Lesson 6: Increasing and decreasing

Goals

- Coordinate statements about “percentage increase” or “percentage decrease” with comparisons to the original amount, e.g., a 20% increase means the new value is 120% of the original value.
- Draw and label a bar model to represent a situation that involves adding or subtracting a percentage of the initial value.
- Explain (orally and in writing) how to calculate the new amount given the original amount and a percentage of increase or decrease.

Learning Targets

- I can draw a bar model that represents a percentage increase or decrease.
- When I know a starting amount and the percentage increase or decrease, I can find the new amount.

Lesson Narrative

This is the first of four lessons about percentage increase and percentage decrease. The goal of this lesson is to understand what is meant by "20% more than" or "10% less than." Students relate this language to the previous two lessons where they talked about "half as much again" and "one third less than." They use bar models to represent percentage increase and percentage decrease, and to solve problems. The contexts in this first lesson are all of the type where you are given the original amount and the percentage increase or decrease and must calculate the final amount.

Students use bar models and their understanding of the language of percentage increase and decrease to reason about different contexts. Students should be able to interpret the meaning of a percentage increase or percentage decrease in the context of a problem.

Building On

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, bar models, double number line diagrams, or equations.

Addressing

- Use proportional relationships to solve multistep ratio and percentage problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Building Towards

- Use proportional relationships to solve multistep ratio and percentage problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time
- Clarify, Critique, Correct
- Discussion Supports
- Notice and Wonder
- Think Pair Share

Student Learning Goals

Let's use percentages to describe increases and decreases.

6.1 Improving Their Game

Warm Up: 10 minutes

The purpose of this warm-up is to encourage students to recognise that in some situations it is helpful to think about a multiplicative increase rather than an additive increase. These situations can be described in terms of **percentage increase** (situations in which an increase is obtained by adding a certain percentage of a quantity to itself). In this situation, total points for each sports team increase from game 1 to game 2, however the increases are different percentages of the first scores. For example, an increase of 8 points from 100 to 108 is not as significant as an increase from 4 points to 12 points, because in the first increase is only 8% of the original value while the second is 200% of the original value.

As students share things they notice, listen for language students use to discuss the significance of the different increases. For example, students may say that the baseball team tripled their score, which would be like the basketball team going from 100 to 300 points in the next game.

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students they will look at a table and think of at least one thing they notice and at least one thing they wonder. Display the table for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have noticed something about the teams' scores. Invite students to share their ideas; record and display their responses for all to see. If no students wonder which team improved the most, direct them to the second question and give them 1 minute to work with a partner.

Anticipated Misconceptions

Students may say that the football team improved the least because the 8 points could have been scored from only 1 touchdown in American football, but it would have to be 3 or 4 baskets in basketball and 8 separate runs in baseball. Prompt students to look at the significance of the 8 additional points in the context of each team's score in the game 1, rather than the mechanics of scoring in each sport.

Student Task Statement

Here are the scores from 3 different sports teams from their last 2 games.

sports team	total points in game 1	total points in game 2
football team	22	30
basketball team	100	108
baseball team	4	12

1. What do you notice about the teams' scores? What do you wonder?

2. Which team improved the most? Explain your reasoning.

Student Response

1. Answers vary. Students may notice: each team improved by 8 points; the basketball team had the most points; and the baseball team's score in game 2 was 3 times as much as in game 1, a three-fold increase.

2. Answers vary. Sample response: The baseball team improved the most because their 8-point increase tripled their previous score, while the other teams' scores were multiplied by a smaller factor: about 1.1 for the basketball team and about 1.4 for the football team.

Activity Synthesis

Poll students on which team they think improved the most. Ask a student who thinks they all improved by the same amount to share their reasoning (each team increased its score by 8 points). Then, ask a few students who said the baseball team improved the most to share their reasoning. There is no need to invoke the phrase "percentage increase" or to express the change as a percentage of the game 1 score at this time, but you want to plant the idea that it sometimes makes sense to describe a change *relative to a starting amount,* instead of just looking at absolute change. In the course of discussion, though, it may be natural to say things like the basketball team improved their score by 8% of their game 1 score, the football team improved by nearly $\frac{1}{3}$ of their game 1 score, while the baseball team tripled their game 1 score.

6.2 More Cereal and a Discounted Shirt

10 minutes

In this activity, students are given a percentage increase and use it to calculate the new value, rather than being given the original and new values to calculate the percentage increase. Students can solve the problems using the double number lines but the discussion that follows will be the connection to what the words specifically mean.

As students work, monitor for students who complete the double number line diagram and any student that uses other methods to correctly reason about the problem.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time followed by partner then whole-class discussion.

Anticipated Misconceptions

Have students use the double number line diagram if they need help figuring out 20% more.

Student Task Statement

1.

A cereal box says that now it contains 20% more. Originally, it came with 18.5 ounces of cereal. How much cereal does the box come with now?

2.

The price of a shirt is £18.50, but you have a coupon that lowers the price by 20%. What is the price of the shirt after using the coupon?

Student Response

1. 22.2 ounces of cereal. Sample Explanations:
   - The cereal box gained 3.7 ounces, because 20% of 18.5 ounces is 0.2×18.5, or 3.7 ounces. That means the cereal box now has 22.2 ounces, because $18.5 + 3.7 = 22.2$.
   - The cereal box now has 120% as many ounces of cereal as it originally had, because $100 + 20 = 120$. Now it has 22.2 ounces of cereal because $18.5 \times 1.2 = 22.2$.
2. £14.80. Sample Explanations:
   - The price drops by £3.70, because 20% of £18.50 is 0.2×18.50, or £3.70. That means the shirt will cost £14.80, because $18.50 - 3.70 = 14.80$.
   - The sale price is 80% of the original price, because $100 - 20 = 80$. The price of the shirt after using the coupon will be £14.80 because $18.50 \times 0.80 = 14.8$.

Activity Synthesis

Select students to share their reasoning for each problem. Start with a student that used the double number line diagram to solve the problem. Then, have students share other methods they used to solve the problem, such as multiplying by 1.2 and 0.8.

Ask students:

- Did the number of ounces of cereal in the cereal box increase or decrease?
- What percentage of the original amount of cereal is the new amount of cereal?
- Did the price of the shirt increase or decrease?
- What percentage of the original price of the shirt is the new price of the shirt?

Tell students that the change on the cereal box is an example of a **percentage increase**. The discount on the shirt is an example of a **percentage decrease**. You might want to mention that in both cases, 100% always corresponds to the original amount before the change; however, future activities will address this concept in depth.

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of percentage increase and percentage decrease.
Supports accessibility for: Conceptual processing; Language Writing: Clarify, Critique, Correct. Before students share their reasoning for each problem, present an incomplete strategy for finding the discount price for the shirt. For example, "The discount is 20%, so the price of the shirt is £3.70." Ask students to critique the reasoning, and work with a partner to write an improved explanation. Listen for and amplify the language students use to make sense of what is displayed (e.g., "I think they ___ because ___."), as well as the mathematical language that students use that strengthen their explanations. This will support student understanding of mathematical language related percentage increase and percentage decrease.
Design Principle(s): Maximise meta-awareness

6.3 Using Bar models

10 minutes

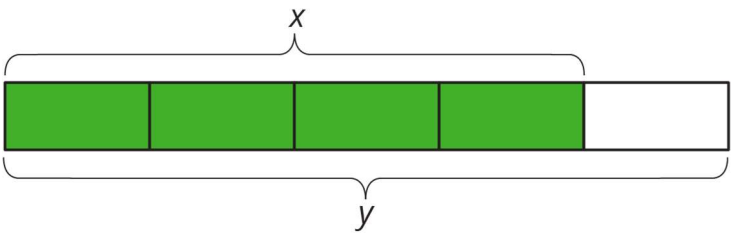
The purpose of this activity is for students to understand that a percentage increase of, say, 15% corresponds to 115% of the original amount, and a percentage decrease of, say, 30% corresponds to 70% of the original amount.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Show students this image.



Say, "Explain how each of these is related to the diagram."

1. $x + \frac{1}{4}x$
2. $y = 1.25x$

3. 125%

4. An increase of 25%

1 minute of quiet think time followed by partner and then whole group discussion.

Representation: Internalise Comprehension. Demonstrate and encourage students to use colour coding and annotations to highlight connections between representations in a problem. For example, annotate the section of diagram B that represents 75% to emphasise that this section represents this year's blueberry harvest.
Supports accessibility for: Visual-spatial processing Speaking: Discussion Supports. As students explain their reason for selecting diagram A or B, press for details in students' explanations by requesting that students challenge an idea, elaborate on an idea, or give an example with values. Provide a sentence frame, such as: "Diagram (A/B) represents the situation because _____." This will help students produce and make sense of the language needed to communicate their own ideas.
Design Principle(s): Support sense-making; Optimise output (for explanation)

Student Task Statement

1. Match each situation to a diagram. Be prepared to explain your reasoning.

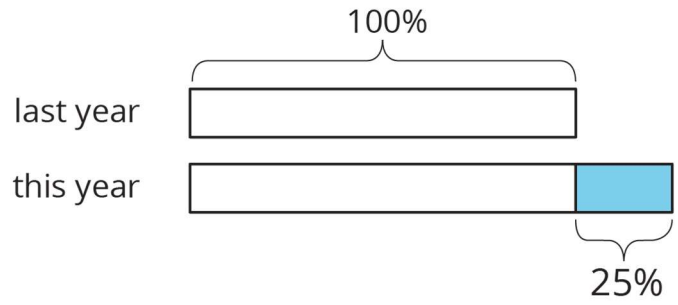
    a. Compared with last year's strawberry harvest, this year's strawberry harvest is a 25% increase.

    b. This year's blueberry harvest is 75% of last year's.

    c. Compared with last year, this year's peach harvest decreased 25%.

    d. This year's plum harvest is 125% of last year's plum harvest.

A



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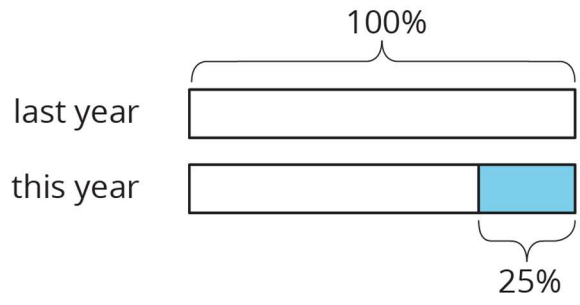


2. Draw a diagram to represent these situations.

   a. The number of ducks living at the pond increased by 40%.

   b. The number of mosquitoes decreased by 80%.

Student Response

1.

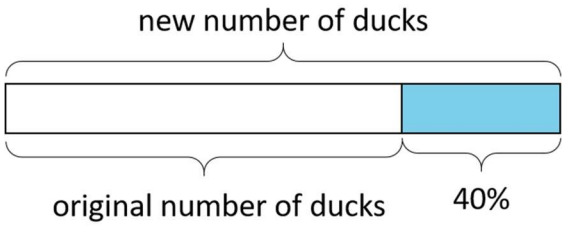
   a. Diagram A. A 25% increase from last year means this year's harvest corresponds to 125% of last year's harvest. The rectangle for this year is 125% of the rectangle for last year.

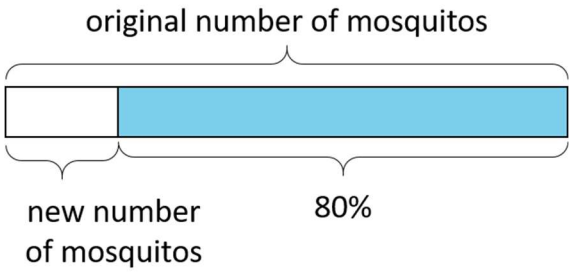
   b. Diagram B. The rectangle for this year is 75% of the rectangle for last year.

   c. Diagram B. A 25% decrease from last year means this year's harvest corresponds to 75% of last year's harvest. The rectangle for this year is 75% of the rectangle for last year.

   d. Diagram A. The rectangle for this year is 125% of the rectangle for last year.

2. Answers vary. Sample diagrams:





Are You Ready for More?

What could it mean to say there is a 100% decrease in a quantity? Give an example of a quantity where this makes sense.

Student Response

It means the quantity is now zero. For example, a person could walk 1 mile one day and 0 miles the next day.

Activity Synthesis

If the amount of fruit increases by 40%, what percentage of the original amount do you have?

If the amount of fruit decreases by 40%, what percentage of the original amount do you have?

6.4 Agree or Disagree: Percentages

Optional: 10 minutes

The purpose of this activity is to for students to evaluate claims about percentages within contexts in which common misunderstandings occur. The first question prompts students to think about the original pay of each employee. Since we do not know the pay for each employee, a higher pay rise percentage does not necessarily mean a higher pound amount. Students could disagree based on the reasoning that employee A makes £10 per hour, so a 50% rise would be an increase of £5 per hour. However, if employee B makes £20 per hour, a 45% rise would be an increase of £9 per hour. On the other hand, students could agree with this statement if they think both employees make the same amount. The second question prompts students to reason about the effect of trying to combine percentages. Ask students to discuss the following with their partner:

- Did you agree or disagree with one another?
- How did you test out your ideas?
- What did you notice to be true after testing out some numbers in the statement?

Instructional Routines

- Stronger and Clearer Each Time
- Clarify, Critique, Correct
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time followed by partner and whole-class discussions. Refer to *Clarify, Critique, Correct* for prompts to build student language for evaluating a statement. One example is to use the "Always-Sometimes-Never" approach for helping students determine the validity of the statements.

Writing, Speaking: Stronger and Clearer Each Time. To help students refine their justifications for whether they agree or disagree with the first statement, give students time to meet with 2-3 partners, sharing their responses. Encourage listeners to press for details and clarity as appropriate based on what each speaker produces. Provide students with prompts for feedback that will help individuals strengthen their ideas and clarify their language (e.g., “Why do you think that?” “How could you use values to show your thinking?”

etc.). Students can borrow ideas and language from each partner to strengthen their final product.
Design Principle(s): Optimise output (for justification)

Anticipated Misconceptions

Students may agree with both statements at first as the statements themselves are common misconceptions. Ask these students to assign values to the pieces of the statement and test them out. For example, assign how much each employee makes in the first statement or how much each shirt costs in the second statement.

Student Task Statement

Do you agree or disagree with each statement? Explain your reasoning.

1. Employee A gets a pay rise of 50%. Employee B gets a pay rise of 45%. So employee A gets the bigger pay rise.
2. Shirts are on sale for 20% off. You buy two of them. As you pay, the cashier says, "20% off of each shirt means 40% off of the total price."

Student Response

1. Agree if both employees make the same amount of money. Disagree if employee A makes enough less than employee B.
2. Disagree. It's still 20% off the total price. If we represent the discount from 20% off shirt A as $0.2a$ and the discount from 20% off shirt B as $0.2b$, then the amount off the total price would be $0.2a + 0.2b$ or by the distributive property, $0.2(a + b)$.

Activity Synthesis

Poll students if they agree or disagree with each statement and ask students to explain their reasoning. Record and display student explanations for all to see. To involve more students in the conversation, consider asking some of the following questions:

- "Do you agree or disagree? Why?"
- "Did anyone think about the statement in the same way but would explain it differently?"
- "Does anyone want to add on to _____'s reasoning?"

Lesson Synthesis

Students should be able to apply their understanding of proportional increases and decreases, from previous lessons, to problems involving percentages.

- "What is another way to describe a 25% percentage increase or decrease?" (when we increase or decrease a quantity by adding or subtracting $\frac{1}{4}$ of the original quantity)

- "When a quantity is increased or decreased, what percentage describes the original or starting value?" (100%)
- "What strategies have we used to help us calculate percentage increase and decrease?" (double number line, table, equation)

6.5 Fish Population

Cool Down:

Anticipated Misconceptions

Some students may answer 15 fish, because they find 25% of 60, but don't realise this is the amount of the decrease, not the final amount.

Some students may answer 75 fish, because they calculate a 25% *increase* instead of a 25% decrease.

Some students may answer 80 fish, because they use 60 as the number *after* the 25% decrease instead of before.

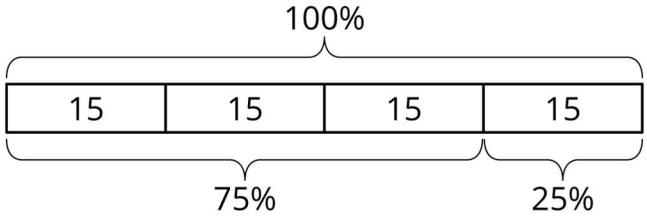
Student Task Statement

The number of fish in a lake decreased by 25% between last year and this year. Last year there were 60 fish in the lake. What is the population this year? If you get stuck, consider drawing a diagram.

Student Response

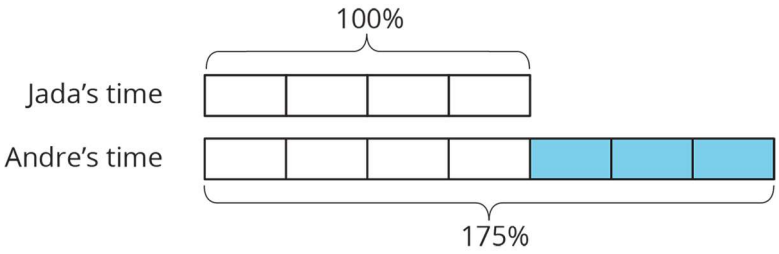
There are 45 fish in the lake this year. Sample explanations:

- The number of fish decreased by 15, because $0.25 \times 60 = 15$. That means there are 45 fish left, because $60 - 15 = 45$.
- There are only 75% as many fish this year, because $100 - 25 = 75$. We can multiply $0.75 \times 60 = 45$.
- Here is a bar model that shows there are 45 fish left:



Student Lesson Summary

Imagine that it takes Andre $\frac{3}{4}$ more than the time it takes Jada to get to school. Then we know that Andre's time is $1\frac{3}{4}$ or 1.75 times Jada's time. We can also describe this in terms of percentages:

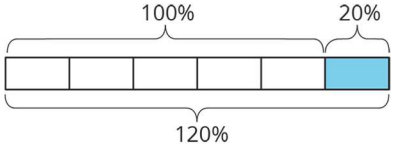


We say that Andre's time is 75% more than Jada's time. We can also see that Andre's time is 175% of Jada's time. In general, the terms **percentage increase** and **percentage decrease** describe an increase or decrease in a quantity as a percentage of the starting amount.

For example, if there were 500 grams of cereal in the original package, then "20% more" means that 20% of 500 grams has been added to the initial amount, $500 + (0.2) \times 500 = 600$, so there are 600 grams of cereal in the new package.

We can see that the new amount is 120% of the initial amount because

$$500 + (0.2) \times 500 = (1 + 0.2)500$$



Glossary

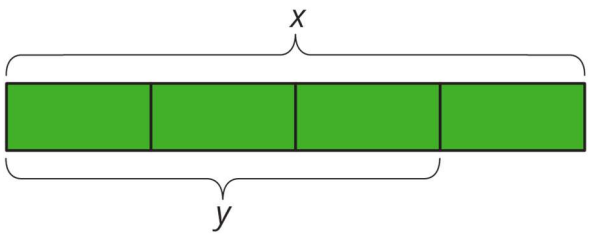
- percentage decrease
- percentage increase

Lesson 6 Practice Problems

Problem 1 Statement

For each diagram, decide if y is an increase or a decrease relative to x. Then determine the percentage increase or decrease.

A



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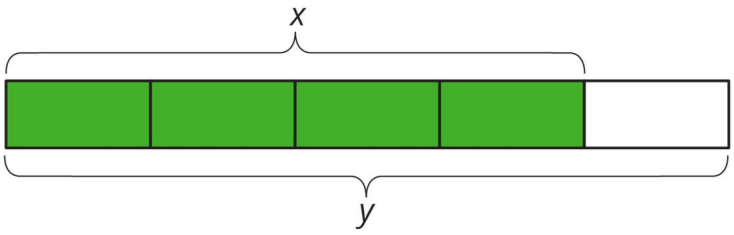


Solution

For A, y is a 25% decrease of x.

For B, y is a 25% increase of x.

Problem 2 Statement

Draw diagrams to represent the following situations.

a. The amount of flour that the bakery used this month was a 50% increase relative to last month.

b. The amount of milk that the bakery used this month was a 75% decrease relative to last month.

Solution

Answers vary.

Problem 3 Statement

Write each percentage increase or decrease as a percentage of the initial amount. The first one is done for you.

a. This year, there was 40% more snow than last year.

The amount of snow this year is 140% of the amount of snow last year.

b. This year, there were 25% fewer sunny days than last year.

c. Compared to last month, there was a 50% increase in the number of houses sold this month.

d. The runner's time to complete the marathon was a 10% less than the time to complete the last marathon.

Solution

a. The amount of snow this year is 140% of the amount of snow last year.

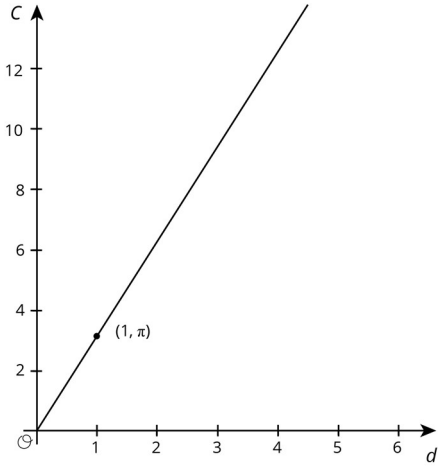
b. The number of sunny days this year is 75% of the number of sunny days last year.

c. The number of houses sold this month is 150% of the number of houses sold last month.

d. The runner's time to complete the marathon was 90% of the time to complete the last marathon.

Problem 4 Statement

The graph shows the relationship between the diameter and the circumference of a circle with the point $(1, \pi)$ shown. Find 3 more points that are on the line.



Solution

Answers vary. Possible answers: $(0,0)$, $(2,2\pi)$, $(3,9.4)$

Problem 5 Statement

Priya bought x grams of flour. Clare bought $\frac{3}{8}$ more than that. Select **all** equations that represent the relationship between the amount of flour that Priya bought, x, and the amount of flour that Clare bought, y.

a. $y = \frac{3}{8}x$

b. $y = \frac{5}{8}x$

c. $y = x + \frac{3}{8}x$

d. $y = x - \frac{3}{8}x$

e. $y = \frac{11}{8}x$

Solution ["C", "E"]