Lesson 2: Keeping the equation balanced

Goals

- Calculate the weight of an unknown object using a balance diagram, and explain (orally) the solution method.
- Comprehend that adding and removing equal items from each side of a balance diagram or multiplying and dividing items on each side of the balance by the same amount are moves that keep the balance balanced.

Learning Targets

- I can add or remove blocks from a balance and keep the balance balanced.
- I can represent balanced balances with equations.

Lesson Narrative

This lesson is the first of a sequence of eight lessons where students learn to work with equations that have variables on each side. In this lesson, students recall a representation that they have seen in prior years: the balanced balance. The balance is balanced because the total weight on each side, hanging at the same distance from the centre, is equal in measure to the total weight on the other side.

In the warm-up, students encounter two real balances, one balanced and one slanted, and notice and wonder about what could cause the balances' appearance. This leads into the first activity where students consider two questions about a balanced balance: first, whether a change of the number of weights keeps the balance in balance, and second, how to find the unknown weight of one of the shapes if the weight of the other shape is known. Students learn that adding or removing the same weight from each side is analogous to writing an equation to represent the balance and adding or subtracting the same amount from each side of the equation. They reason similarly about how halving the weight on each side of the balance is analogous to multiplying by $\frac{1}{2}$ or dividing by 2. In both the balance and the equation, these kinds of moves will produce new balanced balances and equations that ultimately reveal the value of the unknown quantity.

In the second activity, students encounter a balance with an unknown weight that cannot be determined. This situation parallels the situation of an equation where the variable can take on any value and the equation will always be true, which is a topic explored in more depth in later lessons.

Students use concrete quantities to develop their power of abstract reasoning about equations.

Alignments

Addressing

- Analyse and solve linear equations and pairs of simultaneous linear equations.

Building Towards

- Analyse and solve linear equations and pairs of simultaneous linear equations.

Instructional Routines

- Stronger and Clearer Each Time
- Collect and Display
- Discussion Supports
- Notice and Wonder

Student Learning Goals

Let's figure out unknown weights on balanced balances.

2.1 Notice and Wonder: Hanging Socks

Warm Up: 5 minutes

The purpose of this warm-up is to give students an opportunity to ground their understanding of equality in the context of weight, which is a context that will be used throughout the lesson.

Instructional Routines

- Collect and Display
- Notice and Wonder

Launch

Tell students they will look at a picture, and their job is to think of at least one thing they notice and at least one thing they wonder about the picture. Display the problem for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have noticed or wondered about something.

Student Task Statement

What do you notice? What do you wonder?

Student Response

Things students might notice:

- There are four socks / four clips / two hangers.
- One hanger is hanging diagonally and one is straight.
- Half of the socks are blue and half are pink.
- One of the socks looks heavier because it is weighing down that side of its hanger.
- You could fit 20 toes inside of those socks.

Things students might wonder:

- Are the hangers a number line and the socks numbers?
- Is this representing a multiplication problem?
- Would the crooked hanger straighten out if there were two socks on its right side?
- Why is one of the hangers slanted when the socks look identical?
- Did they put something in one blue sock that is making it weigh more than the other sock?

Activity Synthesis

Ask students to share their ideas. Record and display the responses for all to see. In the interest of time, you can ask if anything students wondered was a "why" question, meaning the question begins with the word why. Refer to *Collect and Display*.

If not brought up during the first part of the discussion, ask students why they think the left balance is unbalanced while the right balance is balanced. Students should understand that a balance will only balance if the weight of the unknown objects in both socks is the same. If they are not the same, then the heavier side is lower than the lighter side.

2.2 Hanging Blocks

10 minutes (there is a digital version of this activity)

The purpose of this task is for students to understand and explain why they can add or subtract expressions from each side of an equation and still maintain the equality, even if the value of those expressions are not known. Both problems have shapes with unknown weight on each side to promote students thinking about unknown values in this way before the transition to equations.

While the focus of this activity is on the relationship between both sides of the balance and not equations, some students may start the second problem by writing and solving an equation to find the weight of a square. While students are working, identify those using equations and those not using equations to answer the second problem during the whole-class discussion.

Instructional Routines

- Collect and Display
- Discussion Supports

Launch

Display the problem image for all to see. Tell students that this is a balance problem similar to the one in the warm-up, only instead of the weights hidden inside socks, each block type represents a different weight. Give 5 minutes of quiet work time followed by a whole-class discussion.

If using the digital activity, introduce the balance problem to set the context and connection to the warm-up. Give students individual work time to figure out the weights and use the applet to check their work.

Representation: Internalise Comprehension. Represent the same information through different modalities by using concrete representations. Create a physical model of the balance diagrams using a clothes hanger and weighted objects. Demonstrate how the weights of objects on either side impact whether the balance is balanced or unbalanced. *Supports accessibility for: Conceptual processing; Visual-spatial processing*

Anticipated Misconceptions

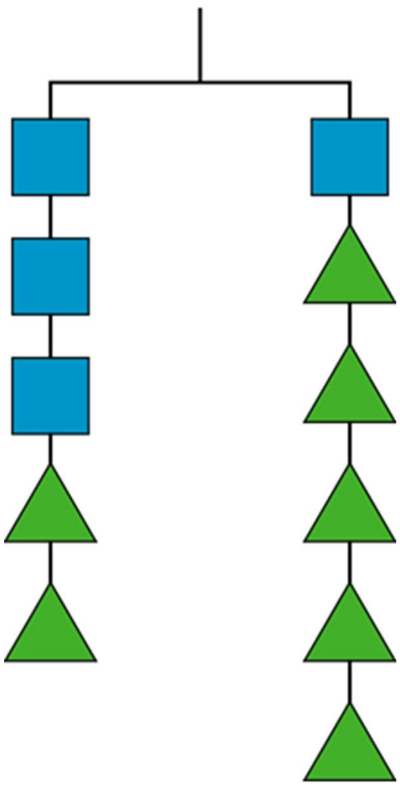
In the first question, students may think the balance will stay in balance since removing the 5 triangles results in three shapes on each side.

Student Task Statement

This picture represents a balance that is balanced because the weight on each side is the same.

1. Elena takes two triangles off of the left side and three triangles off of the right side. Will the balance still be in balance, or will it tip to one side? Which side? Explain how you know.

2. If a triangle weighs 1 gram, how much does a square weigh?



Student Response

1. The balance will tip to the left since only 2 triangles were taken off the left while 3 triangles were taken off the right, which means more weight was taken off the right side making it lighter than the left side.

2. A square weighs $\frac{3}{2}$ grams or equivalent. The balance can be represented by the equation $3x + 2(1) = x + 5(1)$.

Activity Synthesis

Begin the discussion by asking if students think the balance will stay in balance, tip to the left, or tip to the right. Select 2–3 students to explain their vote. Make sure the class understands that removing unequal amounts of weight from the two sides results in the balance tipping before moving on. Use *Collect and Display* to capture student reasoning about it being okay to add or remove terms of the same "size" from both sides of an equation.

For the second question, select previously identified students to explain their answers, with the students who used equations going last. Record and display the specific equations the selected students wrote for all to see, such as $x + x + x + 1 + 1 = x + 1 + 1 + 1 + 1 + 1$ or $3x + 2 = x + 5$, and use it to help the class visualise how that student solved for the weight of a square.

The outcome of this discussion should be that it is okay to add or remove terms of the same "size" from both sides of an equation, and the sides will still be equal. This can be thought of in terms of shapes hanging on balances, where you can remove one square from both sides or add two triangles to both sides, and the balance will stay in balance. Equations are a more abstract representation of this, but the same concept holds: you can remove one x from both sides or add two 3s to both sides and the equation is still true with the left side equal to the right side. Removing equal weights from both sides can leave the balance with 2 squares on the left and 3 triangles (or just 3) on the right. In equation form, this is the same as $2x = 3$. Finally, you can halve the amount of weight on both sides of the balance and keep it in balance, which is the same as multiplying $2x = 3$ by $\frac{1}{2}$ (or dividing both sides by 2).

Speaking: Discussion Supports. During the discussion, use this routine to amplify mathematical uses of language to explain how to balance the balance. To begin the discussion, ask students, "How do you know that the balance is balanced? Explain how this relates to solving an equation." Emphasise words and phrases such as: "each side of the equation," "balance," "same size," and/or "equal weights." Invite students to use these sentence frames in their response: "After the triangles are removed…." and "I can keep the balance/equation balanced by…." This will help students reason and explain that the balancing of an equation removes equal amounts from each side of the equation.
Design Principle(s): Support sense-making

2.3 More Hanging Blocks

15 minutes (there is a digital version of this activity)

Building on the previous activity, students now solve two more balance problems and write equations to represent each balance. In the first problem, the solution is not an integer, which will challenge any student who has been using guess-and-check in the previous activities to look for a more efficient method. In the second problem, the solution is any weight, which is a preview of future lessons when students purposefully study equations with one solution, no solution, and infinite solutions. The goal of this activity is for students to transition their reasoning about solving balances by maintaining the equality of each side to solving equations using the same logic. In future lessons, students will continue to develop this skill as equations grow more complex culminating in solving systems of equations at the end of this unit.

As students work, identify those using strategies to find the weight of one square/pentagon that do not involve an equation. For example, some students may cross out pairs of shapes that are on each side (such as one circle and one square from each side of Balance A) to reason about a simpler problem while others may replace triangles with 3s and circles with 6s first before focusing on the value of 1 square. This type of reasoning should be encouraged and built upon using the language of equations.

Instructional Routines

- Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Give 5 minutes of quiet work time followed by partner discussion. Let students know that they should be prepared to share during the whole-class discussion, so they should make sure their partner understands and agrees with their solution.

If students use the digital activity, the applet provides a way for students to check solutions. Encourage students to work individually (most likely they will need paper/pencil to work these problems) and then check their thinking using the digital applet. After students have had 5 minutes to work alone and with the applet, give them time to discuss their thinking with a partner before the whole-class discussion.

Representation: Internalise Comprehension. Differentiate the degree of difficulty or complexity by beginning with more accessible values. Provide students with a simplified balance, with fewer shapes, to solve first. Encourage students to begin by labelling values they know.
Supports accessibility for: Conceptual processing

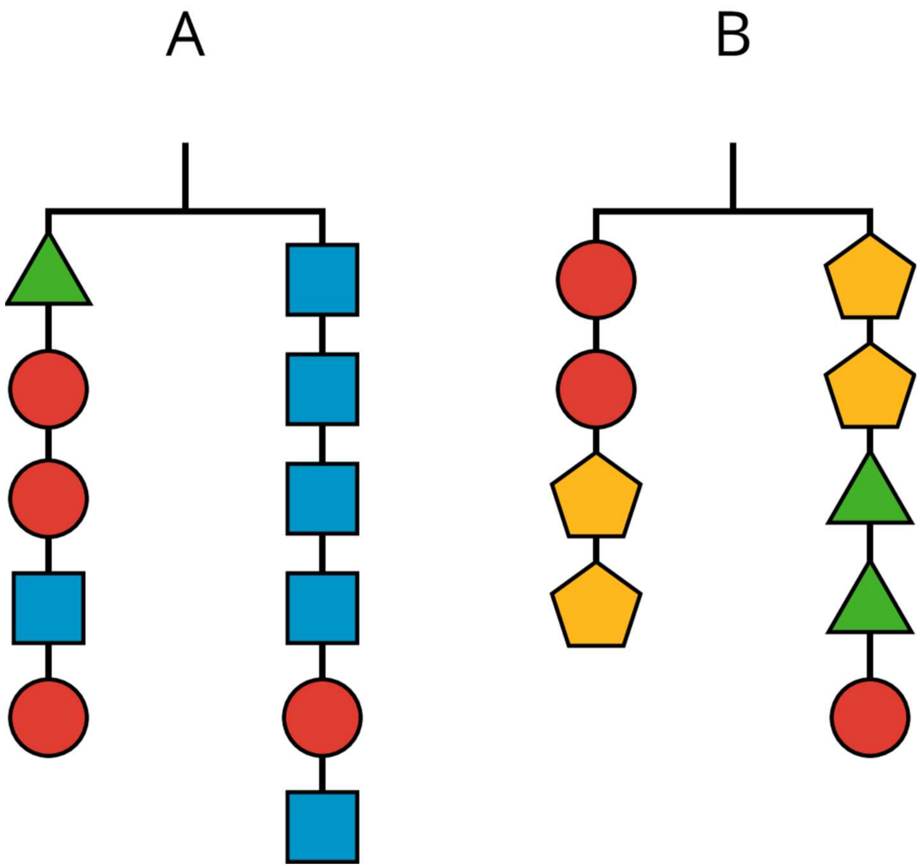
Anticipated Misconceptions

Triangles weigh 3 grams in this activity instead of 1 gram as in the previous activity.

Student Task Statement

A triangle weighs 3 grams and a circle weighs 6 grams.

1. Find the weight of a square in Balance A and the weight of a pentagon in Balance B.
2. Write an equation to represent each balance.

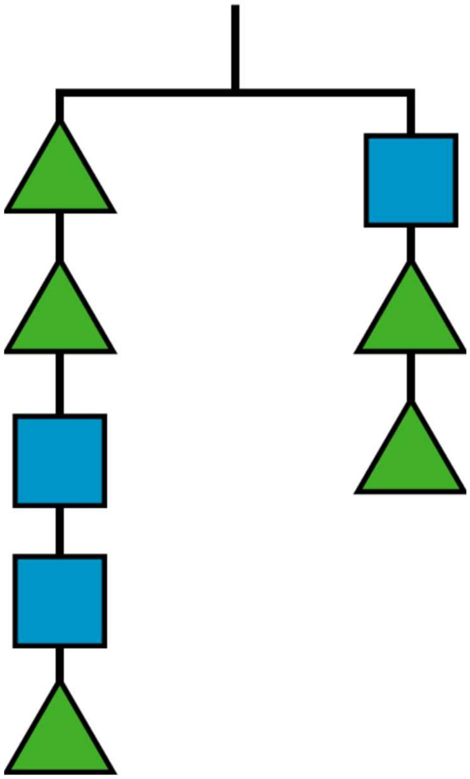


Student Response

1. In Diagram A, each square weighs $\frac{15}{4}$ grams or 3.75 grams or equivalent. In Diagram B, the pentagon's weight cannot be determined. It could be any possible weight.
2. Answers vary. Sample responses: $3 + 18 + x = 5x + 6$ and $12 + 2x = 2x + 3 + 3 + 6$

Are You Ready for More?

What is the weight of a square on this balance if a triangle weighs 3 grams?



Student Response

This balance is not possible since the squares would have to weigh -3 grams for the balance to balance. If the square's weight were a positive value, then the left side would have to be hanging lower than the right side.

Activity Synthesis

Select previously identified students to share their strategies for finding the unknown weight without using an equation. Ask students to be clear how they are changing each side of the balance equally as they share their solutions.

Next, record the equations written by students for each balance and display for all to see in two lists. Assign half the class to the list for Balance A and the other half to the list for Balance B. Give students 1–2 minutes to examine the equations for their assigned balance and be prepared to explain how different pairs of equations are related. The goal here is for student to use the language they developed with the balances (e.g., "remove 6 from each side") on equations.

For example, for Balance A, you might contrast $3 + 6 + 6 + 6 = 4x + 6$ with $21 + x = 6 + 5x$. Possible student responses:

- Removing an x from each side of the second equation would result in the first equation.
- $x = 3.75$ grams makes both equations true.

- You can subtract 6s from the sides of each equation and they are still both true.

For Balance B, examining equations should illuminate why it is impossible to know the weight of the unknown shape. If we start with $6 + 6 + x + x = x + x + 3 + 3 + 6$ and keep removing things of equal weight from each side, we might end up with an equation like $2x = 2x$. Any value of x will work to make this equation true. For example if x is 10, then the equation is $20 = 20$. It is also possible to keep removing things of equal weight from each side and end up with an equation like $6 = 6$, which is always true.

Writing, Conversing: Stronger and Clearer Each Time. Use this routine to give students and opportunity to describe how they wrote equations to represent the Balances. Give students 4 minutes of quiet time to write a response to: "Explain how you created your equation for Balance A or Balance B." Invite to meet with 2–3 partners, to share and get feedback on their responses. Encourage each listener to ask clarifying questions such as, "Why did you subtract __ from both sides?" or "How did you represent the red circles in your equation?" Invite students to write a final draft based on their peer feedback. This will help students reason about solving equations with balancing variables, and prepare them for the whole-class discussion.
Design Principle(s): Optimise output; Cultivate conversation

Lesson Synthesis

The purpose of this discussion is to have students revisit the warm-up and connect it to the activities, reflecting on why the balance is an appropriate and helpful analogy for an equation.

Ask these questions:

- "In the warm-up we wondered why one balance was slanted, whether there were weights in one blue sock that made it heavier than the other, whether the crooked balance would straighten out if another sock was added to the other side (add any other pertinent things your students wondered). How would you answer these questions now?"
- "What is an equation? What does the equal sign in an equation tell you?" (An equation is a statement that two expressions have the same value. The equal sign tells you that the expressions on either side must have the same value, however that value is measured—as a count of objects, a measurement like 10 miles or 6 seconds, or numbers without units.)
- "What features do balanced balances and equations have in common?" (Both representations have sides that are equal in value, even if the actual value of a side is unknown. Each side can contain numbers we do not know in the form of either shapes or variables. Changing the value of one side of a balance or equations means changing the value of the other side by the same amount.)
- "You saw an example of a balance where the unknown weight could not be determined. Can you design your own balance like this one? How would you think

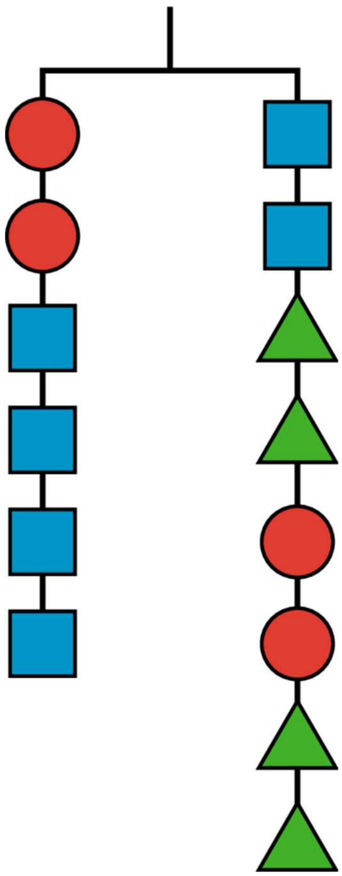
about the weights needed on each side?" (If students completed the extension, you might ask them to also design a balance with no solution.)

2.4 Changing Blocks

Cool Down: 5 minutes

Student Task Statement

Here is a balance that is in balance. We don't know how much any of its shapes weigh. How could you change the number of shapes on it, but keep it in balance? Describe in words or draw a new diagram.

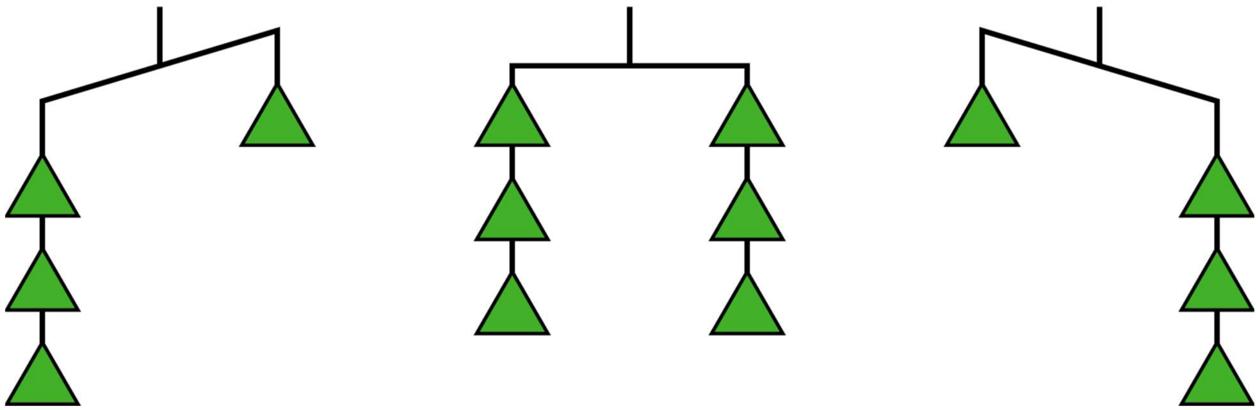


Student Response

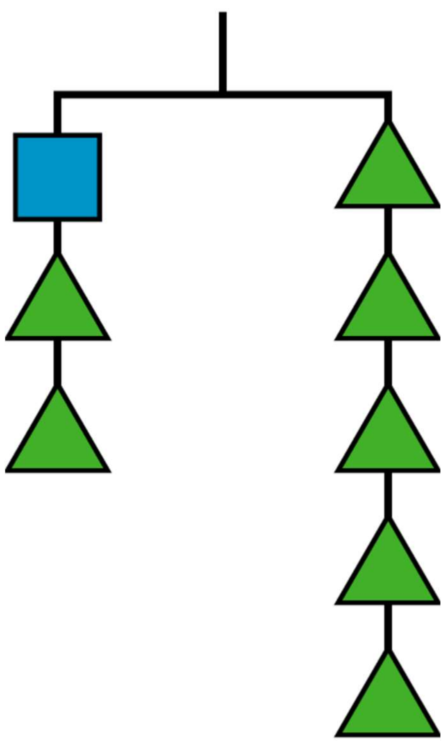
Answers vary. Possible solution: I could remove 2 circles, 2 squares, or all 4 of these shapes from each side of the equation and the balance would still balance. I could also add any number of a specific shape to the left side so long as I added the same amount to the right side and the balance would stay in balance. I could also remove half each type of shape from each side, since there is an even number of each type of shape.

Student Lesson Summary

If we have equal weights on the ends of a balance, then the balance will be in balance. If there is more weight on one side than the other, the balance will tilt to the heavier side.

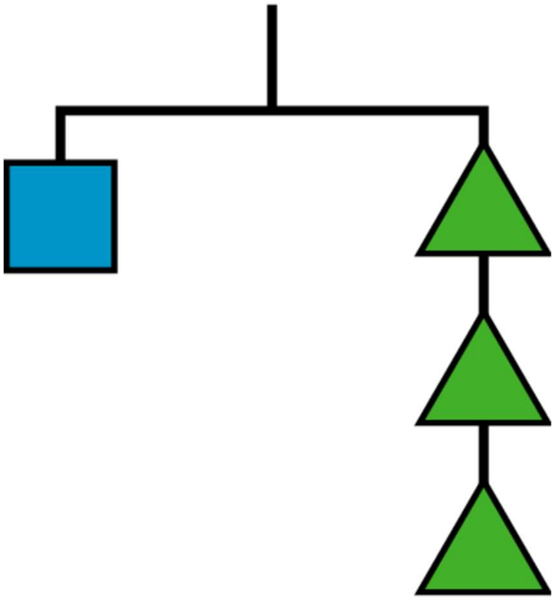


We can think of a balanced balance as a metaphor for an equation. An equation says that the expressions on each side have equal value, just like a balanced balance has equal weights on each side.



$$a + 2b = 5b$$

If we have a balanced balance and add or remove the same amount of weight from each side, the result will still be in balance.



$$a = 3b$$

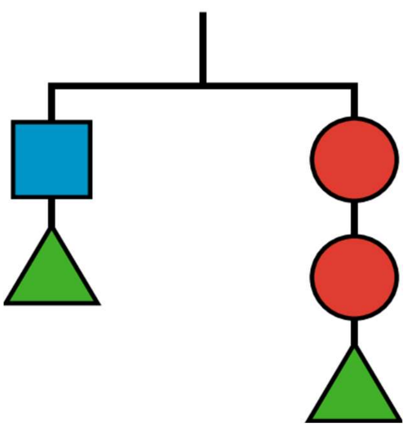
We can do these moves with equations as well: adding or subtracting the same amount from each side of an equation maintains the equality.

Lesson 2 Practice Problems

1. **Problem 1 Statement**

   Which of the changes would keep the balance in balance?

   Select all that apply.



   a. Adding two circles on the left and a square on the right

   b. Adding 2 triangles to each side

c. Adding two circles on the right and a square on the left

d. Adding a circle on the left and a square on the right

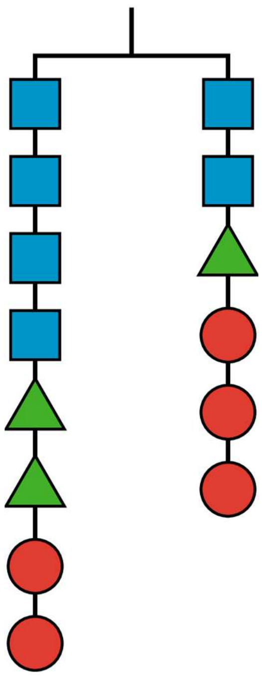
e. Adding a triangle on the left and a square on the right

Solution ["A", "B", "C"]

2. **Problem 2 Statement**

Here is a balanced balance diagram.

Each triangle weighs 2.5 pounds, each circle weighs 3 pounds, and x represents the weight of each square. Select *all* equations that represent the balance.



a. $x + x + x + x + 11 = x + 11.5$

b. $2x = 0.5$

c. $4x + 5 + 6 = 2x + 2.5 + 6$

d. $2x + 2.5 = 3$

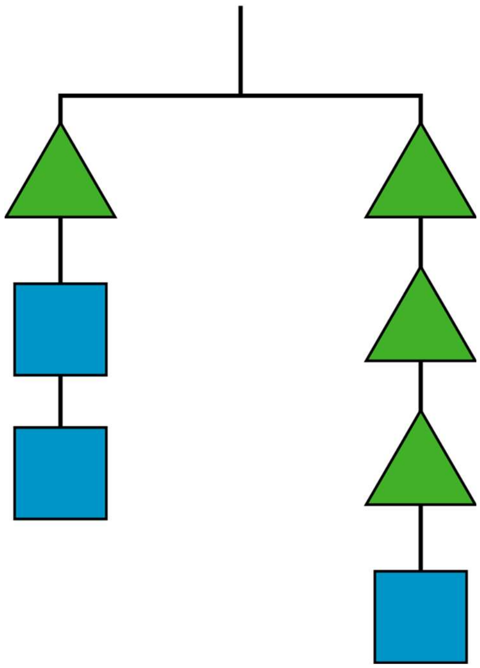
e. $4x + 2.5 + 2.5 + 3 + 3 = 2x + 2.5 + 3 + 3 + 3$

Solution ["B", "D", "E"]

3. **Problem 3 Statement**

What is the weight of a square if a triangle weighs 4 grams?

Explain your reasoning.



Solution

8 grams. There is one more square on the left than on the right and two more triangles on the right than on the left. So the square on the left balances with two triangles on the right.

4. **Problem 4 Statement**

Andre came up with the following puzzle. “I am three years younger than my brother, and I am 2 years older than my sister. My mum's age is one less than three times my brother's age. When you add all our ages, you get 87. What are our ages?”

a. Try to solve the puzzle.

b. Jada writes this equation for the sum of the ages: $(x) + (x + 3) + (x - 2) + 3(x + 3) - 1 = 87$.

   Explain the meaning of the variable and each term of the equation.

c. Write the equation with fewer terms.

d. Solve the puzzle if you haven’t already.

Solution

a. Answers vary.

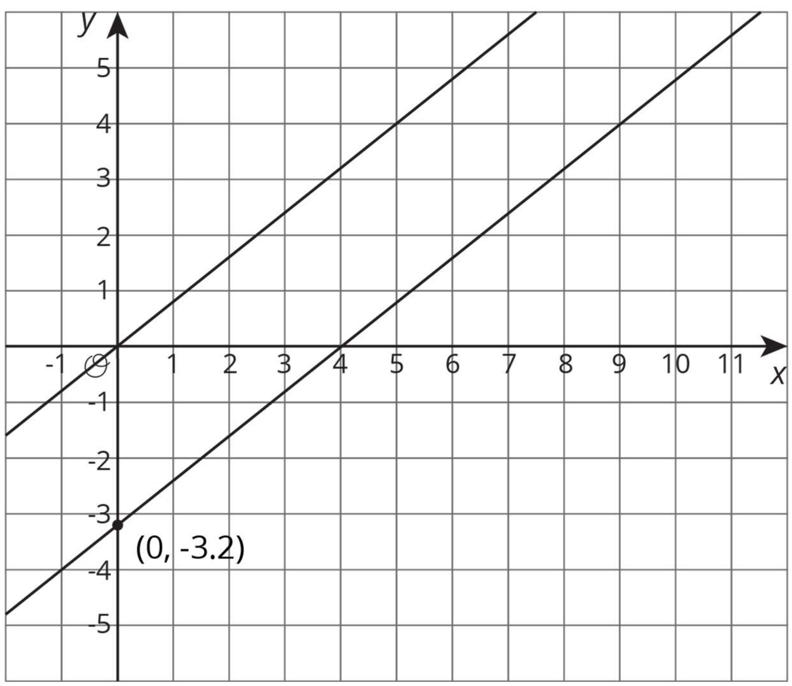
b. x is the age of Andre; $x + 3$ is the age of Andre's brother; $x - 2$ is the age of Andre's sister; $3(x + 3) - 1$ is the age of Andre's mother; 87 is the total of all the ages.

c. Use the distributive property and combine like terms to get $6x + 9 = 87$.

d. Since $6x + 9 = 87$, we also know that $6x = 78$ and $x = 13$ are true. So, Andre is 13, his brother is 16, his sister is 11, and his mum is 47.

5. **Problem 5 Statement**

These two lines are parallel. Write an equation for each.



Solution

Answers vary. Possible responses:

- $y = \frac{4}{5}x$ or $\frac{y}{x} = \frac{4}{5}$, or $\frac{y-4}{x-5} = \frac{4}{5}$
- $y = \frac{4}{5}(x - 4)$ or $\frac{y}{x-4} = \frac{4}{5}$