Lesson 4: Converting units

Goals

- Choose and create a double number line diagram or table to solve problems involving unit conversion.
- Explain (orally) how to use a "unit rate" to solve problems involving unit conversion.
- Recognise that when we measure things in two different units, the pairs of measurements are equivalent ratios.

Learning Targets

- I can convert measurements from one unit to another, using double number lines, tables, or by thinking about "how much for 1."
- I know that when we measure things in two different units, the pairs of measurements are equivalent ratios.

Lesson Narrative

In KS2 students began converting units of measurements by multiplying, and conversion by dividing, but were still restricted to units within the same measurement system. In this lesson, students progress to converting units that may be in different systems of measurement, using ratio reasoning and recently-learned strategies such as double number lines, tables, and multiplication or division of unit rates.

Building On

- Interpret the product $\left(\frac{a}{b}\right) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $\left(\frac{2}{3}\right) \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\left(\frac{2}{3}\right) \times \left(\frac{4}{5}\right) = \frac{8}{15}$. (In general, $\left(\frac{a}{b}\right) \times \left(\frac{c}{d}\right) = \frac{ac}{bd}$.)

Addressing

- Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Collect and Display
- Clarify, Critique, Correct
- Compare and Connect
- Discussion Supports

- Number Talk
- Think Pair Share

Required Materials

Four-function calculators

Student Learning Goals

Let’s convert measurements to different units.

4.1 Number Talk: Fractions of a Number

Warm Up: 10 minutes

This number talk encourages students to think about numbers and rely on what they know about structure, patterns, multiplication, division, and properties of operations to mentally solve a problem. Discussion of strategies is integral to the activity, but it may not be possible to share every possible strategy for each problem given limited time. Consider gathering only two or three different strategies per problem.

The factors in the problems are chosen such that their connections become increasingly more apparent as students progress. If such connections do not arise during discussions, make them explicit. Students should also be able to state that taking a fraction of a number involves multiplication and can be done with either multiplication or division.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Find the values mentally.

$\frac{1}{4}$ of 32

$\frac{3}{4}$ of 32

$\frac{3}{8}$ of 32

$\frac{3}{8}$ of 64

Student Response

- 8. Possible strategies: $32 \div 4$ or 4×8.
- 24. Possible strategies: $32 \div 4 \times 3$, or since one of the factors in the first question tripled and the other remained constant, the product triples (8×3).
- 12. Possible strategies: $32 \div 8 \times 3$, or since $\frac{3}{8}$ is half of $\frac{3}{4}$, the product is half of the product in the second question ($24 \div 2$).
- 24. Possible strategies: $64 \div 8 \times 3$, or since one factor doubled from the previous problem and the other stayed the same, the product doubles (12×2).

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. If not mentioned by students as they discuss the last three problems, ask if or how the given factors impacted their strategy choice.

To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone solve the problem the same way but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?" If time permits, ask students if they notice any connections between the problems. Have them share any relationships they notice.

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimise output (for explanation)

4.2 Road Trip

15 minutes

The purpose of this activity is to help students understand that quantities measured using the same two units of measure form a set of equivalent ratios. All of the strategies and representations they have for reasoning about equivalent ratios can be used for reasoning about converting from one unit of measure to another. Any ratio $a : b$ has two associated unit rates: $\frac{a}{b}$ and $\frac{b}{a}$, with a particular meaning in the context. For example, since there are 8 kilometres in approximately 5 miles, there are $\frac{8}{5}$ kilometres in 1 mile, and there are $\frac{5}{8}$ of a mile in one kilometre. We want students to notice that finding "how much per 1" and reasoning with these unit rates is efficient, but to make sense of these efficient strategies by using familiar representations like double number lines and tables. In a constant speed context, students are explicitly asked to compute each unit rate, and then they are asked to solve a problem where either unit rate can be used. For the second problem, monitor for one student who uses each strategy to solve the problem:

- Creating a double number line or a table to represent the association between miles and the equivalent distance in kilometres as a set of equivalent ratios. (If both of these representations are used, it is fine to include both.)
- Converting 80 kilometres into 50 miles by evaluating $80 \times \frac{5}{8}$ (in order to compare the resulting 50 miles per hour with 75 miles per hour)
- Converting 75 miles into 120 kilometres by evaluating $75 \times \frac{8}{5}$ (in order to compare the resulting 120 kilometres per hour with 80 kilometres per hour)

Continuing to draw connections between representations of equivalent ratios and more efficient methods will help students make sense of the more efficient methods.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Compare and Connect

Launch

Display the image from the task statement for all to see, tell students it is a traffic sign you might see while driving, and ask students to explain what it means. They will likely guess it is a speed limit sign and assume it means 80 miles per hour. If no one brings it up, tell students that this is a sign you might see while driving in Canada or another country that uses the metric system.

Give students 2 minutes of quiet work time and ask them to pause after the first question. Ensure that everyone has correct answers for the first question before proceeding with the second question. Follow with whole-class discussion.

Representation: Internalise Comprehension. Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems and other text-based content.
Supports accessibility for: Language; Conceptual processing

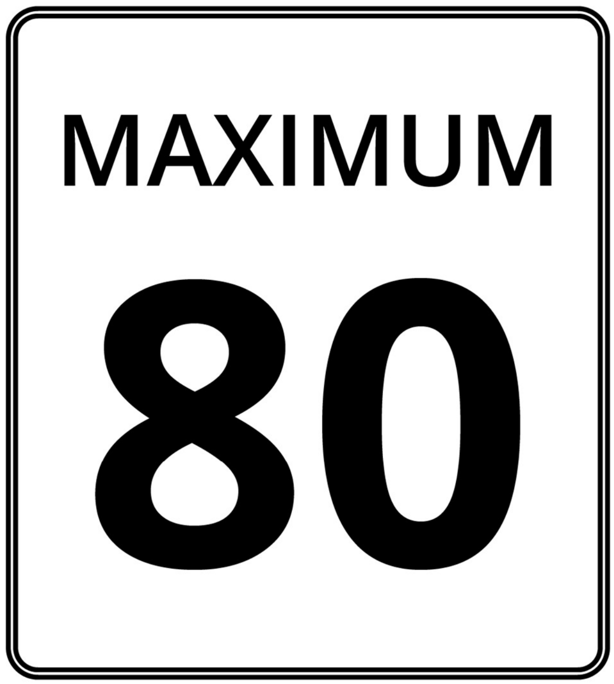
Anticipated Misconceptions

It is acceptable to express the answers to the first question in either fraction or decimal form. If students express uncertainty about carrying out the division of $5 \div 8$ or $8 \div 5$, encourage them to express the quotient in fraction form.

Student Task Statement

Elena and her mum are on a road trip. Elena sees this road sign.

Elena’s mum is driving 75 miles per hour when she gets pulled over for speeding.



1. The police officer explains that 8 kilometres is approximately 5 miles.
    a. How many kilometres are in 1 mile?
    b. How many miles are in 1 kilometre?
2. If the speed limit is 80 kilometres per hour, and Elena’s mum was driving 75 miles per hour, was she speeding? By how much?

Student Response

1.
    a. About $\frac{8}{5}$ or 1.6 kilometres
    b. About $\frac{5}{8}$ or 0.625 miles

2. Yes, she was speeding by about 25 miles per hour or about 40 kilometres per hour. Possible strategies:

    - Convert 80 kilometres per hour into miles per hour: $80 \times \frac{5}{8} = 50$ and $75 - 50 = 25$
    - Convert 75 miles per hour into kilometres per hour: $75 \times \frac{8}{5} = 120$ and $120 - 80 = 40$

Activity Synthesis

Focus discussion on different approaches to the second question. If any students with less-efficient methods were selected, have them go first in the sequence, or present one of these representations yourself. As students are presenting their work, encourage them to explain the meaning of any numbers used and the reason they decided to use particular operations. For example, if a student multiplies 80 by $\frac{5}{8}$, ask them to explain what $\frac{5}{8}$ means in this context and why they decided to multiply 80 by it. It can be handy to have representations like double number lines or tables displayed to facilitate these explanations.

Representing, Listening, Speaking: Compare and Connect. Ask students to display their approaches to determine whether or not Elena's mum was speeding. As students share their work, encourage them to explain the meaning of each quantity they use. For example, if they convert 80 miles per hour into kilometres per hour, where 80 is multiplied by $\frac{5}{8}$, ask what $\frac{5}{8}$ means in this context and why they decided to multiply it by 80. If students used a table or a double number line, ask how these representations connect with other strategies. This will help students make sense of the various approaches to reason about equivalent ratios which can be used for reasoning about converting one unit of measure to another. *Design Principle(s): Cultivate conversation; Maximise meta-awareness*

4.3 Veterinary Weights

10 minutes

This activity is an opportunity to apply insights from the previous activity in a different context. In this activity, students convert between pounds and kilograms. The conversion factor is not given as a unit rate. As a result of the work in the previous activity, some students may compute and use unit rates, and some may still reason using various representations of equivalent ratios. The numbers are also purposely chosen such that the unit rate $\frac{10}{22}$ does not have a convenient decimal equivalent, suggesting that fractions are sometimes much more convenient to work with than decimals. Additionally, while all measurements within this activity are accurate with rounding to the nearest integer, you may choose to point out before or after the task that $\frac{10}{22}$ is a common approximation of the conversion factor from pounds to kilograms and not the true conversion factor.

As students work, identify those who computed and used the unit rates $\frac{10}{22}$ and $\frac{22}{10}$. Highlight these strategies in the discussion, while continuing to refer to other representations to make sense of them as needed.

Instructional Routines

- Collect and Display
- Think Pair Share

Launch

Ask students to recall which is heavier: 1 pound or 1 kilogram? (1 kilogram is heavier.) Tell them that in this activity, they will be given weights in pounds and asked to express it in kilograms, and also the other way around. The pairs of measurements in pounds and kilograms for a set of objects are all equivalent ratios. Encourage students to consider how finding unit rates—how many kilograms in 1 pound and how many pounds in 1 kilogram—can make their work more efficient.

Give students quiet think time to complete the activity, and then time to share their explanation with a partner. Follow with whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organisation and problem solving. For example, ask students to first draw a table or double number line to organise their thinking and then find a ratio for converting pounds to kilograms. Next, ask students to convert the weight of each dog separately. After that, ask students to complete the last question.
Supports accessibility for: Organisation; Attention

Anticipated Misconceptions

Students working with the unit rate $\frac{10}{22}$ may want to convert it to a decimal and get bogged down. Encourage them to work with the fraction, reviewing strategies for multiplying by a fraction as necessary.

Student Task Statement

A veterinarian uses weights in kilograms to figure out what dosages of medicines to prescribe for animals. For every 10 kilograms, there are 22 pounds.

1. Calculate each animal's weight in kilograms. Explain or show your reasoning. If you get stuck, consider drawing a double number line or table.

    a. Fido the Labrador weighs 88 pounds.

    b. Spot the Beagle weighs 33 pounds.

    c. Bella the Chihuahua weighs $5\frac{1}{2}$ pounds.

2. A certain medication says it can only be given to animals over 25 kilograms. How much is this in pounds?

Student Response

1. Sample reasoning for each part: There are $\frac{10}{22}$ kilograms per pound, so I can multiply by $\frac{10}{22}$ to convert from pounds to kilograms.

   a. 40, because $88 \times \frac{10}{22} = 40$

   b. 15, because $33 \times \frac{10}{22} = 15$

   c. $2\frac{1}{2}$ (or equivalent). $5\frac{1}{2} \times \frac{10}{22} = 2\frac{1}{2}$

2. 55 pounds. Sample reasoning: Since 10 kilograms is 22 pounds, 1 kilogram is $\frac{22}{10}$ pounds. Therefore, 25 kilograms is 55 pounds because $\frac{22}{10} \times 25 = 55$.

Here is a table that may be used to organise the work:

weight (pounds)	weight (kilograms)
22	10
1	$\frac{10}{22}$
88	40
33	15
$5\frac{1}{2}$	$2\frac{1}{2}$
$\frac{22}{10}$	1
55	25

Activity Synthesis

Invite one or more students to share who used the unit rates $\frac{10}{22}$ and $\frac{22}{10}$ as part of their work. Display a table of equivalent ratios as needed to help students make sense of this approach, including attending to the meaning of these numbers and the rationale for any operations used.

Listening, Speaking: Collect and Display. As students share their explanations for the first and second questions with a partner followed with whole class discussion, listen for and scribe the words and phrases that students used when converting between pounds and kilograms. Some students may convert to decimals and find this method more challenging, while others may show reasoning using the unit rates $\frac{10}{22}$ and $\frac{22}{10}$. Highlight key vocabulary that students use in the discussion, while continuing to refer to other representations to

make sense of them as needed. This will help students increase sense-making while simultaneously support meta-awareness of language.
Design Principle(s): Support sense-making; Maximise meta-awareness

4.4 Cooking with a Tablespoon

Optional: 15 minutes

This optional activity is an opportunity to practise the methods in this lesson to convert between cups and tablespoons. The conversion factor is given in the form of a unit rate, so students only need to decide whether to multiply or divide. They might, however, choose to create a double number line diagram or a table to support their reasoning. Several of the measurements include fractions, giving students an opportunity to practise multiplying mixed numbers by whole numbers and dividing whole numbers that result in fractions.

Instructional Routines

- Clarify, Critique, Correct
- Think Pair Share

Launch

Tell students they will now convert between tablespoons and cups. Just as with pairs of weights in pounds and kilograms, these pairs of tablespoons and cups can also be thought of as equivalent ratios. Welcome any strategies for reasoning about equivalent ratios, but encourage students to try to find efficient methods using multiplication and division.

Give students quiet think time to complete the activity and then time to share their explanation with a partner. Follow with whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge related to measuring using tablespoons and cups. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Students may answer “zero cups” for the last question, because it is less than one. Ask them to consider what fraction of a cup would be equivalent to 6 tablespoons.

Student Task Statement

Diego is trying to follow a recipe, but he cannot find any measuring cups! He only has a tablespoon. In the cookbook, it says that 1 cup equals 16 tablespoons.

1. How could Diego use the tablespoon to measure out these ingredients?

   $\frac{1}{2}$ cup almonds

$1\frac{1}{4}$ cups of oatmeal

$2\frac{3}{4}$ cups of flour

2. Diego also adds the following ingredients. How many cups of each did he use?

   28 tablespoons of sugar

   6 tablespoons of cocoa powder

Student Response

1. Cup to tablespoon conversions:

   a. 8 tablespoons of almonds, because $\frac{1}{2} \times 16 = 8$.

   b. 20 tablespoons of oatmeal, because $\frac{1}{4} \times 16 = 4$ and $16 + 4 = 20$.

   c. 44 tablespoons of flour, because $2\frac{3}{4} \times 16 = 44$.

2. Tablespoon to cup conversions

   a. $1\frac{3}{4}$ cups of sugar. Possible strategies:

      - From earlier, 20 tablespoons is $1\frac{1}{4}$ cups. For 28 tablespoons, you need an additional 8 tablespoons, or an additional $\frac{1}{2}$ cup. $1\frac{1}{4} + \frac{1}{2} = 1\frac{3}{4}$.
      - 1 tablespoon is $\frac{1}{16}$ of a cup. $\frac{1}{16} \times 28 = \frac{28}{16}$, which is equivalent to $1\frac{12}{16}$ and $1\frac{3}{4}$.

   b. $\frac{3}{8}$ cup of cocoa powder. Possible strategies:

      - Notice that to convert from tablespoons to cups, always divide by 16, so $6 \div 16 = \frac{6}{16} = \frac{3}{8}$.
      - 1 tablespoon is $\frac{1}{16}$ of a cup, so $\frac{1}{16} \times 6 = \frac{6}{16} = \frac{3}{8}$.

Activity Synthesis

As in the previous task, select students to share based on their strategies, sequencing from less efficient to more efficient, being sure to highlight approaches using multiplication or division (by 16, or multiplication by $\frac{1}{16}$). Record the representations or strategies students shared and display them for all to see.

When discussing the last strategy, ask students how they would know whether to multiply or to divide. Highlight that we multiply or divide depending on the information we have. Since 1 cup equals 16 tablespoons, if we know a quantity in cups, we can multiply it by 16 to find the number of tablespoons. On the other hand, if we know a quantity in tablespoons, we can divide it by 16 (or multiply by $\frac{1}{16}$) to find the number of cups.

Reading, Writing, Speaking: Clarify, Critique, Correct. Before students share their answers to the second question, present an incorrect solution and explanation or representation. For example, "Diego used zero cups of cocoa powder because 6 tablespoons is less than 1." Ask students to identify the error(s), analyse the response in light of their own understanding of the problem, and work with a partner to propose an improved response. This will help students understand how a "unit rate" could help convert between units.
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

Lesson Synthesis

In this lesson, students learned two important points:

- Two measurements of the same object in different units form equivalent ratios, and we can use all of our familiar tools to reason about equivalent ratios when we are thinking about converting units of measure.
- If we know a "unit rate" that relates the two units, we can use it to convert one measurement to the other by multiplication or division.

To highlight the first point, choose and display a couple of tables of equivalent ratios from the lesson, each table showing two different units (e.g., pounds and kilograms, or cups and teaspoons). Ask students to explain how pairs of numbers in the table represent equivalent ratios and how to use equivalent ratios to convert between units of measurement.

Then, ask if and how a "unit rate" could help us convert between units. Show examples from the lesson about how multiplying and dividing a unit rate helps us with conversion. For instance, we know that 1 kilogram is 2.2 pounds. With this information, we can convert 5 kilograms into 11 pounds, because $5 \times (2.2) = 11$. We can also convert 220 pounds into 100 kilograms, because $220 \div 2.2 = 100$.

4.5 Buckets

Cool Down: 5 minutes

Student Task Statement

A large bucket holds 5 gallons of water, which is about the same as 19 litres of water.

A small bucket holds 2 gallons of water. About how many litres does it hold?

Student Response

$\frac{38}{5}$ (or 7.6 or equivalent). Possible strategy:

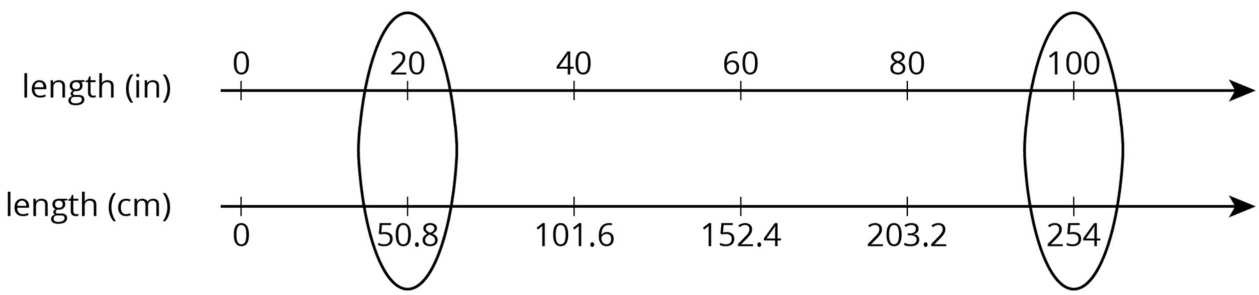
gallons	litres
5	19
1	$\frac{19}{5}$
2	$\frac{38}{5}$

Student Lesson Summary

When we measure something in two different units, the measurements form an equivalent ratio. We can reason with these equivalent ratios to convert measurements from one unit to another.

Suppose you cut off 20 inches of hair. Your friend asks how many centimetres of hair that was. Since 100 inches equal 254 centimetres, we can use equivalent ratios to find out how many centimetres equal 20 inches.

Using a double number line:



Using a table:

length (in)	length (cm)
100	254
1	2.54
20	50.8

One quick way to solve the problem is to start by finding out how many centimetres are in 1 inch. We can then multiply 2.54 and 20 to find that 20 inches equal 50.8 centimetres.

Lesson 4 Practice Problems

Problem 1 Statement

Priya's family exchanged 250 pounds for 4 250 pesos. Priya bought a sweater for 510 pesos. How many pounds did the sweater cost?

pesos	pounds
4 250	250
	25
	1
	3
510	

Solution

30 pounds

Problem 2 Statement

There are 3 785 millilitres in 1 gallon, and there are 4 quarts in 1 gallon. For each question, explain or show your reasoning.

a. How many millilitres are in 3 gallons?

b. How many millilitres are in 1 quart?

Solution

a. 11 355 millilitres, because $3\,785 \times 3 = 11\,355$.

b. 946.25 millilitres, because $3\,785 \div 4 = 946.25$

Problem 3 Statement

Lin knows that there are 4 quarts in a gallon. She wants to convert 6 quarts to gallons, but cannot decide if she should multiply 6 by 4 or divide 6 by 4 to find her answer. What should she do? Explain or show your reasoning. If you get stuck, consider drawing a double number line or using a table.

Solution

Lin should divide 6 by 4. Explanations vary. Sample explanations:

- A gallon is larger than a quart, so there are fewer than 6 gallons in 6 quarts.
- Table:

quarts	gallons
4	1
6	1.5

Problem 4 Statement

Tyler has a baseball bat that weighs 28 ounces. Find this weight in kilograms and in grams. (Note: 1 kilogram $\approx$ 35 ounces)

Solution

0.8 kilograms $(28 \div 35 = 0.8)$ and 800 grams $(0.8 \times 1\,000 = 800)$

Problem 5 Statement

Identify whether each unit measures length, volume, or weight (or mass).

- Mile
- Cup
- Pound
- Centimetre
- Litre
- Gram
- Pint
- Yard
- Kilogram
- Teaspoon
- Millilitre

Solution

a. Length

b. Volume

c. Weight (or mass)

d. Length

e. Volume

f. Weight (or mass)

g. Volume

h. Length

i. Weight (or mass)

j. Volume

k. Volume

Problem 6 Statement

A recipe for trail mix uses 7 ounces of almonds with 5 ounces of raisins. (Almonds and raisins are the only ingredients.) How many ounces of almonds would be in a one-pound bag of this trail mix? Explain or show your reasoning. There are 16 ounces in 1 pound.

Solution

$\frac{28}{3} = 9\frac{1}{3}$, so there are $9\frac{1}{3}$ ounces of almonds. There are multiple ways to find this, and one way is to know the original mix has 12 ounces and multiply by $\frac{16}{12} = \frac{4}{3}$ to produce an equivalent ratio for a 16-ounce mix.

Problem 7 Statement

An ant can travel at a constant speed of 980 inches every 5 minutes.

a. How far does the ant travel in 1 minute?

b. At this rate, how far can the ant travel in 7 minutes?

Solution

a. 196 inches per minute because $980 \div 5 = 196$.

b. 1 372 inches because 196 times 7 is 1 372.