Lesson 20: Combining like terms (Part 1)

Goals

- Apply properties of operations to justify (orally and in writing) that expressions are equivalent.
- Generate an expression that is equivalent to a given expression with fewer terms.
- Interpret different methods for determining whether expressions are equivalent, and evaluate (orally) their usefulness.

Learning Targets

- I can figure out whether two expressions are equivalent to each other.
- When possible, I can write an equivalent expression that has fewer terms.

Lesson Narrative

In this lesson, students have a chance to recall one way of understanding equivalent expressions, that is, the expressions have the same value for any number substituted for a variable. Then they use properties they have studied over the past several lessons to understand how to properly write an equivalent expression using fewer terms. We are gently building up to students being able to fluently *combine like terms*, though that language is not used with students yet.

Building On

- Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

Addressing

- Apply properties of operations as strategies to add, subtract, factorise, and expand linear expressions with rational coefficients.

Building Towards

- Apply properties of operations as strategies to add, subtract, factorise, and expand linear expressions with rational coefficients.

Instructional Routines

- Discussion Supports

Student Learning Goals

Let's see how we can tell that expressions are equivalent.

20.1 Why is it True?

Warm Up: 10 minutes

The purpose of this warm-up is to remind students about a few algebraic moves they have studied in the past several lessons by prompting them to explain the reason the moves are allowed. These moves are important to understand as students work toward fluency in writing expressions with fewer terms. Although this activity isn't properly a Number Talk, a similar routine can be followed.

Launch

Display one statement at a time. Give students 30 seconds of quiet think time for each statement and ask them to give a signal when they have an explanation. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Student Task Statement

Explain why each statement is true.

1. $5+2+3=5+(2+3)$
2. $9a$ is equivalent to $11a-2a$.
3. $7a+4-2a$ is equivalent to $7a+\text{-}2a+4$.
4. $8a-(8a-8)$ is equivalent to 8.

Student Response

Answers vary. Sample responses:

1. Associative property: The convention is to add left to right so $5+2$ is added first, but the associative property says grouping differently with addition gives the same result.
2. Distributive property: $11a-2a=(11-2)a=9a$
3. Subtraction can be written as adding the opposite, and then the order can be switched with the commutative property: $7a+4-2a=7a+4+\text{-}2a=7a+\text{-}2a+4$.
4. Subtracting a negative is the same as adding its opposite, and then the distributive property means $8a-(8a-8)=8a-(8a+\text{-}8)=8a-8a-\text{-}8=8$.

Activity Synthesis

Ask students to share their reasons why each statement is true. Record and display their responses for all to see. Highlight correct use of precise, mathematical language and give students opportunities to revise their response to be more precise.

To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same reason but would explain it differently?”
- “Does anyone want to add on to _____’s reason?” “Do you agree or disagree? Why?”

20.2 A’s and B’s

10 minutes

In this activity students see an example of why applying properties is the only reliable way to decide whether two expressions are equivalent. They begin by substituting a value of the variable into expressions believed to be equivalent, and discover that the expressions are equal for that value. They then substitute other values and find that one of the expressions has a different value than the others. Students follow up by expanding the terms of the expression to consider each instance of the variables individually, and uncover the properties applied in each step of writing the expression with fewer terms.

Instructional Routines

- Discussion Supports

Launch

Display the first part of the task statement for all to see:

Diego and Jada are both trying to write an expression with fewer terms that is equivalent to $7a + 5b - 3a + 4b$

- Jada thinks $10a + 1b$ is equivalent to the original expression.
- Diego thinks $4a + 9b$ is equivalent to the original expression.

Remind students that we can tell whether the expressions are equivalent by substituting some different values for a and b and evaluating the expressions.

First, ask students to substitute the values $a = 4$ and $b = 3$ and evaluate the original expression, Jada’s expression, and Diego’s expression. All expressions evaluate to 43. Perhaps Jada’s and Diego’s expressions are both equivalent to the original expression?

Then, ask students to choose some different values for a and b and evaluate: the original expression, Jada’s expression, and Diego’s expression. For any other values of a and b, Jada and Diego's expressions do not evaluate to the same thing. For example, for $a = 1$ and $b = 1$, the original is 13, Jada's expression is 11, and Diego's is 13.

The outcome of Diego's expression will match the original expressions, and Jada's will not.

Tell students that experimenting with numbers can tell us that two expressions are *not* equivalent, but can't prove that two expressions are equivalent. For example, Jada and Diego's expressions yielded the same outcome for $a = 4$ and $b = 3$, but aren't equivalent. For that, we need to reason about the expressions using the properties that we know.

Arrange students in groups of 2. Allow 6–7 minutes quiet work time and partner discussions followed by a whole class discussion.

Representation: Internalise Comprehension. Begin with a physical demonstration of expanding terms and substituting values into an expression to support connections between new situations and prior understandings. For example, demonstrate how to expand "$7a$" by writing out 7 "a's" with the addition sign between them and then evaluate it using $a = 2$. Ask students "How are these expressions equivalent?" and "How does substituting values help determine when expressions are equivalent?"
Supports accessibility for: Conceptual processing; Visual-spatial processing Speaking. Use sentence frames to support students in producing explanations about why expressions are equivalent. For example, "This row is equivalent to the last row because . . . ."
Design Principle(s): Support sense-making; Optimise output (for explanation)

Anticipated Misconceptions

Students may think the expressions are equivalent after finding them equal for $a = 4$ and $b = 3$. Remind them that equivalent expressions must be equal for every possible value of the variable.

Students might have trouble describing the moves in the last two rows for part 2 and justifying that the expressions are equivalent. Encourage students to closely examine the changes from row to row and consider why they do not change the value of the expression.

Student Task Statement

Diego and Jada are both trying to write an expression with fewer terms that is equivalent to $7a + 5b - 3a + 4b$

- Jada thinks $10a + 1b$ is equivalent to the original expression.
- Diego thinks $4a + 9b$ is equivalent to the original expression.

1. We can show expressions are equivalent by writing out all the variables. Explain why the expression on each row (after the first row) is equivalent to the expression on the row before it.

   $7a + 5b - 3a + 4b$
   $(a + a + a + a + a + a + a) + (b + b + b + b + b) - (a + a + a) + (b + b + b + b)$
   $(a + a + a + a) + (a + a + a) + (b + b + b + b + b) - (a + a + a) + (b + b + b + b)$
   $(a + a + a + a) + (b + b + b + b + b) + (a + a + a) - (a + a + a) + (b + b + b + b)$
   $(a + a + a + a) + (b + b + b + b + b) + (b + b + b + b)$
   $(a + a + a + a) + (b + b + b + b + b + b + b + b + b)$
   $4a + 9b$

2. Here is another way we can rewrite the expressions. Explain why the expression on each row (after the first row) is equivalent to the expression on the row before it.
$7a + 5b - 3a + 4b$
$7a + 5b + (\text{-}3a) + 4b$
$7a + (\text{-}3a) + 5b + 4b$
$(7 + \text{-}3)a + (5 + 4)b$
$4a + 9b$

Student Response

1. Answers vary. Sample responses:
   - First row: Write products as sums (distributive property).
   - Second row: Group first set of a's differently (associative property).
   - Third row: Switch second and third groups of addends (commutative property).
   - Fourth row: Subtract an addend from itself to get 0: $(a + a + a) - (a + a + a)$.
   - Fifth row: Group all the b's together (associative property).
   - Sixth row: Write sums as products (distributive property).
2. Answers vary. Sample responses:
   - First row: Write subtraction as addition.
   - Second row: Switch 2nd and 3rd terms (commutative property).
   - Third Row: Write sums as products (distributive property).
   - Fourth row: Evaluate numerical expressions.

Are You Ready for More?

Follow the instructions for a number puzzle:

- Take the number formed by the first 3 digits of your phone number and multiply it by 40
- Add 1 to the result
- Multiply by 500
- Add the number formed by the last 4 digits of your phone number, and then add it again
- Subtract 500
- Multiply by $\frac{1}{2}$

1. What is the final number?
2. How does this number puzzle work?
3. Can you invent a new number puzzle that gives a surprising result?

Student Response

Explanations vary. Sample response:

- Let x represent the 3-digit number, so $40x$
- $40x + 1$
- $500(40x + 1)$
- Let y represent the 4-digit number, so $500(40x + 1) + y + y$ or $500(40x + 1) + 2y$
- $500(40x + 1) + 2y - 500 = 20,000x + 500 + 2y - 500 = 20,000x + 2y$
- $\frac{20,000x+2}{2} = 10,000x + y$

  $10,000x + y$ means the 3-digit number, x was moved 4 place values to the left followed by 4 zeros and then the 4-digit number was added to the zeros, forming the phone number.

Activity Synthesis

Invite students to justify that the steps taken by Diego do not change the value of the expressions. Emphasise places where he used the distributive property and the commutative property.

Ask students which method they prefer (substituting values or using the properties of operations) for telling whether expressions are equivalent. Explain that while checking values can give us useful information, there is usually no way to check *all* possible values. That is why it is important to have some algebraic methods to rely on.

20.3 Making Sides Equal

15 minutes

In this activity, students use what they have learned so far to find a missing term that makes two expressions equivalent. They have many tools at their disposal to reason about the missing term. For example, for the first problem $6x + ? = 10x$, they might reason as in the last activity and write out the sum of 6 x's and ? on one side, and a string of 10 x's on the other side, and reason that 4 x's are needed to make the sides equivalent. Alternatively, they might reason with the distributive property, and rewrite the left side as $x(6 + ?) = 10x$. These alternative ways of reasoning about equivalent expressions should be highlighted in the discussion.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 2. Explain that they will use what they have learned so far to find a missing term that will make two expressions equivalent. Draw their attention to the instructions, which instruct students to complete the first set of problems, check in with their partner, and then proceed. If desired, you might ask students to pause after the first set for whole-class discussion.

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Display sentence frames to support student conversation such as "To find the missing term, first, I _____ because...", "Why did you...?", "Can you explain or show that another way?" or "I agree/disagree because…"
Supports accessibility for: Language; Social-emotional skills

Student Task Statement

Replace each ? with an expression that will make the left side of the equation equivalent to the right side.

Set A

1. $6x + ? = 10x$

2. $6x + ? = 2x$

3. $6x + ? = -10x$

4. $6x + ? = 0$

5. $6x + ? = 10$

Check your results with your partner and resolve any disagreements. Next move on to Set B.

Set B

1. $6x - ? = 2x$

2. $6x - ? = 10x$

3. $6x - ? = x$

4. $6x - ? = 6$

5. $6x - ? = 4x - 10$

Student Response

Set A

1. $6x + 4x = 10x$
2. $6x + \text{-}4x = 2x$
3. $6x + \text{-}16x = \text{-}10x$
4. $6x + \text{-}6x = 0$
5. $6x + (10 - 6x) = 10$

Set B

1. $6x - 4x = 2x$
2. $6x - (\text{-}4x) = 10x$
3. $6x - 5x = x$
4. $6x - (6x - 6) = 6$
5. $6x - (2x + 10) = 4x - 10$

Activity Synthesis

Ask students to share their expressions for each problem. Record and display their responses for all to see. After each student shares, ask the class if they agree or disagree.

The following questions, when applicable, can be used as students share:

- "Why didn't you combine x terms and numbers?" (Rewriting expressions using the properties of multiplication or the distributive property shows why this doesn't result in an equivalent expression.)
- "How did you decide on the components of the missing term?"
- "Did you use the commutative property?"
- "Did you use the distributive property?"

Speaking, Representing: Discussion Supports. Use this routine to support whole-class discussion. After each student shares their expressions, provide the class with the following sentence frames to help them respond: "I agree because ….” or "I disagree because ….” Encourage students to name a property as part of their explanation for why they agree or disagree. This will help students to connect the properties with the processes they used to confirm equivalent expressions.
Design Principle(s): Optimise output (for explanation)

Lesson Synthesis

Consider asking students to choose one of these questions, think about it quietly for a few minutes, and then explain it to their partner either verbally or in writing. Their partner listens or reads carefully, and asks any clarifying questions if they don't fully understand.

- “What are some ways we can tell that $7x + 2$ is *not* equivalent to $9x$?”
- “Someone is doubtful that $3b - 8b$ is equivalent to $-5b$, but they do understand the distributive property. How could you convince them that these expressions are equivalent?”
- “What are some ways we could rearrange the terms in the expression $-2x + 6y - 6x + 15y$ and create an equivalent expression?”

20.4 Fewer Terms

Cool Down: 5 minutes

Student Task Statement

Write each expression with fewer terms. Show your work or explain your reasoning.

1. $10x - 2x$
2. $10x - 3y + 2x$

Student Response

1. $8x$
2. $12x - 3y$

Student Lesson Summary

There are many ways to write equivalent expressions that may look very different from each other. We have several tools to find out if two expressions are equivalent.

- Two expressions are definitely not equivalent if they have different values when we substitute the same number for the variable. For example, $2(-3 + x) + 8$ and $2x + 5$ are not equivalent because when x is 1, the first expression equals -4 and the second expression equals 7.

- If two expressions are equal for many different values we substitute for the variable, then the expressions *may* be equivalent, but we don't know for sure. It is impossible to compare the two expressions for all values. To know for sure, we use properties of operations. For example, $2(-3 + x) + 8$ is equivalent to $2x + 2$ because:

$2(-3 + x) + 8$	
$-6 + 2x + 8$	by the distributive property
$2x + -6 + 8$	by the commutative property
$2x + (-6 + 8)$	by the associative property
$2x + 2$	

Lesson 20 Practice Problems

1. **Problem 1 Statement**

   Andre says that $10x + 6$ and $5x + 11$ are equivalent because they both equal 16 when x is 1. Do you agree with Andre? Explain your reasoning.

   Solution

   No, equivalent expressions are equal for any value of their variable. When x is 0, they are not equal.

2. **Problem 2 Statement**

   Select **all** expressions that can be subtracted from $9x$ to result in the expression $3x + 5$.

   a. $-5 + 6x$

   b. $5 - 6x$

   c. $6x + 5$

   d. $6x - 5$

   e. $-6x + 5$

   Solution ["A", "D"]

3. **Problem 3 Statement**

   Select **all** the statements that are true for any value of x.

   a. $7x + (2x + 7) = 9x + 7$

   b. $7x + (2x - 1) = 9x + 1$

c. $\frac{1}{2}x + (3 - \frac{1}{2}x) = 3$

d. $5x - (8 - 6x) = \text{-}x - 8$

e. $0.4x - (0.2x + 8) = 0.2x - 8$

f. $6x - (2x - 4) = 4x + 4$

Solution ["A", "C", "E", "F"]

4. **Problem 4 Statement**

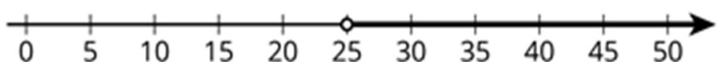
For each situation, would you describe it with $x < 25$, $x > 25$, $x \leq 25$, or $x \geq 25$?

a. The library is having a party for any student who read at least 25 books over the summer. Priya read x books and was invited to the party.

b. Kiran read x books over the summer but was not invited to the party.

c.

d.



Solution

a. $x \geq 25$

b. $x < 25$

c. $x \leq 25$

d. $x > 25$

5. **Problem 5 Statement**

Consider the problem: A water bucket is being filled with water from a water tap at a constant rate. When will the bucket be full? What information would you need to be able to solve the problem?

Solution

Answers vary. Possible response:

a. How big is the bucket?

b. What is the rate of water flow?

c. How high is the bucket?

d. How high is the water in the bucket after 1 minute?