Grades 11-12 (A) Duration: 40 min Tools: one 9 pcs Set / class Individual / Pair work Keywords: Transformations, GeoGebra	534 - Upside down Blocks in GeoGebra **MATHS / TRANSFORMATIONS**	LOGIFACES METHODOLOGY Erasmus+ **TEACHER** Logifaces 2019-1-HU01-KA201-0612722019-1

DESCRIPTION

LEVEL 1 Students start with the GeoGebra model of the block 123 (see exercises 526 - Calculate the Coordinates and 527 - Coordinates in GeoGebra). The task is to find transformations to move the block in such a way that the top face lies in the xy-plane, one edge lies on the x-axis, one vertex is in the origin and the whole block is contained in the upper half-space.

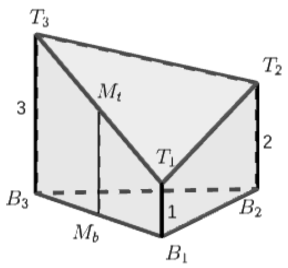
HINTS

- Recommended steps of transformations: reflection, rotation around a line, translation
- It is recommended to start with the model where the vertical edge of length 1 has coordinates x=0, y=0, and the vertical edge of height 3 has coordinates x=4, y=0.

LEVEL 2 Students calculate the exact values of the coordinates of the block placed as in the Level 1 exercise. They can use the transformations to help the calculations.

SOLUTIONS / EXAMPLES

The following notations are used for the vertices in the solution:



The points M_b and M_t are the midpoints of the segments B_1B_3 and T_1T_3 respectively.
The same notations are used after the reflection. After the rotations, the notations T'_i are used, and after the translation, the notations T''_i are used.

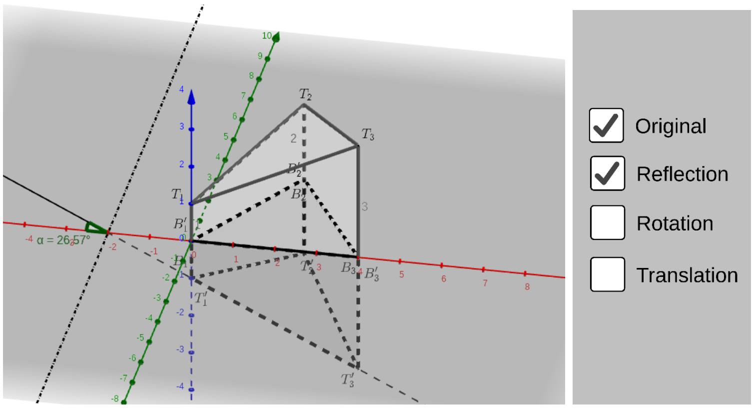
LEVEL 1 The task can be performed in three steps, as the diagrams show below (see also the following GeoGebra link: https://www.geogebra.org/classic/sdhgtdg4):

1. Reflection in the xy-plane

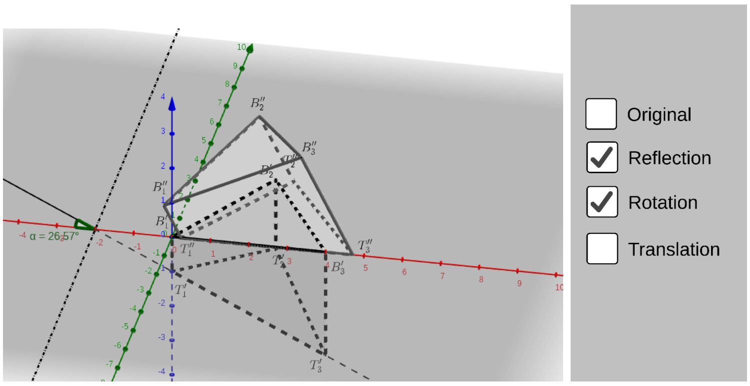
2. Rotation around the intersection line of the planes of the top and base planes with the angle of the two planes. The rotation can be performed as follows: the x-axis and the line $T'_1T'_3$ are both perpendicular to the line of intersection of the planes of the top and base faces. The angle of the two planes can be measured as the angle of these two lines.
After this transformation the top face lies in the xy-plane, the whole block lies in the upper half-plane, but there is no vertex in the origin.

3. Translation by the vector $\overrightarrow{T_1''O}$. This transformation moves the vertex T'_1 to the origin.

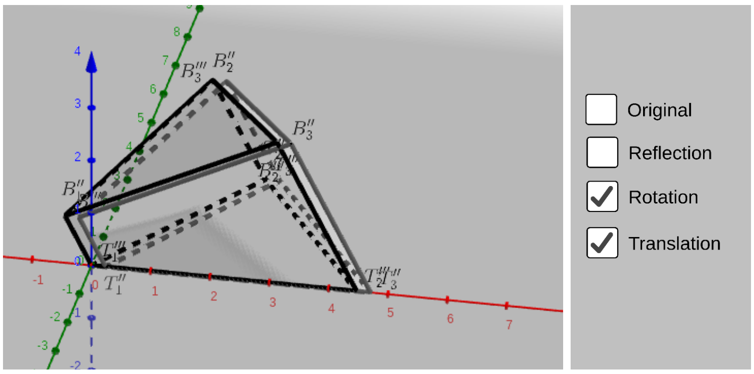
STEP 1 Reflexion to the xy-plane



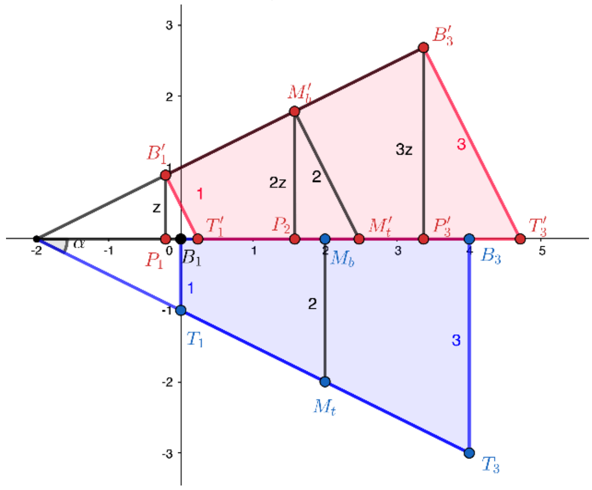
STEP 2 Rotation



STEP 3 Translation:



LEVEL 2 The diagram below shows the xz-cross-section of the rotation step.



We use the notation α for the angle between the planes of the top and base faces. We do not need the exact value of α, only the values of the trigonometric ratios of it.
The lengths of the segments on the diagram are $AB_1 = B_1M_b = M_bB_3 = 2$ and $AT_1 = T_1M_t = M_tT_3 = \sqrt{5}$ by prior calculations and similarity, and the same holds for their images by rotation.
By the definitions of the trigonometric ratios in the right-angled triangle:
$sin(\alpha) = \frac{B_1T_1}{AT_1} = \frac{1}{\sqrt{5}}$ and $cos(\alpha) = \frac{AB_1}{AT_1} = \frac{2}{\sqrt{5}}$, hence the lengths z and AP_1 needed for the coordinates can be calculated as follows:
$sin(\alpha) = \frac{z}{AB'_1} = \frac{z}{2} = \frac{1}{\sqrt{5}} \Rightarrow z = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ and $cos(\alpha) = \frac{AP_1}{AB'_1} = \frac{AP_1}{2} = \frac{2}{\sqrt{5}} \Rightarrow AP_1 = \frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$.

The coordinates of the image of the block after the reflection and the rotation are the following:

$T'_1\left(\sqrt{5} - 2, 0, 0\right)$, $T'_3\left(3\sqrt{5} - 2, 0, 0\right)$, $B'_1\left(\frac{4\sqrt{5}}{5} - 2, 0, \frac{2\sqrt{5}}{5}\right)$, $B'_3\left(\frac{12\sqrt{5}}{5} - 2, 0, \frac{6\sqrt{5}}{5}\right)$
The vertices T_2 and B_2 lie on the plane perpendicular to the y-axes through the point $\left(0, 2\sqrt{3}, 0\right)$ during the reflection and rotation, because the plane of the reflection and the axis of rotation are perpendicular to that plane. Furthermore, the x and z coordinates of the points T_2 and B_2 are the same as that of M'_t and M'_b, respectively, so the coordinates are: $T'_2\left(2\sqrt{5} - 2, 2\sqrt{3}, 0\right)$ and $B'_2\left(\frac{8\sqrt{5}}{5} - 2, 2\sqrt{3}, \frac{4\sqrt{5}}{5}\right)$.

Finally, a translation is needed by the vector $\overrightarrow{T'_1B_1}$, which is parallel to the x-axis, points toward the negative direction and has length $\sqrt{5} - 2$. This results in the coordinates
$T''_1(0, 0, 0)$, $T''_3\left(2\sqrt{5}, 0, 0\right)$, $T''_2\left(\sqrt{5}, 2\sqrt{3}, 0\right)$
and $B''_1\left(-\frac{\sqrt{5}}{5}, 0, \frac{2\sqrt{5}}{5}\right)$, $B''_3\left(\frac{7\sqrt{5}}{5}, 0, \frac{6\sqrt{5}}{5}\right)$, $B''_2\left(\frac{3\sqrt{5}}{5}, 2\sqrt{3}, \frac{4\sqrt{5}}{5}\right)$

PRIOR KNOWLEDGE

Reflection in a plane, Rotation around an axis, Transformations in GeoGebra

RECOMMENDATIONS / COMMENTS

Exercises 526 - Calculate the Coordinates and 527 - Coordinates in GeoGebra are recommended before this exercise.