Lesson 10: Changing scales in scale drawings

Goals

- Determine how much actual area is represented by one square unit in a scale drawing.
- Generalise (orally) that as the actual distance represented by one unit on the drawing increases, the size of the scale drawing decreases.
- Reproduce a scale drawing at a different scale and explain (orally) the solution method.

Learning Targets

- Given a scale drawing, I can create another scale drawing that shows the same thing at a different scale.
- I can use a scale drawing to find actual areas.

Lesson Narrative

In the previous lesson, students created multiple scale drawings using different scales. In this lesson, students are given a scale drawing and asked to recreate it at a different scale. Two possible strategies to produce these drawings are:

- Calculating the actual lengths and then using the new scale to find lengths on the new scale drawing.
- Relating the two scales and calculating the lengths for the new scale drawing using corresponding lengths on the given drawing.

In addition, students previously saw that the area of a scaled copy can be found by multiplying the area of the original figure by $(\text{scale factor})^2$. In this lesson, they extend this work in two ways:

- They compare areas of scale drawings of the same object with different scales.
- They examine how much area, on the actual object, is represented by 1 square centimetre on the scale drawing. For example, if the scale is 1 cm to 50 m, then 1 cm^2 represents 50×50, or 2 500 m^2.

Throughout this lesson, students observe and explain structure, both when they reproduce a scale drawing at a different scale and when they study how the area of a scale drawing depends on the scale.

Building On

- Measure and estimate lengths in standard units.
- Find the area of right-angled triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other

shapes; apply these techniques in the context of solving real-world and mathematical problems.

Addressing

- Solve problems involving scale drawings of geometric shapes, including calculating actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- Analyse proportional relationships and use them to solve real-world and mathematical problems.
- Use proportional relationships to solve multistep ratio and percentage problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Collect and Display
- Discussion Supports

Required Materials

Geometry toolkits
tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Pre-printed slips, cut from copies of the blackline master

Same Plot, Different Drawings 1 cm to 5 m	Same Plot, Different Drawings 1 cm to 10 m	Same Plot, Different Drawings 1 cm to 15 m
Same Plot, Different Drawings 1 cm to 20 m	Same Plot, Different Drawings 1 cm to 30 m	Same Plot, Different Drawings 1 cm to 50 m

Required Preparation

Print and cut the scales for the Same Plot, Different Drawings activity from the blackline master (1 set of scales per group of 5–6 students).

Ensure students have access to their geometry toolkits, especially centimetre rulers.

Student Learning Goals

Let’s explore different scale drawings of the same actual thing.

10.1 Appropriate Measurements

Warm Up: 5 minutes

This warm-up prompts students to attend to precision in measurements, which will be important in upcoming work.

Launch

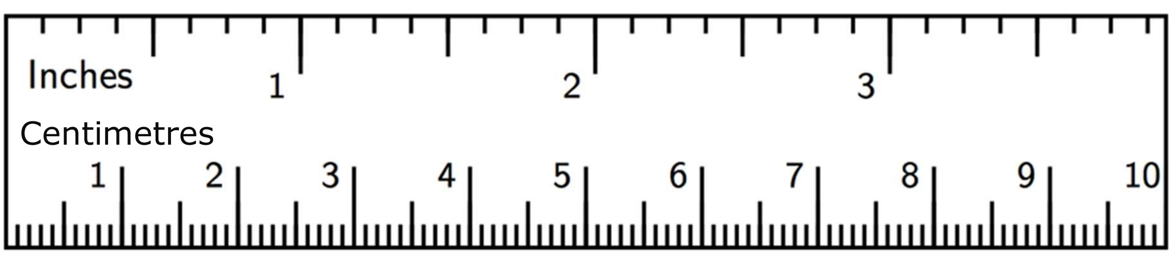
Arrange students in groups of 2. Give students 1 minute of quiet think time to estimate the size of their own foot in centimetres or inches, and a moment to share their estimate with a partner. Then, ask them to complete the task.

Anticipated Misconceptions

Some students may say the large foot is about $3\frac{1}{2}$ inches or about 9 centimetres long, because they assume the ruler shown in the first question is at the same scale as the feet shown in the second question. Explain that the images are drawn at different scales.

Student Task Statement

1. If a student uses a ruler like this to measure the length of their foot, which choices would be appropriate measurements? Select **all** that apply. Be prepared to explain your reasoning.



   a. $9\frac{1}{4}$ inches

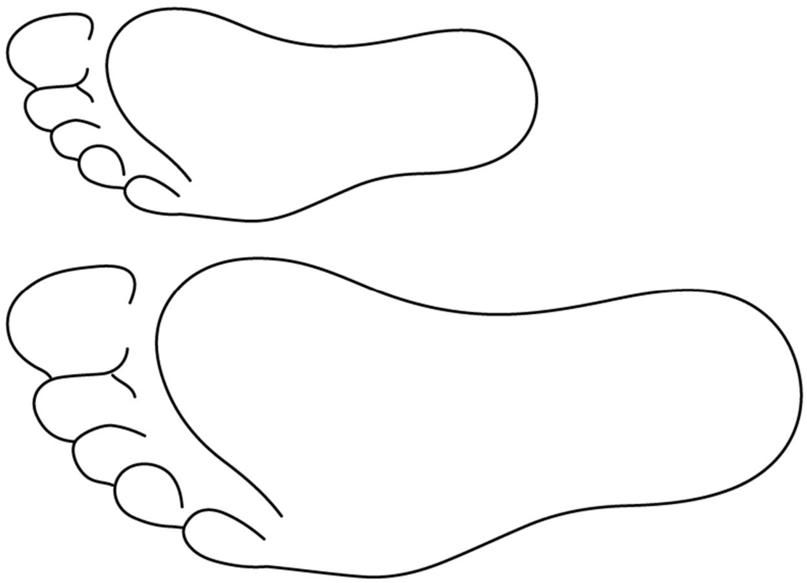
   b. $9\frac{5}{64}$ inches

   c. 23.47659 centimetres

d. 23.5 centimetres

e. 23.48 centimetres

2. Here is a scale drawing of an average seventh-grade student's foot next to a scale drawing of a foot belonging to the person with the largest feet in the world. Estimate the length of the larger foot.



Student Response

1. A and D would be the only appropriate measurements based on the markings on the given ruler. Since the ruler is only marked in $\frac{1}{8}$ inches and $\frac{1}{10}$ centimetre, we could not get measurements as precise as B, C, or E.

2. The largest foot in the world is about 1.5 times as long as the average year 8's foot. My foot is about 10 inches long, so the largest foot is about 15 inches or 1 foot and 3 inches long.

Activity Synthesis

Select a few students to share the measurements they think would be appropriate based on the given ruler. Consider displaying the picture of the ruler for all to see and recording students' responses on it. After each response, poll the class on whether they agree or disagree.

If students consider B, C, or E to be an appropriate measurement, ask them to share how to get such a level of precision on the ruler. Make sure students understand that reporting measurements to the nearest $\frac{1}{64}$ of an inch or to the hundred-thousandths of a centimetre would not be appropriate (i.e., show that the ruler does not allow for these levels of precision).

Choice E of 23.48 cm may merit specific attention. With the ruler, it is possible to *guess* that the hundredths place is an 8. This may even be correct. The problem with reporting the measurement in this way is that someone who sees this might misinterpret it and imagine that an extremely accurate measuring device was used to measure the foot, rather than this ruler. The way a measurement is reported reflects how the measurement was taken.

Next, invite students to share their estimates for the length of the large foot. Since it is difficult to measure the length of these feet very precisely, these measurements should not be reported with a high level of precision; the nearest centimetre would be appropriate.

10.2 Same Plot, Different Drawings

15 minutes

This activity serves several purposes: to allow students to practice creating scale drawings at given scales, to draw attention to the size of the scale drawing as one of the values in the scale changes, and to explore more fully the relationship between scaled area and actual area.

Each group member uses a different scale to calculate scaled lengths of the same plot of land, draw a scale drawing, and calculate its scaled area. The group then orders the different drawings and analyses them. They think about how many square metres of actual area are represented by one square centimetre on each drawing. Students are likely to determine this value in two ways:

- By visualising what a 1×1 centimetre square represents at a given scale (e.g., at a scale of 1 cm to 5 m, each 1 cm^2 represents 5×5, or 25 m^2).
- By dividing the actual area represented by the scale drawing by the area of their scale drawing

The relationships between scale, lengths in scale drawings, and area in scale drawings are all important examples of the mathematical structure of scale drawings.

You will need the Same Plot, Different Drawings blackline master for this activity.

Instructional Routines

- Collect and Display

Launch

Display this map of a neighbourhood in Philadelphia for all to see. Tell students that they are going to reproduce a map of the triangular piece of land at a different scale.

Tell students that the actual base of the triangle is 120 m and its actual height is 90 m. Ask, “What is the area of the plot of land?” (5 400 square metres—one half the base times the height of the triangle.)

Arrange students in groups of 5–6 and provide access to centimetre graph paper. Assign each student in a group a different scale (from the blackline master) to use to create a scale drawing. Give students 3–4 minutes of quiet work time to answer the first 3 questions, and then 1–2 minutes to work on the last question in their groups.

Remind students to include the units in their measurements.

Representation: Internalise Comprehension. Activate or supply background knowledge about finding area of a triangle. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Here is a map showing a plot of land in the shape of a right-angled triangle.

1. Your teacher will assign you a scale to use. On centimetre graph paper, make a scale drawing of the plot of land. Make sure to write your scale on your drawing.

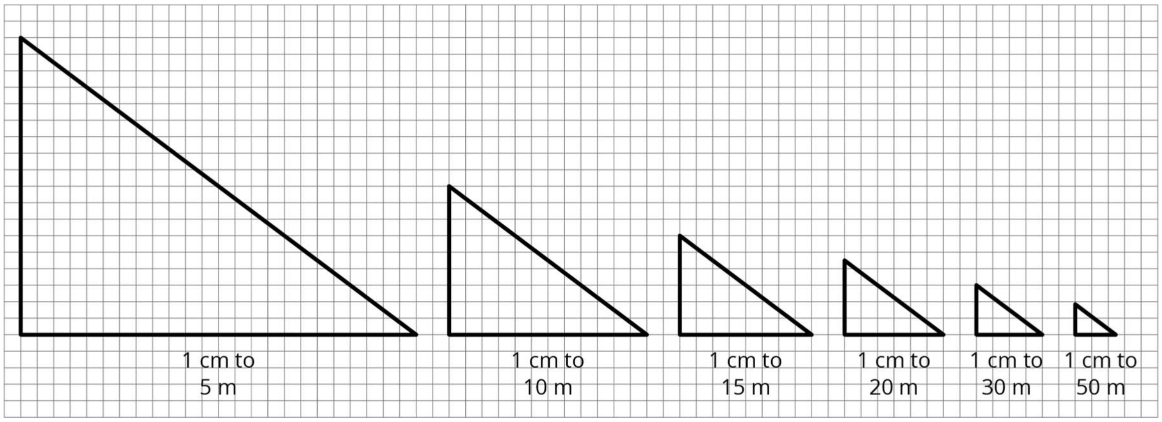
2. What is the area of the triangle you drew? Explain or show your reasoning.

3. How many square metres are represented by 1 square centimetre in your drawing?

4. After everyone in your group is finished, order the scale drawings from largest to smallest. What do you notice about the scales when your drawings are placed in this order?

Student Response

1. Right-angled triangles of various sizes:



2. Answers vary depending on the assigned scale. Possible solutions:

   - 216 cm^2, because $\frac{1}{2} \times 24 \times 18 = 216$.
   - 54 cm^2, because $\frac{1}{2} \times 12 \times 9 = 54$.
   - 24 cm^2, because $\frac{1}{2} \times 8 \times 6 = 24$.
   - 13.5 cm^2, because $\frac{1}{2} \times 6 \times (4.5) = 13.5$.
   - 6 cm^2, because $\frac{1}{2} \times 4 \times 3 = 6$.
   - 2.16 cm^2, because $\frac{1}{2} \times (2.4) \times (1.8) = 2.16$.

3. Answers vary depending on the assigned scale. Possible solutions:

   - 25 m^2, because $5400 \div 216 = 25$.
   - 100 m^2, because $5400 \div 54 = 100$.
   - 225 m^2, because $5400 \div 24 = 225$.
   - 400 m^2, because $5400 \div 13.5 = 400$.
   - 900 m^2, because $5400 \div 6 = 900$.
   - $2\,500 \text{ m}^2$, because $5400 \div 2.16 = 2\,500$.

4. The smaller the number of metres represented by one centimetre, the larger the scale drawing is.

Are You Ready for More?

Noah and Elena each make a scale drawing of the same triangular plot of land, using the following scales. Make a prediction about the size of each drawing. How would they compare to the scale drawings made by your group?

1. Noah uses the scale 1 cm to 200 m.

2. Elena uses the scale 2 cm to 25 m.

Student Response

1. Noah's drawing will be smaller than all the other drawings. The scale that created the smallest drawing so far was 1 cm to 50 m. Each length in a drawing done at 1 cm to 200 m will be 4 times as small as in the 1 cm-to-50 m drawing because every centimetre represents 4 times as much length.

2. The scale 2 cm to 25 m is equivalent to 1 cm to 12.5 m, so Elena's drawing will be larger than the 1 cm to 15 m drawing but smaller than the 1 cm to 10 m drawing.

Activity Synthesis

Focus the discussion on patterns or features students noticed in the different scale drawings. Ask questions such as:

- How does a change in the scale influence the size of the drawings?
  (As the length being represented by 1 cm gets larger, the size of the drawing decreases.)

- How do the lengths of the scale drawing where 1 cm represents 5 metres compare to the lengths of the drawing where 1 cm represents 15 metres? (They are three times as long.)

- How do the lengths of the scale drawing where 1 cm represents 5 metres compare to the lengths of the drawing where 1 cm represents 50 metres? (They are ten times as long.)

- How does the area of the scale drawing where 1 cm represents 5 metres compare to the area of the drawing where 1 cm represents 15 metres? (It is 9 times as great.)

- How does the area of the scale drawing where 1 cm represents 5 metres compare to the area of the drawing where 1 cm represents 50 metres? (It is 100 times as great.)

Help students to observe and formulate these patterns:

- As the number of metres represented by one centimetre increases, the lengths in the scale drawing decrease.

- As the number of metres represented by one centimetre increases, the area of the scale drawing also decreases, but it decreases by the square of the factor for the lengths (because finding the area means multiplying the length and width, both of which decrease by the same factor).

Conversing, Reading: Collect and Display. In this routine, the teacher circulates and listens to student talk while jotting down words, phrases, drawings, or writing students use. The language collected is displayed visually for the whole class to use throughout the lesson and unit. Generally, the display contains different examples of students using features of the disciplinary language functions, such as interpreting, justifying, or comparing. The purpose of this routine is to capture a variety of students' words and phrases in a display that students can refer to, build on, or make connections with during future discussions, and to increase students' awareness of language used in mathematics conversations.
Design Principle(s): Support sense-making; Maximise meta-awareness

How It Happens:

1. As students share their ideas in their groups about what they notice in the scale drawings, write down the language they use to describe how the scale affects the size of the scale drawing. Listen for the language students use to compare the lengths and areas of scale drawings with different scales.

   To support the discussion, provide these sentence frames: "As the value of the scale increases, the size of the drawing ___ because ___.", "When I compare the lengths of (choose two scale drawings), I notice that ___.", and "When I compare the areas of (choose two scale drawings), I notice that ___."

2. As groups close their conversation, display the language collected for all to reference.

3. Next, facilitate a whole-class discussion encouraging students to ask and respond to clarifying questions about the meaning of a word or phrase on the display.

   To prompt discussion, ask students, "What word or phrase is unclear? From the language I collected, what part does not make sense to you?"

   Here is an example:

   Student A: The phrase 'the triangle is 9 times greater' is unclear to me. I'm not sure what 9 times greater means.

   Teacher: [Point to the phrase on the display] Can someone clarify this phrase with specific details?

   Student B: The area of the triangle with a 1 cm to 5 m scale is 9 times greater than the area of the triangle with a 1 cm to 15 m scale.

   Teacher: (pressing for more detail) Why is it 9 times greater? Can someone different explain or illustrate what 9 times greater means in this case?

   Student C: [Student adds a sketch of both triangles to the display next to the phrase; teacher adds labels/arrows/calculations to the sketch while student C explains] It's 9 times greater because 15 m divided by 5 m is 3, and since we're talking about area, you then calculate 3 times 3 to give you 9.

4. As time permits, continue this discussion until all questions have been addressed. If it did not arise through student-led questions and responses, help students generalise that as the number of metres represented by one centimetre increases, the lengths and areas of the scale drawing decreases.

   If students are having difficulty, consider using multiple examples from the activity to build up to the generalisation. You may say, "Let's look at the case where we are comparing…." or "Can someone demonstrate the steps they took to compare…?"

5. Close this conversation by posting the display in the front of the classroom for students to reference for the remainder of the lesson, and then have students move on to the next activity.

10.3 A New Drawing of the Playground

15 minutes

Earlier, students created scale drawings given the actual dimensions and different scales. In this activity, instead of being given the actual dimensions, they are given a scale drawing to reproduce at a different scale.

There are two different types of reasoning students may apply. Monitor for students who:

- Use the scale drawing to find the dimensions of the actual school playground and then use those measurements to find the dimensions of the new scale drawing.
- Notice that in the given drawing, 1 centimetre represents 30 m, and in the new drawing, 1 centimetre represents 20 m. That means that each centimetre in the new drawing represents $\frac{2}{3}$ centimetres in the given one. So in the new drawing, the length of each side needs to be multiplied by a factor of $\frac{3}{2}$.

Select students using each strategy to share during the discussion, sequenced in this order.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports

Launch

Tell students that they are going to reproduce a scale drawing using a different scale. The scale for the given drawing is 1 cm to 30 metres, and they are going to make a new scale drawing at a scale of 1 cm to 20 metres. Ask them if they think the new drawing will be larger or smaller than the given one.

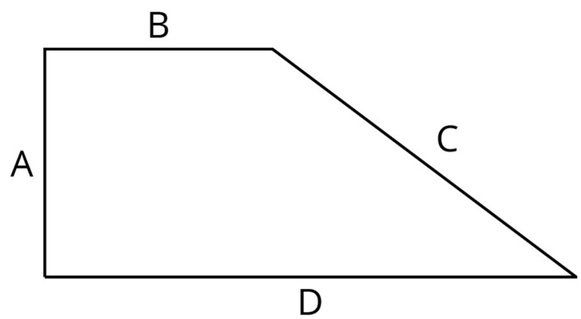
Arrange students in groups of 3. Make sure students have access to their geometry toolkits. Give students 5 minutes of quiet work time, followed by 3 minutes of group discussion.

Anticipated Misconceptions

Some students may not know how to begin the task. Prompt them to start by calculating the actual length of each side of the playground.

Student Task Statement

Here is a scale drawing of a playground.



The scale is 1 centimetre to 30 metres.

1. Make another scale drawing of the same playground at a scale of 1 centimetre to 20 metres.

2. How do the two scale drawings compare?

Student Response

1. Scaled copy of the drawing where each edge is 1.5 times as long as in the drawing.

2. The new drawing is larger. Sample explanation: When 1 cm represents 20 m, it takes 1.5 cm to represent 30 m. So the length measurements on the 1 cm to 20 m scale are 1.5 times as long as they are with the 1 cm to 30 m scale. The area measurements are 2.25 (1.5×1.5) times as large.

Activity Synthesis

Invite selected students to share their work producing the new scale drawing. Ask students how the two scale drawings compare. Make sure that they recognise the shapes are the same (both represent the same playground) but the sizes are different.

Ask students if their prediction about which scale drawing would be larger was correct. Ask them to explain why the drawing at a scale of 1 cm to 20 m is larger than the drawing at a scale of 1 cm to 30 m. The important idea here is that when 1 cm on the scale drawing represents a *greater* distance, it takes fewer of those centimetres to describe the object. So the scale drawing at a scale of 1 cm to 30 m is smaller than the scale drawing at a scale of 1 cm to 20 m. Consider doing a demonstration in which you zoom in on a map with the scale showing.

To encourage students to think about the areas of the scale drawings like they did in the previous activity, consider asking questions like the following:

- "On the original map with the scale of 1 cm to 30 m, how much area does one square centimetre represent?" (900 cm^2)
- "On the new map with the scale of 1 cm to 20 m, how much area does one square centimetre represent?" (400 cm^2)
- "How many times as large as the original map is the new map?" (1.5 times for side lengths; 1.5×1.5, or 2.25, times for area)

Speaking, Listening: Discussion Supports. Use this routine to support whole-class discussion. For each strategy that is shared, ask another student to restate what they heard using precise mathematical language. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others. This will promote students' use of mathematical language as they make connections between the various ways to reproduce a scale drawing at a different scale.
Design Principle(s): Support sense-making

Lesson Synthesis

Sometimes we have a scale drawing and want to reproduce it at a different scale. Two common approaches are:

1. Using the original scale drawing to calculate the actual lengths and then using the actual lengths and the new scale to calculate the corresponding lengths on the new drawing.
2. Scaling lengths in the original scale drawing by a factor that relates the scales of the two drawings.

Suppose you have a map that uses the scale 1 cm to 200 m. You draw a new map of the same place using the scale 1 cm to 20 m.

- How does your new map compare to your original map? (The lengths are 10 times as long and the area is 100 times as large.)
- How much actual area does 1 cm^2 on your new map represent? (400 m^2)
- How much actual area did 1 cm^2 on your original map represent? (40 000 m^2)

10.4 Window Frame

Cool Down: 5 minutes

Student Task Statement

Here is a scale drawing of a window frame that uses a scale of 1 cm to 6 inches.

Create another scale drawing of the window frame that uses a scale of 1 cm to 12 inches.

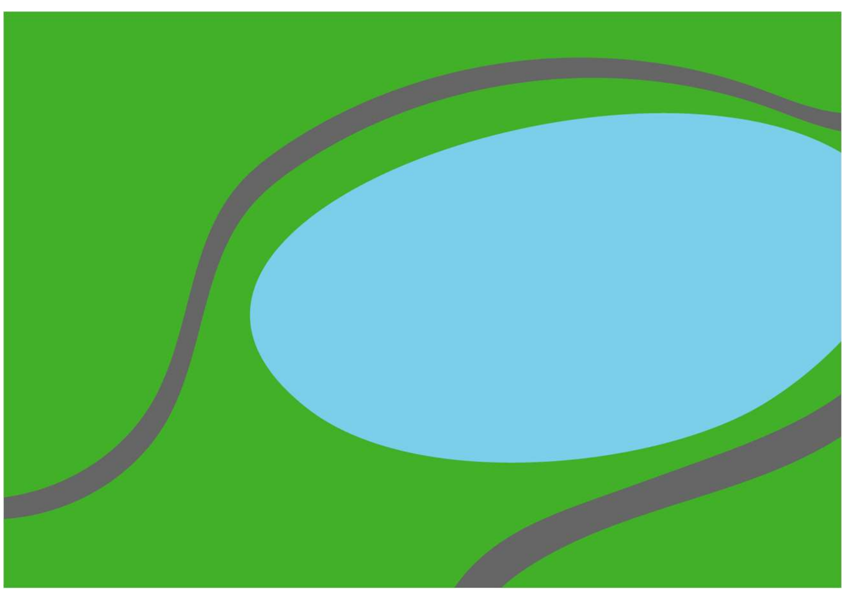
Student Response

Scaled copy of the drawing where each length is half as long as in the original.

Student Lesson Summary

Sometimes we have a scale drawing of something, and we want to create another scale drawing of it that uses a different scale. We can use the original scale drawing to find the size of the actual object. Then we can use the size of the actual object to figure out the size of our new scale drawing.

For example, here is a scale drawing of a park where the scale is 1 cm to 90 m.



The rectangle is 10 cm by 4 cm, so the actual dimensions of the park are 900 m by 360 m, because $10 \times 90 = 900$ and $4 \times 90 = 360$.

Suppose we want to make another scale drawing of the park where the scale is 1 cm to 30 metres. This new scale drawing should be 30 cm by 12 cm, because $900 \div 30 = 30$ and $360 \div 30 = 12$.

Another way to find this answer is to think about how the two different scales are related to each other. In the first scale drawing, 1 cm represented 90 m. In the new drawing, we would need 3 cm to represent 90 m. That means each length in the new scale drawing should be 3 times as long as it was in the original drawing. The new scale drawing should be 30 cm by 12 cm, because $3 \times 10 = 30$ and $3 \times 4 = 12$.

Since the length and width are 3 times as long, the area of the new scale drawing will be 9 times as large as the area of the original scale drawing, because $3^2 = 9$.

Lesson 10 Practice Problems

Problem 1 Statement

Here is a scale drawing of a swimming pool where 1 cm represents 1 m.

a. How long and how wide is the actual swimming pool?

b. Will a scale drawing where 1 cm represents 2 m be larger or smaller than this drawing?

c. Make a scale drawing of the swimming pool where 1 cm represents 2 m.

Solution

a. Answers vary. Sample response: The scale drawing is 10 cm long and 5 cm wide so the actual swimming pool is 10 m long and 5 m wide.

b. It will be smaller. Each centimetre will represent a larger distance so it will take fewer centimetres to represent the width and length of the swimming pool.

c. Answers vary. Sample response: The length and width will each be half as long as the given scale drawing. So the new scale drawing of the swimming pool will be 5 cm long and 2.5 cm wide.

Problem 2 Statement

A map of a park has a scale of 1 inch to 1 000 feet. Another map of the same park has a scale of 1 inch to 500 feet. Which map is larger? Explain or show your reasoning.

Solution

The map with a scale of 1 inch to 500 feet. It takes twice the number of units on this map to represent the same actual distance covered by the other map. For example, on the 1 inch to 1 000 feet map, it takes 1 inch to represent 1 000 feet in the actual park. On the 1 inch to 500 feet map, it takes 2 inches to represent the same 1 000 feet in the park.

Problem 3 Statement

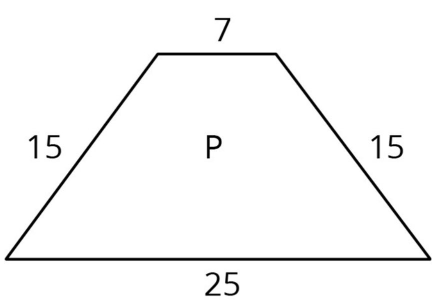
On a map with a scale of 1 inch to 12 feet, the area of a restaurant is 60 in^2. Han says that the actual area of the restaurant is 720 ft^2. Do you agree or disagree? Explain your reasoning.

Solution

I disagree. Sample reasoning: At the scale of 1 inch to 12 feet, every 1 square inch represents 144 square feet, since $12 \times 12 = 144$. The actual area of the restaurant should be 8 640 square feet, because $60 \times 144 = 8\,640$.

Problem 4 Statement

If quadrilateral Q is a scaled copy of quadrilateral P created with a scale factor of 3, what is the perimeter of Q?



Solution

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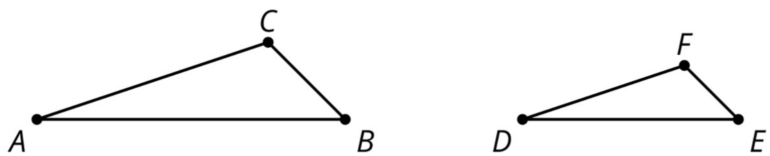
Problem 5 Statement

Triangle DEF is a scaled copy of triangle ABC. For each of the following parts of triangle ABC, identify the corresponding part of triangle DEF.

- angle ABC

- angle BCA
- line segment AC
- line segment BA



Solution

- angle DEF
- angle EFD
- line segment DF
- line segment ED

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