To highlight the connection between the different expressions, ask:

- "What do all these expressions have in common that make the number puzzle work?" (All of them are equivalent to $x-6$.)
- "How would you justify that, for example, $\frac{1}{6}((3x-7)\times 2-22)=x-6$?" (I would simplify the left side of the equation until it was $x-6$.)
- "What does it mean if we have an equation that says $x-6=x-6$?" (The two sides of this equation are always the same.)

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation Representing: Compare and Connect. Use this routine when students write an expression to represent Tyler's number puzzle. In pairs or groups, ask students to switch their expressions and compare them. Prompt students to share and explain their strategy and then discuss what is the same or different about their approaches. Monitor student discussion, then clarify how operations should be correctly arranged and grouped to help produce $x-6$. Allow time for students to share with the whole class. This will help students make connections between different expressions, using appropriate mathematical language to detail their steps.
Design Principle(s): Optimise output (for explanation)

Lesson Synthesis

Give students 2–3 minutes to think about all the equations they solved in today's lesson and to write down any errors they made or observed. Discuss and consider creating a permanent display showing:

- different approaches for different structures of equations
- types of errors to look out for

5.4 Check It

Cool Down: 5 minutes

Student Task Statement

Noah wanted to check his solution of $x = \frac{14}{5}$ for the equation $\frac{1}{2}(7x - 6) = 6x - 10$. Substituting $\frac{14}{5}$ for x, he writes the following:

$$\frac{1}{2}\left(7\left(\frac{14}{5}\right) - 6\right) = 6\left(\frac{14}{5}\right) - 10$$
$$\left(7\left(\frac{14}{5}\right) - 6\right) = 12\left(\frac{14}{5}\right) - 20$$
$$5\left(7\left(\frac{14}{5}\right) - 6\right) = 5\left(12\left(\frac{14}{5}\right) - 20\right)$$
$$7 \times 14 - 6 = 12 \times 14 - 20$$
$$98 - 6 = 168 - 20$$
$$92 = 148$$

Find the incorrect step in Noah's work and explain why it is incorrect.

Student Response

Noah made a mistake between these two lines:

$5\left(7\left(\frac{14}{5}\right) - 6\right) = 5\left(12\left(\frac{14}{5}\right) - 20\right)$

$7 \times 14 - 6 = 12 \times 14 - 20$

Noah should have multiplied through by the 5 through to each term inside the brackets so that the second line read like this: $7 \times 14 - 30 = 12 \times 14 - 100$

Then, the solution would finish like this: $98 - 30 = 168 - 100$, $68 = 68$

Student Lesson Summary

When we have an equation in one variable, there are many different ways to solve it. We generally want to make moves that get us closer to an equation like

variable = *some number*.

For example, $x = 5$ or $t = \frac{7}{3}$. Since there are many ways to do this, it helps to choose moves that leave fewer **terms** or factors. If we have an equation like $3t + 5 = 7$, adding -5 to each side will leave us with fewer terms. The equation then becomes $3t = 2$.

Dividing each side of this equation by 3 will leave us with t by itself on the left and that $t = \frac{2}{3}$. Or, if we have an equation like $4(5 - a) = 12$, dividing each side by 4 will leave us with fewer factors on the left, $5 - a = 3$.

Some people use the following steps to solve a linear equation in one variable:

1. Use the distributive property so that all the expressions no longer have brackets.
2. Collect like terms on each side of the equation.
3. Add or subtract an expression so that there is a variable on just one side.
4. Add or subtract an expression so that there is just a number on the other side.
5. Multiply or divide by a number so that you have an equation that looks like *variable* = *some number*.

For example, suppose we want to solve $9 - 2b + 6 = -3(b + 5) + 4b$.

$9 - 2b + 6$	$= -3b - 15 + 4b$	Use the distributive property
$15 - 2b$	$= b - 15$	Gather like terms
15	$= 3b - 15$	Add $2b$ to each side
30	$= 3b$	Add 15 to each side
10	$= b$	Divide each side by 3

Following these steps will always work, although it may not be the most efficient method. From lots of experience, we learn when to use different approaches.

Glossary

- Term

Lesson 5 Practice Problems

1. **Problem 1 Statement**

   Solve each of these equations. Explain or show your reasoning.

   $2(x + 5) = 3x + 1$

   $3y - 4 = 6 - 2y$

   $3(n + 2) = 9(6 - n)$

Solution

a. $x = 9$. Responses vary. Sample response: Multiply through by 2 on the left side, add -1 to each side, then add -2x to each side.

b. $y = 2$. Responses vary. Sample response: Multiply through by 2 on the right side, add 2y to each side, add 4 to each side, then divide each side by 5.

c. $n = 4$. Responses vary. Sample response: Divide each side by 3, multiply through by 3 on the right side, subtract 2 from each side, add 3n to each side, then divide each side by 4.

2. **Problem 2 Statement**

Clare was solving an equation, but when she checked her answer she saw her solution was incorrect. She knows she made a mistake, but she can't find it. Where is Clare's mistake and what is the solution to the equation?

$$\begin{aligned} 12(5+2y) &= 4y-(5-9y) \\ 72+24y &= 4y-5-9y \\ 72+24y &= -5y-5 \\ 24y &= -5y-77 \\ 29y &= -77 \\ y &= \frac{-77}{29} \end{aligned}$$

Solution

Clare's mistakes occurred in the transition from the 1st line to the 2nd line. She wrote $4y - {-9y}$ as $4y - 9y$ instead of $4y + 9y$ and $12(5) = 72$ instead of $12(5) = 60$. The correct solution is $y = -\frac{65}{11}$.

3. **Problem 3 Statement**

Solve each equation, and check your solution.

$\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)$

$-4(r + 2) = 4(2 - 2r)$

$12(5 + 2y) = 4y - (6 - 9y)$

Solution

a. $m = -7$

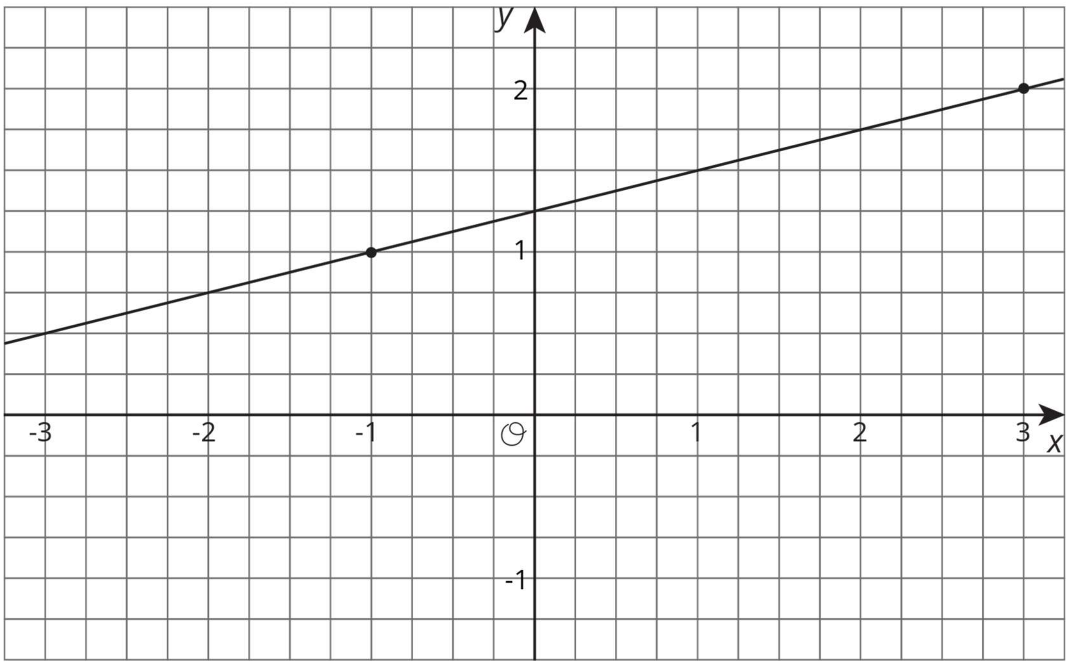
b. $r = 4$

c. $y = -6$

4. **Problem 4 Statement**

Here is the graph of a linear equation.

Select **all** true statements about the line and its equation.



a. One solution of the equation is (3,2).

b. One solution of the equation is (-1,1).

c. One solution of the equation is $\left(1, \frac{3}{2}\right)$.

d. There are 2 solutions.

e. There are infinitely many solutions.

f. The equation of the line is $y = \frac{1}{4}x + \frac{5}{4}$.

g. The equation of the line is $y = \frac{5}{4}x + \frac{1}{4}$.

Solution ["A", "B", "C", "E", "F"]

5. **Problem 5 Statement**

A participant in a 21-mile walkathon walks at a steady rate of 3 miles per hour. He thinks, "The relationship between the number of miles left to walk and the number of hours I already walked can be represented by a line with gradient -3." Do you agree with his claim? Explain your reasoning.

Solution

Yes. Explanations vary. Sample response: The walker completes 3 miles each hour, so 3 is subtracted for each 1 hour walked. Another sample response: Points on the graph of remaining miles (y) and hours walked (x) could be (0,21), (1,18), (2,15), (3,12), etc., so the line slopes down. Another sample response: The number of miles remaining decreases by 3 for every increase of 1 in the hours walked.