Lesson 4: Solving and representing situations with equations

Goals

- Interpret and coordinate sentences, equations, and diagrams that represent the same addition or multiplication situation.
- Solve equations of the form $x + p = q$ or $px = q$ and explain (in writing) the solution method.

Learning Targets

- I can explain why different equations can describe the same situation.
- I can solve equations that have whole numbers, fractions, and decimals.

Lesson Narrative

In this lesson, students consolidate their equation writing and solving skills. In the first activity they solve a variety of equations with different structures, and in the second they work to match equations to situations and solve them. Students may choose any strategy to solve equations, including drawing diagrams to reason about unknown quantities, looking at the structure of the equation, or doing the same thing to each side of the equation. They choose efficient tools and strategies for specific problems. This will help students develop flexibility and fluency in writing and solving equations.

Alignments

Addressing

- Reason about and solve one-variable equations and inequalities.
- Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- Collect and Display
- Three Reads
- Discussion Supports
- Number Talk

Student Learning Goals

Let's solve equations by doing the same to each side.

4.1 Number Talk: Subtracting From Five

Warm Up: 5 minutes

The purpose of this number talk is to have students recall subtraction where regrouping needs to happen in preparation for the problems they will solve in the lesson.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Find the value of each expression mentally.

$5 - 2$

$5 - 2.1$

$5 - 2.17$

$5 - 2\frac{7}{8}$

Student Response

- 3

- 2.9
- 2.83
- $2\frac{1}{8}$

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

4.2 Row Game: Solving Equations Practice

15 minutes

The purpose of this activity is for students to practice solving equations. Some students may use the "do the same to each side" strategy they developed in their work with balanced balances. Others may use strategies like substituting values until they find a value that makes the equation true, or asking themselves questions like "2 times what is 18?" As students progress through the activity, the equations become more difficult to solve by strategies other than "do the same thing to each side."

Instructional Routines

- Collect and Display

Launch

Display an equation like $2x = 12$ or similar. Ask students to think about the balanced balances of the last lesson and to recall how that helped us solve equations by doing the same to each side. Tell students that, after obtaining a solution via algebraic means, we end up with a variable on one side of the equal sign and a number on the other, e.g. $x = 6$. We can easily read the solution—in this case 6—from an equation with a letter on one side and a number on the other, and we often write solutions in this way. Tell students that the act of finding an equation's solution is sometimes called *solving* the equation.

Arrange students in groups of 2, and ensure that everyone understands how the row game works before students start working. Allow students 10 minutes of partner work followed by a whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts. For example, after students have completed the first four rows of the table, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.
Supports accessibility for: Organisation; Attention Conversing, Representing, Writing: Collect and Display. While pairs are working, circulate, collect and make a visual display of vocabulary, phrases and representations students use as they solve each situation. Make connections between how similar ideas might be communicated and represented in different ways. Look for and amplify phrases such as "I did the same thing to each side" or "I subtracted the same amount from both sides." This helps students use mathematical language during paired and whole-group discussions.
Design Principle(s): Support sense-making

Student Task Statement

Solve the equations in one column. Your partner will work on the other column.

Check in with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error and correct it.

column A	Answer	column B
$18 = 2x$		$36 = 4x$
$17 = x + 9$		$13 = x + 5$
$8x = 56$		$3x = 21$
$21 = \frac{1}{4}x$		$28 = \frac{1}{3}x$
$6x = 45$		$8x = 60$
$x + 4\frac{5}{6} = 9$		$x + 3\frac{5}{6} = 8$

$\frac{5}{7}x = 55$		$\frac{3}{7}x = 33$
$\frac{1}{5} = 6x$		$\frac{1}{3} = 10x$
$2.17 + x = 5$		$6.17 + x = 9$
$\frac{20}{3} = \frac{10}{9}x$		$\frac{14}{5} = \frac{7}{15}x$
$14.88 + x = 17.05$		$3.91 + x = 6.08$
$3\frac{3}{4}x = 1\frac{1}{4}$		$\frac{7}{5}x = \frac{7}{15}$

Student Response

1. $x = 9$
2. $x = 8$
3. $x = 7$
4. $x = 84$
5. $x = 7\frac{1}{2}$
6. $x = 4\frac{1}{6}$
7. $x = 77$
8. $x = \frac{1}{30}$
9. $x = 2.83$
10. $x = 6$
11. $x = 2.17$
12. $x = \frac{1}{3}$

Activity Synthesis

Draw students' attention to $21 = \frac{1}{4}x$, and ask selected students to explain how they thought about solving this equation. Some may share strategies like “one-fourth of what number is 21?” Ideally, one student will say “divide each side by $\frac{1}{4}$” and another will say “multiply each side by 4.” From their studies in earlier units, students should understand that multiplying by 4 has the same result as dividing by $\frac{1}{4}$. Next, turn students attention to $\frac{5}{7}x = 55$ and ask them to describe the two ways to think about solving it. “Divide each side by $\frac{5}{7}$” gives the same result as “Multiply each side by $\frac{7}{5}$.”

4.3 Choosing Equations to Match Situations

15 minutes

The purpose of this activity is for students to practice matching equations to situations and then solving those equations using their new strategies. Monitor for students who draw diagrams (bar models, balances, or their own creations) that describe the relationships and those who solve the equations by doing the same to each side of one or more equations.

Instructional Routines

- Three Reads

Launch

Allow students 10 minutes of quiet work time, followed by a whole-class discussion.

Reading: Three Reads. Demonstrate this routine with the first situation to support reading comprehension. Use the first read to help students understand context. Ask, "What is this situation about?" (e.g., Clare and Mai each have a different number of books). After the second read, ask students “What are the quantities in the situation?” (e.g., the number of books Mai has, the number of books Clare has). After the third read, ask students to brainstorm possible strategies to connect the situation with the appropriate equation(s). Encourage students to repeat this routine themselves for each situation. This helps students connect the language in the word problem with the equation(s) while keeping the intended level of cognitive demand in the task.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Students who continue to focus on key words misidentify the relationship in each situation. Encourage students to express the relationships in their own words and draw diagrams comparing the given quantities. For example, in the situation with Clare and Mai, they can draw a long rectangle representing Mai's books subdivided into two pieces. Filling in the information given in the story will help clear up the relationships; Clare's rectangle is labelled x, and she has 8 fewer books than Mai, so Mai's rectangle is labelled $x + 8$ and also 26. Alternatively, they can show that the piece labelled x must equal $26 - 8$.

Student Task Statement

Circle **all** of the equations that describe each situation. If you get stuck, consider drawing a diagram. Then find the solution for each situation.

1. Clare has 8 fewer books than Mai. If Mai has 26 books, how many books does Clare have?

   - $26 - x = 8$
   - $x = 26 + 8$
   - $x + 8 = 26$
   - $26 - 8 = x$

   $x =$ ______

2. A coach formed teams of 8 from all the players in a soccer league. There are 14 teams. How many players are in the league?

   - $y = 14 \div 8$
   - $\frac{y}{8} = 14$
   - $\frac{1}{8}y = 14$
   - $y = 14 \times 8$

   $y =$ ______

3. Kiran scored 223 more points in a computer game than Tyler. If Kiran scored 409 points, how many points did Tyler score?

   - $223 = 409 - z$
   - $409 - 223 = z$
   - $409 + 223 = z$
   - $409 = 223 + z$

   $z =$ ______

4. Mai ran 27 miles last week, which was three times as far as Jada ran. How far did Jada run?

   - $3w = 27$

- $w = \frac{1}{3} \times 27$
- $w = 27 \div 3$
- $w = 3 \times 27$
  $w =$ _____

Student Response

1. $26 - x = 8, x + 8 = 26, 26 - 8 = x; x = 18$
2. $\frac{y}{8} = 14, \frac{1}{8}y = 14, y = 14 \times 8; y = 112$
3. $223 = 409 - z, 409 - 223 = z, 409 = 223 + z; z = 186$
4. $3w = 27, w = \frac{1}{3} \times 27, w = 27 \div 3; w = 9$

Are You Ready for More?

Mai's mother was 28 when Mai was born. Mai is now 12 years old. In how many years will Mai's mother be twice Mai's age? How old will they be then?

Student Response

16 years; Mai will be 28 and her mother will be 56.

Activity Synthesis

Invite students to share their strategies for matching equations to the stories and for solving those equations. Include students who drew bar models, balances, or other types of diagrams to help them understand and reason about the relationships. Record the diagrams and strategies and have students compare them. Ask where they see information from the story in the parts of the diagrams and equations.

If no students bring it up, ask if any of the situations have a similar structure.

- The Clare/Mai and Kiran/Tyler situations share a similar structure where both the larger quantity and the difference between the smaller and larger quantities are known while the smaller quantity is unknown. Note that the first relationship is expressed with "fewer" and the second with "more." This provides an opportunity for students to reason about the quantities, decontextualising to see the similar structure and then contextualising to understand the situations and answer questions.
- The soccer teams and Mai/Jada situations share similar structures in that equal parts add to a whole. The two problems differ in which quantities are known and unknown. In the soccer situation, the size of each group (8 players per team) and number of groups (14 teams) are known while the total is unknown. In the Mai/Jada

multiplicative comparison situation, a total is known (27 miles) and the number of groups is known (3 times as many) but the size of each group is unknown.

Focusing on structure in this way helps students reason about the relationships between quantities in a situation, rather than focus on the words in the problem as hints to the operations needed in the equations.

For students who solved for the unknown by using the equations, ask which of the chosen equations they decided to solve and why.

Lesson Synthesis

The end of this lesson is a good place for students to take a moment and reflect on the learning of the past four lessons. Some questions to guide the discussion:

- "Describe some ways to understand how a situation can be represented mathematically."
- "What have you learned about equations that surprised you?"
- "Share your thoughts about using diagrams to help understand relationships. Where have you seen diagrams used earlier this year? Where were they most helpful to you? Least helpful?"
- "Describe any connections you see between the types of diagrams used in the last four lessons."

4.4 More Storytime

Cool Down: 5 minutes

Student Task Statement

1. Write a story to match the equation $x + 2\frac{1}{2} = 10$.
2. Explain what x represents in your story.
3. Solve the equation. Explain or show your reasoning.

Student Response

Answers vary. Sample responses:

1. Lin likes to bake batches of muffins and share them with friends and family. She needs 10 cups of flour for her next batch, but only has $2\frac{1}{2}$ cups left. How much more flour does she need?
2. x represents the number of cups of flour Lin needs.

3. $x = 7\frac{1}{2}$. I subtracted $2\frac{1}{2}$ from each side of the equation.

Student Lesson Summary

Writing and solving equations can help us answer questions about situations.

Suppose a scientist has 13.68 litres of acid and needs 16.05 litres for an experiment. How many more litres of acid does she need for the experiment?

- We can represent this situation with the equation:

$13.68 + x = 16.05$

- When working with balances, we saw that the solution can be found by subtracting 13.68 from each side. This gives us some new equations that also represent the situation:

$x = 16.05 - 13.68$

$x = 2.37$

- Finding a solution in this way leads to a variable on one side of the equal sign and a number on the other. We can easily read the solution—in this case, 2.37—from an equation with a letter on one side and a number on the other. We often write solutions in this way.

Let's say a food pantry takes a 54-pound bag of rice and splits it into portions that each weigh $\frac{3}{4}$ of a pound. How many portions can they make from this bag?

- We can represent this situation with the equation:

$\frac{3}{4}x = 54$

- We can find the value of x by dividing each side by $\frac{3}{4}$. This gives us some new equations that represent the same situation:

$x = 54 \div \frac{3}{4}$

$x = 72$

- The solution is 72 portions.

Lesson 4 Practice Problems

1. **Problem 1 Statement**

   Select **all** the equations that describe each situation and then find the solution.

a. Kiran's backpack weighs 3 pounds less than Clare's backpack. Clare's backpack weighs 14 pounds. How much does Kiran's backpack weigh?

- $x + 3 = 14$
- $3x = 14$
- $x = 14 - 3$
- $x = 14 \div 3$

b. Each notebook contains 60 sheets of paper. Andre has 5 notebooks. How many sheets of paper do Andre's notebooks contain?

- $y = 60 \div 5$
- $y = 5 \times 60$
- $\frac{y}{5} = 60$
- $5y = 60$

Solution

a. $x + 3 = 14, x = 14 - 3; x = 11$, 11 pounds

b. $y = 5 \times 60, \frac{y}{5} = 60;\ y = 300$, 300 sheets

2. **Problem 2 Statement**

Solve each equation.

a. $2x = 5$

b. $y + 1.8 = 14.7$

c. $6 = \frac{1}{2}z$

d. $3\frac{1}{4} = \frac{1}{2} + w$

e. $2.5t = 10$

Solution

a. $x = \frac{5}{2}$ (or equivalent)

b. $y = 12.9$

c. $z = 12$

d. $w = 2\frac{3}{4}$ (or equivalent)

e. $t = 4$

3. **Problem 3 Statement**

For each equation, draw a bar model that represents the equation.

a. $3 \times x = 18$

b. $3 + x = 18$

c. $17 - 6 = x$

Solution

a. A bar model showing 3 groups labelled x and a total of 18.

b. A bar model showing one part labelled 3 and another labelled x and a total of 18.

c. A bar model showing one part labelled 6 and another labelled x and a total of 17.

4. **Problem 4 Statement**

Find each product.

$(21.2) \times (0.02)$

$(2.05) \times (0.004)$

Solution

$(21.2) \times (0.02) = 0.424$

$(2.05) \times (0.004) = 0.0082$

5. **Problem 5 Statement**

For a science experiment, students need to find 25% of 60 grams.

- Jada says, "I can find this by calculating $\frac{1}{4}$ of 60."
- Andre says, "25% of 60 means $\frac{25}{100} \times 60$."

Do you agree with either of them? Explain your reasoning.

Solution

Both are correct. Andre is right that 25% of a number means $\frac{25}{100}$ of that number. Jada is also right because $\frac{25}{100} = \frac{1}{4}$.