Lesson 13: Reintroducing inequalities

Goals

- Comprehend the terms "less than or equal to" and "greater than or equal to" (in spoken and written language) and the symbols ≤ and ≥ (in written language).
- Recognise that more than one value for a variable makes the same inequality true.
- Use substitution to determine whether a given value for a variable makes an inequality true, and justify (orally) the answer.

Learning Targets

- I can explain what the symbols $\leq$ and $\geq$ mean.
- I can represent an inequality on a number line.
- I understand what it means for a number to make an inequality true.

Lesson Narrative

Earlier in KS3, students learned how to write and interpret inequalities of the form $x < c$ and $x > c$. In this lesson, students begin to investigate inequalities of the form $px < q$ and $x + p < q$.

First, they are reintroduced to the notation < and > and reminded how inequalities can be expressed algebraically and graphically on a number line. A context is used to help students make sense of inequalities. The symbols $\leq$ and $\geq$ are introduced, which are the relevant symbols to use in many of the modelling problems they will see later on. Then they use substitution to check whether given values of x satisfy inequalities.

Building On

- Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Addressing

- Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Building Towards

- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Instructional Routines

- Co-Craft Questions
- Discussion Supports
- Think Pair Share

Student Learning Goals

Let's work with inequalities.

13.1 Greater Than One

Warm Up: 5 minutes

The purpose of this activity is for students to substitute values into an inequality and check to see if each value satisfies the inequality based on a number line representation. This is the first opportunity (of many to come) to practise this type of substitution with inequalities.

Instructional Routines

- Think Pair Share

Launch

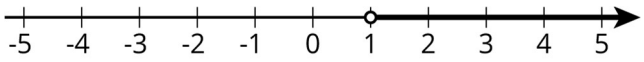
Arrange students in groups of 2. Since it may have been a while since students encountered this notation, remind students that $x > 1$ is read "x is greater than 1." Give students 2 minutes of quiet work time followed by 1 minute to share their responses with a partner. During the partner discussion, tell students to compare their answers for the first question and see if they agree with their partner's chosen values for the second question. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students may think 700 is not a solution to $x > 1$. Tell students that since there is an arrow at the end of the dark line, it includes all values that would fall on that line, even the ones not shown.

Student Task Statement

The number line shows values of x that make the inequality $x > 1$ true.



1. Select **all** the values of x from this list that make the inequality $x > 1$ true.
    a. 3
    b. -3

c. 1

d. 700

e. 1.05

2. Name two more values of x that are solutions to the inequality.

Student Response

1. a (3), d (700), and e (1.05)

2. Answers vary. Sample response: 4 and 10.

Activity Synthesis

Ask a few students to share their responses for the last question. After each student shares, ask the class whether they agree or disagree. If students focus solely on the number line representation, be sure to tell them that to test whether a value makes an inequality true, you can substitute the value for the variable. Highlight the fact that "greater than 1" does not include 1.

Here are some questions for discussion:

- What does the open circle at 1 mean? (It means 1 is not included.)
- Is 1 a solution to the inequality $x > 1$?

13.2 The Roller Coaster

15 minutes

The purpose of this activity is to remind students that the symbol < is read "is less than" and the symbol > is read "is greater than." Also, remind students of the use of an open circle or closed circle to indicate whether the boundary point is included. Then, the symbols $\leq$ and $\geq$ are introduced.

Monitor for students who express the answer to the last question using words or using symbols. The responses to the last question will be used to introduce the new notation.

Instructional Routines

- Co-Craft Questions
- Think Pair Share

Launch

Allow students 5–10 minutes quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Representation: Access for Perception. Read all problems aloud. Students who both listen to and read the information will benefit from extra processing time.

Supports accessibility for: Language Conversing, Writing: Co-craft Questions. Use this routine to help students consider the context of the first problem and to increase awareness about language used to describe situations involving inequalities. Begin by displaying only the initial text and photo of the roller coaster, without revealing the follow-up questions. In groups of 2, invite students to write down mathematical questions they have about this situation. Ask pairs to share their questions with the whole class. Amplify questions that highlight the mathematical language of "at least."
Design Principle(s): Cultivate conversation; Maximise meta-awareness

Anticipated Misconceptions

If students are having trouble interpreting the first three questions or articulating their responses, encourage them to make use of the number line that appears in question 4.

Student Task Statement

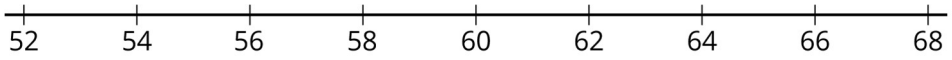
A sign next to a roller coaster at an amusement park says, "You must be at least 60 inches tall to ride." Noah is happy to know that he is tall enough to ride.

1. Noah is x inches tall. Which of the following can be true: $x > 60$, $x = 60$, or $x < 60$? Explain how you know.

2. Noah's friend is 2 inches shorter than Noah. Can you tell if Noah's friend is tall enough to go on the ride? Explain or show your reasoning.

3. List one possible height for Noah that means that his friend is tall enough to go on the ride, and another that means that his friend is too short for the ride.

4. On the number line below, show all the possible heights that Noah's friend could be.



5. Noah's friend is y inches tall. Use y and any of the symbols $<, =, >$ to express this height.

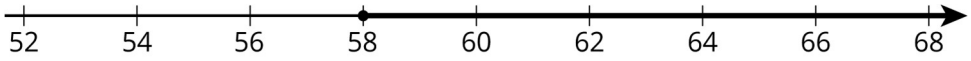
Student Response

1. $x > 60$ and $x = 60$. Explanations vary. Sample response: "At least" means that Noah must be 60 inches or taller.

2. No, we don't know if Noah's friend is tall enough to go on the ride. Explanations vary. Sample response: Since we don't know whether Noah's height is exactly 60, within 2 inches of 60, or more than 2 inches above 60, we can't know if 2 less than his height is at least 60.

3. Answers vary. Sample response: Noah could be 63 inches tall, which means his friend is 61 inches tall and can ride. Noah can be 61 inches tall, which means his friend is 59 inches tall and cannot ride.

4.



5. $y > 58$ or $y = 58$.

Activity Synthesis

Ask selected students to share their response to the last question. They are likely to write something like "$y = 58$ or $y > 58$." The < and > symbols are not enough to capture what we need here with a single mathematical statement. In other units, students saw graphs of inequalities using open and closed circles. Now, we can introduce new symbols $\leq$ and $\geq$, that mean "less than or equal to" and "greater than or equal to."

13.3 Is the Inequality True or False?

15 minutes

The purpose of this task is for students to interpret the notation $<, >, \leq$, and $\geq$ to evaluate whether different values make an inequality true. It is not expected that students will solve the inequalities generally. The work in this activity sets students up for success in later

lessons when they test points to determine the direction of the inequality in the process of solving.

Notice students who use the following approaches:

- Substituting a value in for x, evaluating the expression, thinking about whether the statement is true. For example, "Is -300 less than 75? Yes, so this is true."
- Drawing a number line for each inequality and using it to reason about different values.

Instructional Routines

- Discussion Supports

Launch

Keep students in the same groups. Give 5–10 minutes quiet and partner work time followed by a whole-class discussion.

Engagement: Internalise Self Regulation. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. After students have completed the first 1–2 rows of the table, consider pausing for a brief class discussion before moving on. *Supports accessibility for: Organisation; Attention*

Anticipated Misconceptions

Students who try to apply what they know about solving equations to solve the inequalities algebraically may come up with incorrect solutions. For instance, $100 < 4x$ may at first glance look equivalent to $x < 25$, since the "less than" sign appears. Students may incorrectly think that $-3x > -75$ is equivalent to $x > 25$. Ask these students, for example, what the solution to $100 = 4x$ means (25 is the value of x that makes $4x$ equal to 100). Then encourage these students to test values like 24 and 26 to see whether they are solutions to $100 < 4x$. This will be covered in greater detail in a later lesson, so this understanding does not need to be solidified at this time.

Student Task Statement

The table shows four inequalities and four possible values for x. Decide whether each value makes each inequality true, and complete the table with "true" or "false." Discuss your thinking with your partner. If you disagree, work to reach an agreement.

x	0	100	-100	25
$x \leq 25$				
$100 < 4x$				
$-3x > -75$				
$10 \geq 35 - x$				

Student Response

x	0	100	-100	25
$x \leq 25$	true	false	true	true
$100 < 4x$	false	true	false	false
$-3x > -75$	true	false	true	false
$10 \geq 35 - x$	false	true	false	true

Are You Ready for More?

Find an example of an inequality used in the real world and describe it using a number line.

Student Response

Answers vary.

Activity Synthesis

The purpose of the discussion is to note the consequences of an inequality using $\leq$ versus $<$, and $\geq$ versus $>$. Direct students' attention to $100 < 4x$ and $10 \geq 35 - x$. Substituting 25 for x in each of these inequalities gives $100 < 100$, which is false, and $10 \geq 10$, which is true. The key distinction is that $\geq$ and $\leq$ inequalities are considered true when both sides are equal, whereas $<$ and $>$ inequalities are considered false when both sides are equal. Ask students to present different strategies for determining whether a value makes an inequality true. Ask whether students were surprised by or initially incorrect about any of the answers. Emphasise that substituting a value in for x, and thinking about whether the resulting inequality is saying something true, is the most direct way to check whether the value is a solution.

Speaking, Conversing: Discussion Supports. Provide students with sentence frames that will help them define what each inequality symbol means. For example, "___ is greater than or equal to ___.", "___ is less than or equal to _____.", " _____ is greater than _____." and "_____ is less than _____." Create a chart with the matching symbols for each frame and encourage students to refer to this chart when they explain whether they think each inequality is "true" or "false."
Design Principle(s): Support sense-making

Lesson Synthesis

By the end of this lesson, students should be able to interpret the symbols $\geq$ and $\leq$ and be able to test whether a given value makes an inequality true. Ask students to write an equality to which -5 is a solution, then trade with their partner to see if their partner agrees.

13.4 Some Values, All Values

Cool Down: 5 minutes

The purpose of this activity is for students to use any of the strategies they developed in this lesson to graph the solution to an inequality and contrast < with ≤.

Anticipated Misconceptions

Students may divide both sides of the inequality by -2 to arrive at the incorrect solution $x > -5$.

Student Task Statement

Here is an inequality: $-2x > 10$.

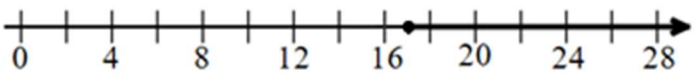
1. List some values for x that would make this inequality true.
2. How are the solutions to the inequality $-2x \geq 10$ different from the solutions to $-2x > 10$? Explain your reasoning.

Student Response

1. Any number less than -5 is a solution.
2. Responses vary. Sample response: The solutions to $-2x > 10$ and $-2x \geq 10$ are the same, except when x is -5. The number -5 would be included as a solution to $10 \leq -2x$ because $10 \leq -2(-5)$ is a true statement.

Student Lesson Summary

We use inequalities to describe a range of numbers. In many places, you are allowed to get a driver's license when you are at least 17 years old. When checking if someone is old enough to get a license, we want to know if their age is at least 17. If h is the age of a person, then we can check if they are allowed to get a driver's license by checking if their age makes the inequality $h > 17$ (they are older than 17) or the equation $h = 17$ (they are 17) true. The symbol ≥, pronounced "greater than or equal to," combines these two cases and we can just check if $h \geq 17$ (their age is greater than or equal to 17). The inequality $h \geq 17$ can be represented on a number line:



Lesson 13 Practice Problems

1. **Problem 1 Statement**

   For each inequality, find two values for x that make the inequality true and two values that make it false.

   a. $x + 3 > 70$

b. $x + 3 < 70$

c. $-5x < 2$

d. $5x < 2$

Solution

Answers vary. Sample response:

a. True: $x = 70$ and $x = 100$, false: $x = 0$ and $x = -10$

b. True: $x = 60$ and $x = 0$, false: $x = 70$ and $x = 100$

c. True: $x = 1$ and $x = 2$, false: $x = -1$ and $x = -2$

d. True: $x = 0$ and $x = -1$, false: $x = 1$ and $x = 100$

2. **Problem 2 Statement**

Here is an inequality: $-3x > 18$.

a. List some values for x that would make this inequality true.

b. How are the solutions to the inequality $-3x \geq 18$ different from the solutions to $-3x > 18$? Explain your reasoning.

Solution

a. Any value less than -6

b. The inequalities have almost the same solutions, but the first includes -6 and the second does not.

3. **Problem 3 Statement**

Here are the prices for cheese pizza at a certain pizzeria:

pizza size	price in pounds
small	11.60
medium	
large	16.25

a. You had a coupon that made the price of a large pizza £13.00. For what percent off was the coupon?

b. Your friend purchased a medium pizza for £10.31 with a 30% off coupon. What is the price of a medium pizza without a coupon?

c. Your friend has a 15% off coupon and £10. What is the largest pizza that your friend can afford, and how much money will be left over after the purchase?

Solution

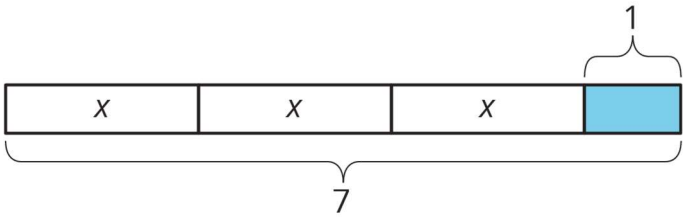
a. 20%

b. £14.73

c. Small, £0.14

4. **Problem 4 Statement**

Select **all** the stories that can be represented by the diagram.



a. Andre studies 7 hours this week for end-of-year exams. He spends 1 hour on English and an equal number of hours each on Maths, Science, and History.

b. Lin spends £3 on 7 markers and a £1 pen.

c. Diego spends £1 on 7 stickers and 3 marbles.

d. Noah shares 7 grapes with 3 friends. He eats 1 and gives each friend the same number of grapes.

e. Elena spends £7 on 3 notebooks and a £1 pen.

Solution ["A", "D", "E"]