Lesson 14: Multiplying, dividing, and estimating with standard form

Goals

- Generalise (orally and in writing) a process of multiplying and dividing numbers in standard form.
- Use standard form and estimation to compare quantities and interpret (orally and in writing) results in context.

Learning Targets

- I can multiply and divide numbers given in standard form.
- I can use standard form and estimation to compare very large or very small numbers.

Lesson Narrative

Students perform operations with numbers expressed in standard form, use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and express how many times as much one quantity is than the other. Students interpret their results in context.

Addressing

- Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.
- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.
- Perform operations with numbers expressed in standard form, including problems where both decimal and standard form are used. Use standard form and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimetres per year for seafloor spreading). Interpret standard form that has been generated by technology.

Building Towards

- Perform operations with numbers expressed in standard form, including problems where both decimal and standard form are used. Use standard form and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimetres per year for seafloor spreading). Interpret standard form that has been generated by technology.

Instructional Routines

- Stronger and Clearer Each Time

- Information Gap Cards
- Co-Craft Questions
- Notice and Wonder
- True or False

Required Materials

Pre-printed slips, cut from copies of the blackline master

Info Gap: Distances in the Solar System

Problem Card 1

Estimate:

1. How many Earths side by side would have the same width as the Sun?

2. How many Earths would it take to equal the mass of the Sun?

Info Gap: Distances in the Solar System

Data Card 1

- The distance from Earth to the Sun is approximately 1.496×10^8 km.
- The diameter of the Sun is 1.392×10^6 km.
- The diameter of Earth is 1.28×10^4 km.
- The mass of the Sun is 1.989×10^{30} kg.
- The mass of Earth is 5.98×10^{24} kg.

Info Gap: Distances in the Solar System

Problem Card 2

Estimate:

1. How many times as far away from Earth is the planet Neptune compared to Venus?

2. How many copies of the planet Mercury would it take to equal the mass of Neptune?

Info Gap: Distances in the Solar System

Data Card 2

- The average distance from Earth . . .
  - to Mercury is 7.73×10^7 km.
  - to Venus is 4×10^7 km.
  - to Neptune is 4.3×10^9 km.
- The mass of Mercury is 3.3×10^{23} kg.
- The mass of Venus is 4.87×10^{24} kg.
- The mass of Neptune is 1.024×10^{26} kg.

Required Preparation

Consider finding an attention-grabbing image of cats and people for the launch of the "Biomass" activity to get students thinking about how the global human population compares to the global cat population.

Print and cut up slips from the Info Gap: Distances in the Solar System blackline master. One copy of the blackline master is needed for every 4 students. A class set could be re-used if you have more than one class.

Student Learning Goals

Let’s multiply and divide with standard form to answer questions about animals, careers, and planets.

14.1 True or False: Equations

Warm Up: 5 minutes

The purpose of this warm-up is to encourage students to apply the properties of integer exponents to reason about equivalent expressions. While students may evaluate each side of the equation to determine if it is true or false, encourage students to reason about the properties of exponents and operations in their solution.

Instructional Routines

- True or False

Launch

Display one problem at a time. Tell students to give a signal when they have decided if the equation is true or false. Give students 2 minutes of quiet think time followed by a whole-class discussion.

Student Task Statement

Is each equation true or false? Explain your reasoning.

1. $4 \times 10^5 \times 4 \times 10^4 = 4 \times 10^{20}$
2. $\frac{7\times10^6}{2\times10^4} = (7 \div 2) \times 10^{(6-4)}$
3. $8.4 \times 10^3 \times 2 = (8.4 \times 2) \times 10^{(3\times2)}$

Student Response

1. False. Sample explanation: The exponents were multiplied when they should have been added.
2. True. Sample explanation: When dividing, the exponent in the denominator is subtracted from the numerator.
3. False. Sample explanation: Multiplication can be done in any order, but the multiplication in this problem doesn’t affect the power of 10.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Do you agree or disagree? Why?”
- “Who can restate ___’s reasoning in a different way?”
- “Does anyone want to add on to ____’s reasoning?”

After each false equation, ask students how the problem could be changed to make the equation true.

14.2 Biomass

15 minutes

In this activity, students answer questions about quantities in context. They use standard form as a tool for working with small and large numbers—to describe quantities, make estimates, and make comparisons (e.g., to express how many times as much one is as the other).

Instructional Routines

- Co-Craft Questions

Launch

If possible, display an attention-grabbing image of cats and people for all to see. Ask students, "How many humans do you think there are for each cat in the world?" Ask for estimates that are too high, too low, and as reasonable as possible.

Explain that large numbers like populations are often estimated using standard form. There are an estimated 7.5×10^9 humans and 6×10^8 cats in the world. Guide students through the example $\frac{7.5\times10^9}{6\times10^8} = \frac{7.5}{6} \times 10^{9-8} = 1.25 \times 10^1 = 12.5$. So there are roughly 12.5 humans for each cat.

Arrange students in groups of 2 to allow partner discussions as they work. Select a student to read the first paragraph before telling students to answer the questions. Tell students that estimation will help answer these questions much more easily, so if they get stuck computing, they should try to make reasonable estimates. For example, estimating the number of humans for each cat could have looked like $\frac{7.5\times10^9}{6\times10^8} \approx \frac{6\times10^9}{6\times10^8} = 1 \times 10^1 = 10$. In this case, the final estimate is within 20% of the original calculation (not very accurate, but within a power of 10).

Give groups 10–12 minutes to work, followed by a brief whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge of multiplication and repeated factors. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing Writing, Conversing: Co-Craft Questions. Display only the table, and invite groups to write a list of mathematical questions that could be answered using the data in the table. Select 2–3 groups to share their questions with the class. Look for questions that ask students to compare quantities. Next, reveal the questions of the activity. This routine allows students to produce the language of mathematical questions and talk about the quantities in this task that are represented in

standard form prior to being asked to solve questions based on the values.
Design Principle(s): Maximise meta-awareness; Support sense-making

Anticipated Misconceptions

Students may need help remembering how to estimate using a single digit times a power of 10. Remind these students that 1.9×10^5, for example, could be estimated as 2×10^5 to make calculation easier.

Student Task Statement

Use the table to answer questions about different creatures on the planet. Be prepared to explain your reasoning.

creature	number	mass of one individual (kg)
humans	7.5×10^9	6.2×10^1
cows	1.3×10^9	4×10^2
sheep	1.75×10^9	6×10^1
chickens	2.4×10^{10}	2×10^0
ants	5×10^{16}	3×10^{-6}
blue whales	4.7×10^3	1.9×10^5
Antarctic krill	7.8×10^{14}	4.86×10^{-4}
zooplankton	1×10^{20}	5×10^{-8}
bacteria	5×10^{30}	1×10^{-12}

1. Which creature is least numerous? Estimate how many times more ants there are.
2. Which creature is the least massive? Estimate how many times more massive a human is.
3. Which is more massive, the total mass of all the humans or the total mass of all the ants? About how many times more massive is it?
4. Which is more massive, the total mass of all the krill or the total mass of all the blue whales? About how many times more massive is it?

Student Response

1. Blue whales are the least numerous. There are about 10^{13}, or 10 trillion, times as many ants as blue whales because $\frac{5\times10^{16}}{5\times10^{3}}$. This uses 5×10^3 to estimate the number of blue whales.
2. Bacteria is the least massive. A human is about 60 trillion times more massive because $\frac{6.2\times10^{1}}{1\times10^{-12}} = 6.2 \times 10^{13}$.

3. The total human mass is about 3 times as massive as the total ant mass. The total human mass is 7.5×10^9 times 6.2×10^1 kg per human, which is approximately 45×10^{10} kg. The total ant mass is 5×10^{16} times 3×10^{-6}, which is 15×10^{10} kg.

4. The total mass of krill is about 400 times the total mass of blue whales. The total krill mass is 7.8×10^{14} times 4.86×10^{-4}, which is approximately 8×10^{14} times 5×10^{-4} or 400 billion kg. The total mass of blue whales is 4.7×10^3 times 1.9×10^5, which is approximately 5×10^3 times 2×10^5 or 1 billion kg.

Activity Synthesis

Select students to explain how they used standard form and estimation to compare quantities. Record and display their reasoning for all to see. After each student presents, ask others if they reasoned the same way and if there are other approaches.

Students should see that being able to multiply and divide quantities in standard form is particularly helpful for reasoning and estimating about very large or very small quantities, which would be challenging to work with in their decimal representations.

14.3 Info Gap: Distances in the Solar System

15 minutes (there is a digital version of this activity)

In this info gap activity, students continue to use standard form as a tool for working with small and large numbers—to describe quantities, make estimates, and make comparisons (e.g., to express how many times as much one is as the other).

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

As students work, identify those who use slightly different estimations to compare during whole-class discussion. While students may have slightly different results based on their estimations, the results should be relatively close in value compared to the power of 10.

Here is the text of the cards for reference and planning:

Info Gap: Distances in the Solar System

Problem Card 1

Estimate:

1. How many Earths side by side would have the same width as the Sun?

2. How many Earths would it take to equal the mass of the Sun?

Info Gap: Distances in the Solar System

Data Card 1

- The distance from Earth to the Sun is approximately 1.496×10^8 km.
- The diameter of the Sun is 1.392×10^6 km.
- The diameter of Earth is 1.28×10^4 km.
- The mass of the Sun is 1.989×10^{30} kg.
- The mass of Earth is 5.98×10^{24} kg.

Info Gap: Distances in the Solar System

Problem Card 2

Estimate:

1. How many times as far away from Earth is the planet Neptune compared to Venus?

2. How many copies of the planet Mercury would it take to equal the mass of Neptune?

Info Gap: Distances in the Solar System

Data Card 2

- The average distance from Earth . . .
  - to Mercury is 7.73×10^7 km.
  - to Venus is 4×10^7 km.
  - to Neptune is 4.3×10^9 km.
- The mass of Mercury is 3.3×10^{23} kg.
- The mass of Venus is 4.87×10^{24} kg.
- The mass of Neptune is 1.024×10^{26} kg.

Instructional Routines

- Information Gap Cards

Launch

Arrange students in groups of 2. In each group, distribute a problem card to one student and a data card to the other student.

The digital version of this activity includes an extension that challenges students to create their own scale drawing of the planets. They need to calculate values for the radii and follow the simple steps in the activity.

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.
Supports accessibility for: Memory; Organisation Conversing: This activity uses Information Gap to give students a purpose for discussing information necessary to solve problems involving standard form. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?”
Design Principle(s): Cultivate Conversation

Anticipated Misconceptions

Some students may not recognize that "width" and "diameter" refer to the same measurement in this context. If needed, prompt students with Data Card 1 to give the information about the diameter when their partner asks for the width.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain to your partner how you are using the information to solve the problem.
4. Solve the problem and explain your reasoning to your partner.

If your teacher gives you the *data card*:

1. Silently read the information on your card.
2. Ask your partner "What specific information do you need?" and wait for your partner to *ask* for information. *Only* give information that is on your card. (Do not figure out anything for your partner!)
3. Before telling your partner the information, ask "Why do you need that information?"
4. After your partner solves the problem, ask them to explain their reasoning and listen to their explanation.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response

Card 1

1. The Sun is about 100 times the width of Earth because $\frac{1.392 \times 10^6}{1.28 \times 10^4} \approx \frac{10^6}{10^4} = 10^2$.
2. The Sun is about 300 000 times as massive as Earth because $\frac{1.989 \times 10^{30}}{5.98 \times 10^{24}} \approx \frac{2 \times 10^{30}}{6 \times 10^{24}} = \frac{1}{3} \times 10^6$, which is $333\,333.\overline{3}$.

Card 2

1. Neptune is about 100 times as far from Earth as Venus because $\frac{4.3\times10^9}{4\times10^7} \approx \frac{10^9}{10^7} = 10^2$.

2. It would take around 300 Mercuries to equal the mass of Neptune because $\frac{1.024\times10^{26}}{3.3\times10^{23}} \approx \frac{1\times10^{26}}{3\times10^{23}} = \frac{1}{3} \times 10^3$, which is $333.\overline{3}$.

Activity Synthesis

Select students to explain how they used standard form and estimation to compare the sizes of objects in the solar system. Poll the class on whether they estimated the same way. Ask previously identified students (who had slightly different ways of estimating) to compare their estimates and to show that any reasonable estimate will differ by an amount that is much smaller than the given quantities.

If time allows, discuss how standard form was useful in answering these questions.

14.4 Professions in the United States

Optional: 15 minutes

This activity gives students additional practice using standard form to work with small and large numbers and answering questions about quantities in context. Students express numbers in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, as well as to determine how many times as much one is than the other.

Instructional Routines

- Stronger and Clearer Each Time
- Notice and Wonder

Launch

Give students a minute of quiet time to observe the values in the table. Ask them to be prepared to share at least one thing they notice and one thing they wonder. Then, invite students to share their observations and questions. Consider recording their responses for all to see and using students' observations or questions about standard form, order of magnitude, and place values to orient students to the work in the activity.

Tell students that estimation will make it much easier to answer these questions. Give students 10–12 minutes to work, followed by a brief whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge of working with small and large numbers. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing Writing, Speaking: Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their response to the last question. Ask each student to meet with 2–3 other partners

in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., "What did you do first?", "How do you know…?", "How did you compare the two values?", etc.). Students can borrow ideas and language from each partner to strengthen their final version.
Design Principle(s): Optimise output (for explanation)

Student Task Statement

Use the table to answer questions about professions in the United States as of 2012.

profession	number	typical annual salary (U.S. dollars)
architect	1.074×10^5	7.3×10^4
artist	5.14×10^4	4.4×10^4
programmer	1.36×10^6	8.85×10^4
doctor	6.9×10^5	1.87×10^5
engineer	6.17×10^5	8.6×10^4
firefighter	3.07×10^5	4.5×10^4
military—enlisted	1.16×10^6	4.38×10^4
military—officer	2.5×10^5	1×10^5
nurse	3.45×10^6	6.03×10^4
police officer	7.8×10^5	5.7×10^4
college professor	1.27×10^6	6.9×10^4
retail sales	4.67×10^6	2.14×10^4
truck driver	1.7×10^6	3.82×10^4

Answer the following questions about professions in the United States. Express each answer in standard form.

1. Estimate how many times more nurses there are than doctors.
2. Estimate how much money all doctors make put together.
3. Estimate how much money all police officers make put together.
4. Who makes more money, all enlisted military put together or all military officers put together? Estimate how many times more.

Student Response

1. There are about 5 times as many nurses as doctors because $\frac{3.45\times10^6}{6.9\times10^5} \approx \frac{3.5\times10^6}{7\times10^5} = \frac{1}{2} \times 10^1 = 5$.

2. Doctors make about \$140 billion put together, because 6.9×10^5 individuals times 1.87×10^5 per doctor is roughly equal to $7 \times 10^5 \times 2 \times 10^5$, which is equal to 14×10^{10}.

3. Police make about \$48 billion put together, because 7.8×10^5 individuals times 5.7×10^4 per police is roughly equal to $8 \times 10^5 \times 6 \times 10^4$, which is equal to 48×10^9.

4. All enlisted military put together make about twice as much as all officers put together. All enlisted officers together make 1.16×10^6 times 4.38×10^4, which is roughly 5×10^{10}. All officers make 2.5×10^5 times 1×10^5, which is 2.5×10^{10}.

Activity Synthesis

Select students who use slightly different estimations and discuss how their answers are different, but very close compared to the size of the numbers they are working with. The main idea is that students can use a single digit times a power of 10 to estimate numbers and use what they know about exponent arithmetic to compare how many times larger one value is than another.

Lesson Synthesis

The discussion should focus on why standard form is helpful in making multiplicative comparisons of numbers.

- "What did you notice about the numbers in the tables you saw today?" (They were all written in standard form.)
- "A number is expressed in standard form if it is the product of an integer power of 10 and a number greater than or equal to 1 but less than 10. How does the convention about the numeric factor help us quickly get an idea about the size of a number? How does it help us compare numbers?" (Since the factor is always less than 10, the exponent gives most of the information about size of the number. Numbers can be compared by quickly examining the exponents and rounding the numerical factor.)
- "Suppose we had a table where the numbers were not in standard form. Would you be able to take a quick look at the table and have a feel for the relative sizes of the numbers? Describe how the process of answering 'how many times as great' questions would be different for such a table than the work you did with today's problems." (If the numbers were written using different powers of 10, we would need to look at both the first factor and the power of 10 to gauge the size of each number. If the numbers were written as decimals, it would be even more challenging, as it would require counting zeros or decimal places. Multiplying and dividing these numbers—to answer 'how many times as great' questions—would also be much more cumbersome and prone to error.)

14.5 Estimating with Standard form

Cool Down: 5 minutes

This cool-down checks how students make decisions about estimating with standard form.

Student Task Statement

1. Estimate how many times larger 6.1×10^7 is than 2.1×10^{-4}.

2. Estimate how many times larger 1.9×10^{-8} is than 4.2×10^{-13}.

Student Response

1. 6.1×10^7 is about 300 billion times larger than 2.1×10^{-4} because $\frac{6.1\times10^7}{2.1\times10^{-4}} \approx \frac{6\times10^7}{2\times10^{-4}} = 3 \times 10^{7-(-4)} = 3 \times 10^{11}$.

2. 1.9×10^{-8} is about 50 000 times larger than 4.2×10^{-13} because $\frac{1.9\times10^{-8}}{4.2\times10^{-13}} \approx \frac{2\times10^{-8}}{4\times10^{-13}} = 0.5 \times 10^5 = 5 \times 10^4$.

Student Lesson Summary

Multiplying numbers in standard form extends what we do when we multiply regular decimal numbers. For example, one way to find (80)(60) is to view 80 as 8 tens and to view 60 as 6 tens. The product (80)(60) is 48 hundreds or 4 800. Using standard form, we can write this calculation as $(8 \times 10^1)(6 \times 10^1) = 48 \times 10^2$. To express the product in standard form, we would rewrite it as 4.8×10^3.

Calculating using standard form is especially useful when dealing with very large or very small numbers. For example, there are about 39 million or 3.9×10^7 residents in California. Each Californian uses about 180 gallons of water a day. To find how many gallons of water Californians use in a day, we can find the product $(180)(3.9 \times 10^7) = 702 \times 10^7$, which is equal to 7.02×10^9. That's about 7 billion gallons of water each day!

Comparing very large or very small numbers by estimation also becomes easier with standard form. For example, how many ants are there for every human? There are 5×10^{16} ants and 7×10^9 humans. To find the number of ants per human, look at $\frac{5\times10^{16}}{7\times10^9}$. Rewriting the numerator to have the number 50 instead of 5, we get $\frac{50\times10^{15}}{7\times10^9}$. This gives us $\frac{50}{7} \times 10^6$. Since $\frac{50}{7}$ is roughly equal to 7, there are about 7×10^6 or 7 million ants per person!

Lesson 14 Practice Problems

1. **Problem 1 Statement**

   Evaluate each expression. Use standard form to express your answer.

   a. $(1.5 \times 10^2)(5 \times 10^{10})$

b. $\frac{4.8\times10^{-8}}{3\times10^{-3}}$

c. $(5 \times 10^8)(4 \times 10^3)$

d. $(7.2 \times 10^3) \div (1.2 \times 10^5)$

Solution

a. 7.5×10^{12}

b. 1.6×10^{-5}

c. 2×10^{12}

d. 6×10^{-2}

2. **Problem 2 Statement**

How many bucketloads would it take to bucket out the world's oceans? Write your answer in standard form.

Some useful information:

– The world's oceans hold roughly 1.4×10^9 cubic kilometres of water.

– A typical bucket holds roughly 20 000 cubic centimetres of water.

– There are 10^{15} cubic centimetres in a cubic kilometre.

Solution

7×10^{19}. The world's oceans hold 1.4×10^{24} cubic centimetres of water, found by multiplying 1.4×10^9 by 10^{15}. Then divide by 2×10^4 to get 0.7×10^{20}. In standard form, this quotient is 7×10^{19}.

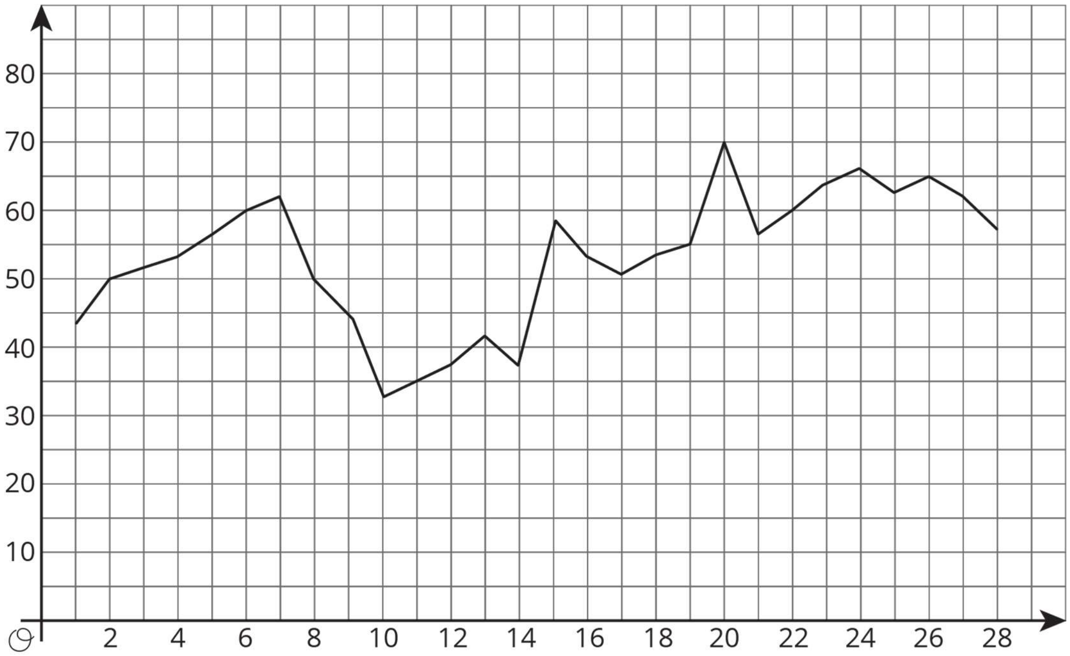
3. **Problem 3 Statement**

The graph represents the closing price per share of stock for a company each day for 28 days.

a. What variable is represented on the horizontal axis?

b. In the first week, was the stock price generally increasing or decreasing?

c. During which period did the closing price of the stock decrease for at least 3 days in a row?



Solution

a. The day

b. Increasing

c. Days 7 to 10

4. **Problem 4 Statement**

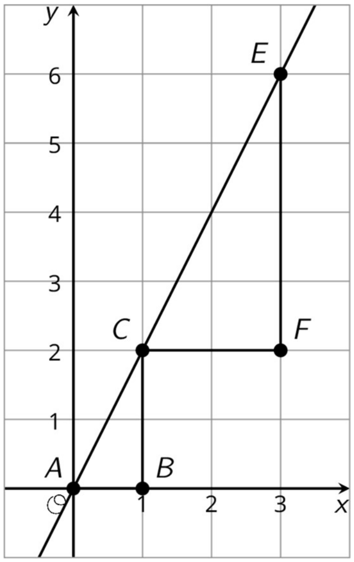
Write an equation for the line that passes through (-8.5,11) and (5,-2.5).

Solution

$y = -x + 2.5$

5. **Problem 5 Statement**

Explain why triangle ABC is similar to triangle CFE.



Solution

Answers vary. Sample responses:

- Translate C to A, and then enlarge with centre A by a factor of $\frac{1}{2}$.
- Enlarge with centre A by a factor of 2, then translate A to C.