FINAL EXAMINATION - 2078

Class: 9   **Subject:** Opt.Mathematics   **F.M.:** 100
Time: 3 hours   **P.M. :** 40

Attempt all the questions. All the working must be shown.

Group 'A' $[10 \times 1 = 10]$

1. (a) If $(a, b) = (c, d)$, write the relation between a and c.

   (b) What is the degree of the expression $xy^2 + xy + 2$?

2. (a) Write $x \to a$ into mathematical sentence.

   (b) A matrix A has x rows and y columns. Write the order of the matrix.

3. (a) Write the formula to calculate the distance between the points (x_1, y_1) and (x_2, y_2).

   (b) Write down the equation of a straight line parallel to x-axis and at a distance of a units.

4. (a) Write the relation among degree, minute and second according to sexagesimal system.

   (b) Which triogonometric ratio represents the ratio between adjacent side and hypotenuse?

5. (a) What do m and n represent in $\vec{a} = \begin{pmatrix} m \\ n \end{pmatrix}$

   (b) What is the image of a point (m, n) when reflected in the line $y = x$?

Group 'B' $[13 \times 2 = 26]$

6. (a) What element in the domain has the image 6 under the function $g(x) = \dfrac{2x}{3}$?

   (b) If the polynomials $x^3 + (a + 2)x^2 + x + 8$ and $x^3 + 4x^2 + (b - 1)x + 8$ are equals, find the values of a and b.

   (c) Express the first 10 terms of a sequence 2, 5, 8, 11 ...... in sigma notation.

7. (a) If $R = \begin{pmatrix} 5 & 3 \\ 2 & -1 \end{pmatrix}$ and $S = \begin{pmatrix} 0 & -4 \\ 2 & 6 \end{pmatrix}$, find $(2R + 3S)^T$.

   (b) If $P = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $K = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, find the value of PK.

8. (a) Find the distance between the parallel lines $2x - y + 4 = 0$ and $6x - 3y = 5$.

   (b) A(2, – 1), B(5, – 2) and C(4, 3) are the vertices of $\triangle$ABC. Find the area of $\triangle$ABC.

9. (a) The given figure is a part of the circle with centre O and the arc AB. If arc AB = 44 cm and $\angle AOB = \left(\dfrac{7\pi^c}{18} \right)$. Find the length of the radius OA.

   (b) Prove that: $\tan^2 A + \cot^2 A = \sec^2 A.\csc^2 A - 2$

   (c) Find the value without using calculator or table: $\sin 75°$

10. (a) Find the unit vector along the direction of $\overrightarrow{MN}$ where M(3,-2) and N(1,5).

    (b) If $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DE}$ and $\overrightarrow{EA}$ are the sides of pentagon, prove that:

    $$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = 0$$

    (c) In an individual series madian is 50, N = 8 and $\Sigma(x{-}md) = 59$. Find the mean deviation from median and its coefficient.

Group 'C' $[11 \times 4 = 44]$

11. If $g(x) = x^2 + 4x + 5$, find $g(x + 1)$ and the value of x if $g(x) = g(x + 1)$.

12. What should be multiplied to $x^2 - 6x + 23$ to get product $x^3 - 2x^2 - x + 92$?

13. The general term of a numerical sequence is given by $t_n = 2n^2$. Find the first 4 terms of the sequence. Also, find the limit value of the sequence.

14. If $2A+B = \begin{pmatrix} 5 & 8 \\ 11 & 5 \end{pmatrix}$ and $2A-B = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}$ find the matrices A and B.

15. If a moving point $P(x, y)$ moves in such a way that the ratio of the distances from the points $(-7, 0)$ and $(7, 0)$ to its is $4 : 5$, find the locus of the point P.

16. The angles of a triangle are $\left(\dfrac{\pi x}{36}\right)^c$, $(7x)^\circ$ and $\left(\dfrac{20x}{3}\right)^g$. Find the measure of each angle in degree.

17. Prove that: $\tan^4 \alpha + \sec^4 \alpha = 1 + \dfrac{2\tan^2 \alpha}{\cos^2 \alpha}$

18. If $8\sin\theta = 4 + \cos\theta$, find the value of $\tan\theta$.

19. E(2, 0), F(4, 0), G(2, 3) and H(4, 5) are the vertices of a quadrilateral EFGH. Write the co–ordinates of the image under the rotation through -270° about the origin.

20. Find the quartile deviation and its coefficient of the following data:

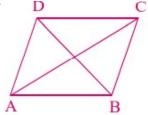| Marks	10	40	30	20	60	50
Frequency	4	3	2	1	6	3

21. Find the standard deviation and its coefficient of the given data: 5, 10, 20, 15, 25

Group 'D' $[4 \times 5 = 20]$

22. If $S_n = 8n^2 + 16n + 12$, find the value of $t_2 + t_4 + t_6$.

23. Reduce the equation $y = mx+c$ in the form $x\cos\alpha+y\sin\alpha = p$ and prove that: $\dfrac{1}{p^2} = \dfrac{m^2}{c^2} + \dfrac{1}{c^2}$

24. AC and BD are the diagonals of a parallelogram ABCD, prove that: $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{BC}$

25. Enlarge the $\triangle ABC$ having the vertices A(3, 4) B(– 2, 6) and C(1, – 5) with the centre (1, 2) and scale factor of – 2 so that the $\triangle A'B'C'$ which is the image of $\triangle ABC$ is formed. Find the coordinates of A', B' and C'. Also present the $\triangle ABC$ and $\triangle A'B'C'$ on the same graph paper.

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