

Lesson 1: Introducing ratios and ratio language

Goals

- Comprehend the word “ratio” (in written and spoken language) and the notation $a : b$ (in written language) to refer to an association between quantities.
- Describe (orally and in writing) associations between quantities using the language “For every a of these, there are b of those” and “The ratio of these to those is $a : b$ (or a to b).”

Learning Targets

- I can write or say a sentence that describes a ratio.
- I know how to say words and numbers in the correct order to accurately describe the ratio.

Lesson Narrative

In this lesson, students use collections of objects to make sense of and use **ratio** language. Students see that there are several different ways to describe a situation using ratio language. For example, if we have 12 squares and 4 circles, we can say the ratio of squares to circles is $12 : 4$ and the ratio of circles to squares is 4 to 12. We may also see a structure that prompts us to regroup them and say that there are 6 squares for every 2 circles, or 3 squares for every one circle.

Expressing associations of quantities in a context—as students will be doing in this lesson—requires students to use ratio language with care. Making groups of physical objects that correspond with “for every” language is a concrete way for students to make sense of the problem.

It is important that in this first lesson students have physical objects they can move around. Later, they will draw diagrams that reflect the same structures and learn to reason with and interpret abstract representations like double number line diagrams and tables. Working with objects that can be physically rearranged in the beginning of the unit can help students make sense of increasingly abstract representations they will encounter as the unit progresses. Students will continue to develop ratio language throughout the unit and will learn about equivalent ratios in a future lesson.

Building On

- Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Addressing

- Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the

bird house at the zoo was 2 : 1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- Compare and Connect
- Think Pair Share

Required Materials

Students’ collections of objects

Teacher’s collection of objects

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

A few days before this lesson, ask students to bring a personal collection of 10–50 small objects. Examples include rocks, seashells, cards, or coins. Bring in your personal collection and display it ahead of time. Think of possible ways to sort your collection. (See the Launch section of the first activity for details.) Prepare a few extra collections for students who don’t bring one.

Student Learning Goals

Let’s describe two quantities at the same time.

1.1 What Kind and How Many?

Warm Up: 5 minutes

In this warm-up, students compare shapes and sort them into categories.

Instructional Routines

- Think Pair Share

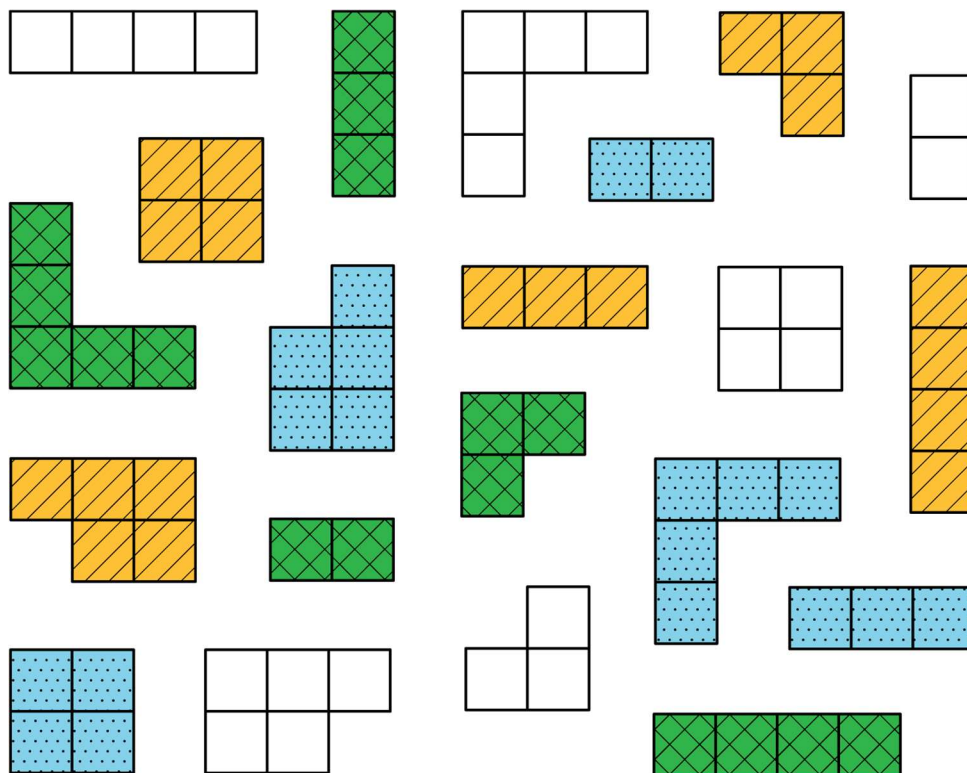
Launch

Display the image for all to see. Give students 1 minute of quiet think time followed by 2 minutes of partner discussion.

Anticipated Misconceptions

If students struggle to create their own categories, prompt them to consider a specific attribute of the shapes, such as the size, colour, or shape.

Student Task Statement



Think of different ways you could sort these shapes. What categories could you use? How many groups would you have?

Student Response

Answers vary. Sample responses:

By area: Four groups: 2, 3, 4, and 5 square units

By colour: Four groups: blue, green, yellow, and white

By pattern: Four groups: striped, dotted, cross-hatch, and blank

By shape:

- Two groups: rectangles and non-rectangles.
- Three groups: rectangles, two different squares glued together, and L-shapes.
- Four groups: squares, rectangles, two different squares glued together, and L-shapes.
- Seven groups: small, medium, and large rectangles, 2 by 2 squares, small L, big L, and a small and a big square glued together.

Activity Synthesis

Record all the ways students answered the question for all to see. Ask a student to explain how they sorted the shapes. Ask if anyone saw it a different way until all the different ways of seeing the shapes have been shared. Emphasise that the important thing is to describe the way they sorted them clearly enough that everyone agrees that it is a reasonable way to sort them. Tell students we will be looking at different ways of seeing the same set of objects in the next activity.

1.2 The Teacher's Collection

10 minutes

This activity introduces students to ratio language and notation through examples based on a collection of everyday objects. Students learn that a ratio is an association between quantities, and that this association can be expressed in multiple ways.

After discussing examples of ratio language and notation for one way of categorising the objects in the collection, students write ratios to describe the quantities for another way of categorising objects in the collection.

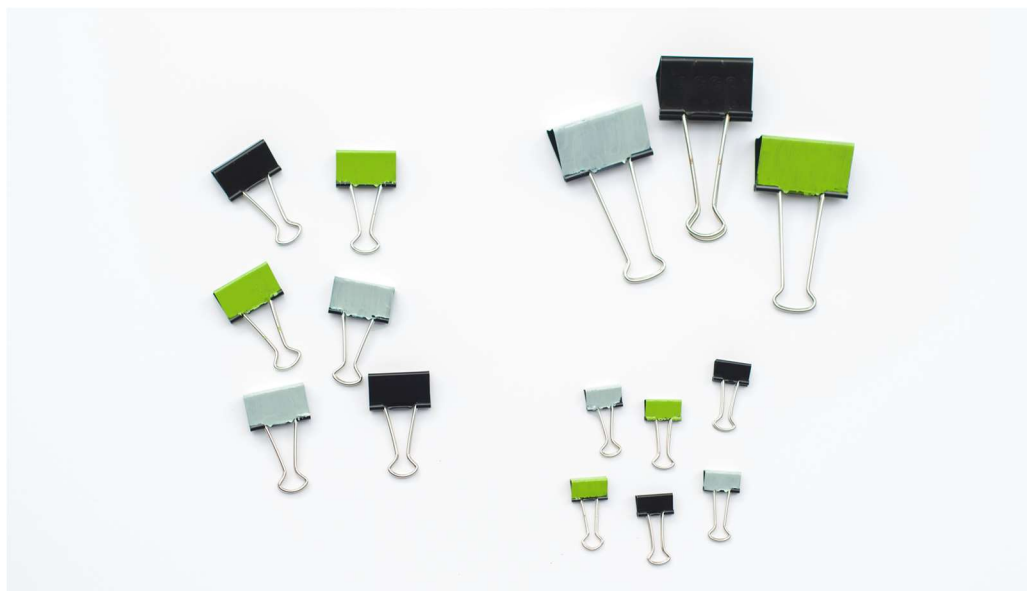
As students work, circulate and identify those who:

- Create different categories from the given collection.
- Create categories whose quantities can be rearranged into smaller groups (e.g. 6 A's and 4 B's can be expressed as "for every 3 A's there are 2 B's").
- Express the same ratio in opposite order or by using different words (e.g. "the ratio of A to B is 7 to 3," and "for every 7 A's there are 3 B's").

Have a collection of objects ready to display for the launch. Make sure there are different ways the collection can be sorted. For example, the dinosaurs below can be categorised by colour (green, orange, and purple), by the number of legs they stand on (standing on 4 legs or on 2 legs), or by the features along their backs (crest, white stripe, or nothing).



Familiar classroom objects such as binder clips or pattern blocks can also be used to form collections. This picture shows a collection of binder clips that could be categorised by size (small, medium, and large) or by colour (black, green, and blue) .



Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Display a collection of objects for all to see. Give students 2 minutes of quiet think time to come up with as many different categories for sorting the collection as they can think of. Record students' categories for all to see. Sort the collection into one of the student-suggested categories and count the number of items in each. Record the number of objects in each category and display for all to see. For example:

category A: green	category B: orange	category C: purple
3	2	4

Explain that we can talk about the quantities in the different categories using something called **ratios**. Tell students: "A **ratio** is an association between two or more quantities." We use a colon, or the word "to," between two values we are associating.

Share the following examples (adapt them to suit your collection) and display them for all to see. Keep the examples visible for the duration of the lesson.

- The ratio of purple to orange dinosaurs is 4 to 2.
- The ratio of purple to orange dinosaurs is 4 : 2.
- The ratio of orange to purple dinosaurs is 2 to 4.

- The ratio of orange to purple dinosaurs is 2 : 4

Explain that we can also associate two quantities using the phrase “for every a of these, there are b of those.” Add the following examples to the display.

- For every 3 green dinosaurs there are 4 purple dinosaurs.
- There are 4 purple dinosaurs for every 2 orange dinosaurs.

Finally, find two categories whose items can be rearranged into smaller groups, e.g. 4 purple dinosaurs to 2 orange dinosaurs. Point out that in some cases we can associate the same categories using different numbers. Share the following example and add it to the display.

- For every 2 purple dinosaurs, there is 1 orange dinosaur.



Have students write two or three sentences to describe ratios between the categories they suggested.

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: ratio and category.

Supports accessibility for: Conceptual processing; Language

Anticipated Misconceptions

Students may write ratios with no descriptive words. $8 : 2$ is a good start, but part of writing a ratio is stating what those numbers mean. Draw students' attention to the sentence stems in the task statement; encourage them to use those words.

Student Task Statement

1. Think of a way to sort your teacher's collection into two or three categories. Count the items in each category, and record the information in the table.

category name

category amount

Pause here so your teacher can review your work.

2. Write at least two sentences that describe **ratios** in the collection. Remember, there are many ways to write a ratio:
 - The ratio of *one category* to *another category* is _____ to _____.
 - The ratio of *one category* to *another category* is _____ : _____.
 - There are _____ of *one category* for every _____ of *another category*.

Student Response

Answers vary depending on the particulars of the teacher’s collection and the choices made by students.

Activity Synthesis

Invite several students to share their categories and sentences. Display them for all to see, attending to correct ratio language. Be sure to include students who express the same categories in reverse order, in different words, or with a different set of numbers (which students will later call an equivalent ratio). Leave several sentences displayed for students to see and use as a reference while working on the next task.

1.3 The Student’s Collection

20 minutes

In this activity, students write ratios to describe objects in their own collection. They create a display of objects and circulate to look at their classmates’ work. Students see that there are several ways to write ratios to describe the same situation.

Instructional Routines

- Group Presentations
- Compare and Connect

Launch

Invite students to share what types of items are in their personal collections. If students did not bring in a collection, pair them with another student, or provide them with an extra collection that you have brought in for that purpose.

Provide access to tools for creating a visual display. Tell students they will pause their work before creating a visual display to get their sentences approved.

Representing, Conversing: Compare and Connect. Use this routine to help students consider audience when preparing to display their work. Display the list of items that should be included on the display and ask students, “what kinds of details could you include on your

display to help a reader understand the ratios you've used to describe the objects in your collection?" Record ideas and display for all to see. Examples of these types of details or annotations include: the order in which representations are organised on the display, attaching written notes or details to certain representations, using specific vocabulary or phrases, or using colour or arrows to show connections between representations. If time allows, ask students to describe specific examples of additional details that other groups used that helped them to interpret and understand their displays.

Design Principle(s): Maximise meta-awareness; Optimise output

Anticipated Misconceptions

Watch for students simply writing a numerical ratio, such as 3 : 7, without any descriptive words. Draw their attention to the sentence stems in the task statement.

Student Task Statement

1. Sort your collection into three categories. You can experiment with different ways of arranging these categories. Then, count the items in each category, and record the information in the table.

category name

category amount

2. Write at least two sentences that describe **ratios** in the collection. Remember, there are many ways to write a ratio:
 - The ratio of *one category* to *another category* is _____ to _____.
 - The ratio of *one category* to *another category* is _____ : _____.
 - There are _____ of *one category* for every _____ of *another category*.

Pause here so your teacher can review your sentences.

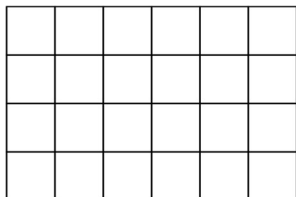
3. Make a visual display of your items that clearly shows one of your statements. Be prepared to share your display with the class.

Student Response

Answers vary.

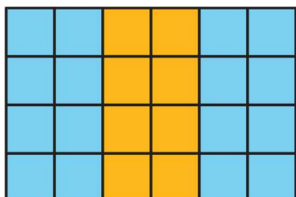
Are You Ready for More?

1. Use two colours to shade the rectangle so there are 2 square units of one colour for every 1 square unit of the other colour.
2. The rectangle you just coloured has an area of 24 square units. Draw a different shape that does *not* have an area of 24 square units, but that can also be shaded with two colours in a 2 : 1 ratio. Shade your new shape using two colours.



Student Response

1. A correct answer will have 16 square units of one colour and 8 square units of the other colour. Here's one way:



2. Answers vary.

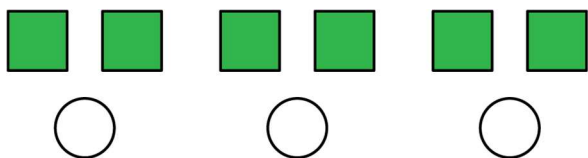
Activity Synthesis

Once students have had enough time to create their displays, circulate through each display and listen to how students describe their ratios.

As students present their displays, point out the various ways that students chose to showcase their work, including different ways to say the same ratio. Ask students who used two sets of numbers to describe the same categories (e.g., 8 to 2 and “4 for every 1”) to demonstrate the two ways of grouping the objects.

Lesson Synthesis

This lesson is all about how to use **ratio** language and notation to describe an association between two or more quantities. Wrap up the lesson by drawing a diagram for all to see of, say, 6 squares and 3 circles.



Say, “One way to write this ratio is, there are 6 squares for every 3 circles. What are some other ways to write this ratio?” Some correct options might be:

- The ratio of squares to circles is 6 : 3.
- The ratio of circles to squares is 3 to 6.
- There are 2 squares for every 1 circle.

Display this diagram and the associated sentences the class comes up with somewhere in the classroom so students can refer back to the correct ratio and rate language during subsequent lessons.

Consider posing some more general questions, such as:

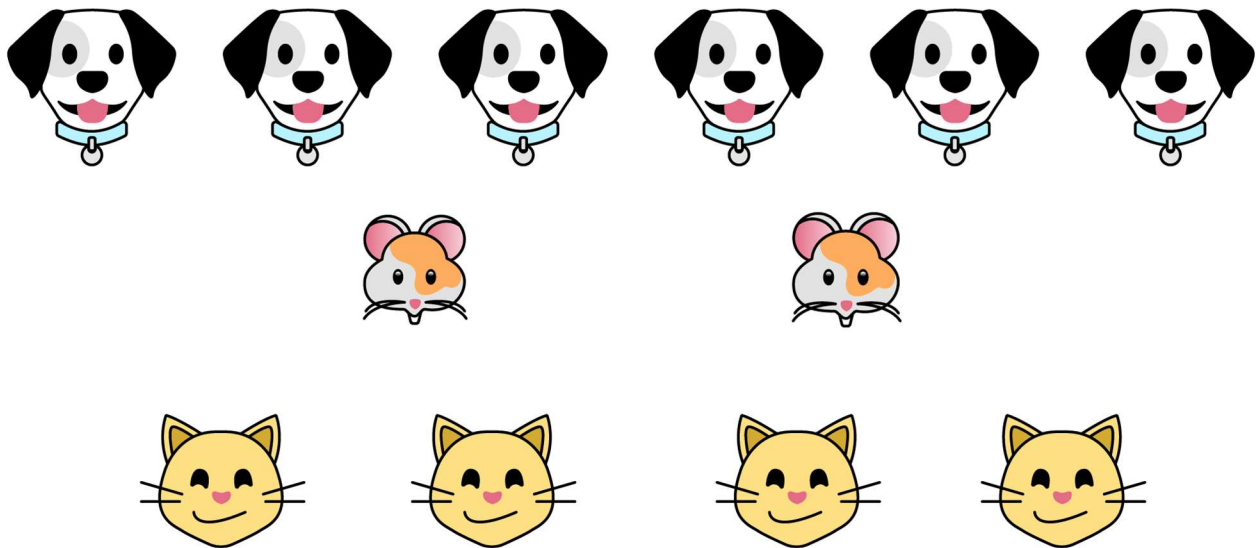
- Explain what a ratio is in your own words.
- What things must you pay attention to when writing a ratio?
- What are some words and phrases that are used to write a ratio?

1.4 A Collection of Animals

Cool Down: 5 minutes

Student Task Statement

Here is a collection of dogs, mice, and cats:



Write *two* sentences that describe a ratio of types of animals in this collection.

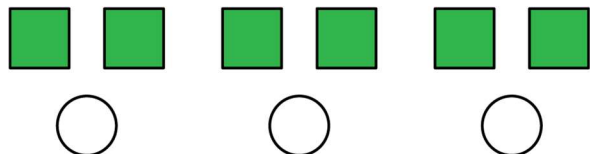
Student Response

Answers vary. Sample responses:

- The ratio of dogs to cats is 6 : 4.
- There are 3 dogs for every 2 cats.
- There is 1 mouse for every 2 cats.
- The ratio of cats to mice is 4 : 2.

Student Lesson Summary

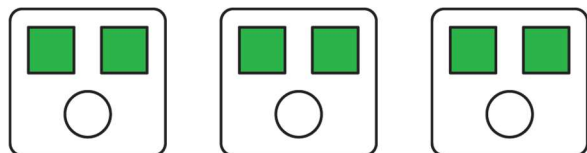
A **ratio** is an association between two or more quantities. There are many ways to describe a situation in terms of ratios. For example, look at this collection:



Here are some correct ways to describe the collection:

- The ratio of squares to circles is 6 : 3.
- The ratio of circles to squares is 3 to 6.

Notice that the shapes can be arranged in equal groups, which allow us to describe the shapes using other numbers.



- There are 2 squares for every 1 circle.
- There is 1 circle for every 2 squares.

Glossary

- ratio

Lesson 1 Practice Problems

1. Problem 1 Statement

In a fruit basket there are 9 bananas, 4 apples, and 3 plums.

- The ratio of bananas to apples is _____ : _____.
- The ratio of plums to apples is _____ to _____.
- For every _____ apples, there are _____ plums.
- For every 3 bananas there is one _____.

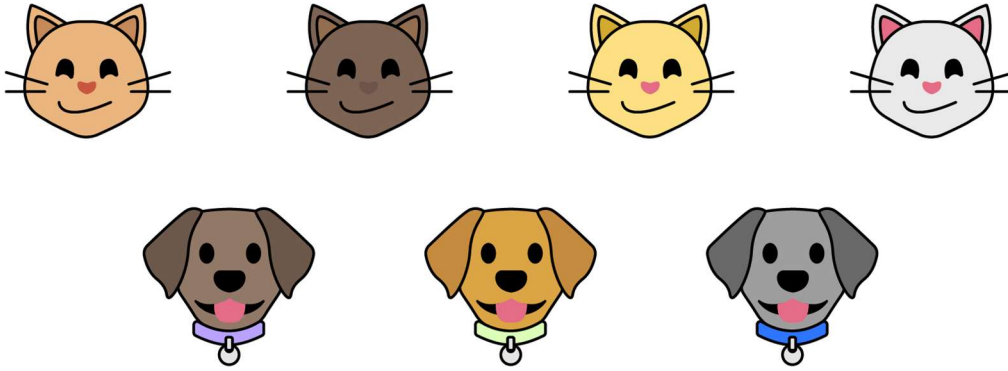
Solution

- 9, 4
- 3, 4

- c. 4, 3
- d. plum

2. Problem 2 Statement

Complete the sentences to describe this picture.



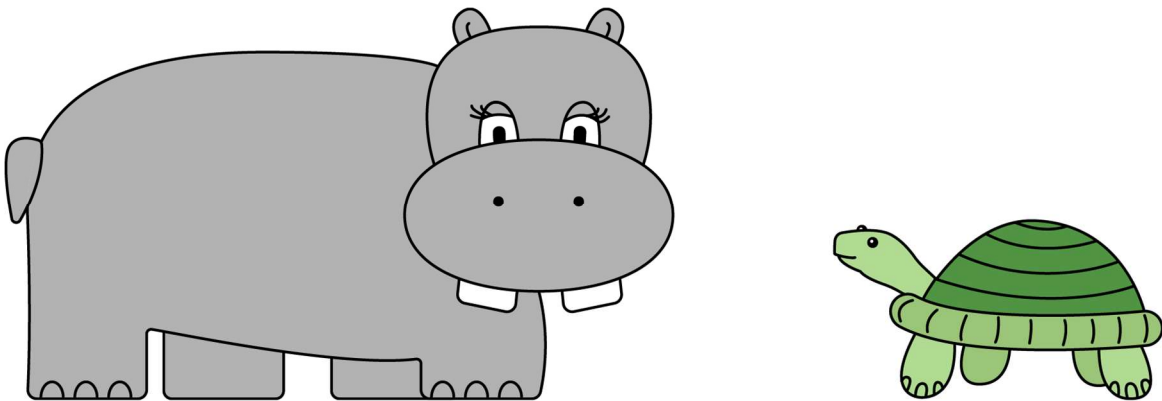
- a. The ratio of dogs to cats is _____.
- b. For every _____ dogs, there are _____ cats.

Solution

- a. 3 to 4
- b. 3, 4

3. Problem 3 Statement

Write two different sentences that use ratios to describe the number of eyes and legs in this picture.



Solution

Answers vary. Sample responses:

- The ratio of legs to eyes is 8 to 4.
- The ratio of eyes to legs is 4 : 8.
- There are 2 legs for every eye.
- There are 4 legs for every 2 eyes.

4. Problem 4 Statement

Choose an appropriate unit of measurement for each quantity.

- a. area of a rectangle
 - b. volume of a cuboid
 - c. side of a square
 - d. area of a square
 - e. volume of a cube
- cm
 - cm^3
 - cm^2

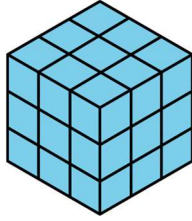
Solution

- a. cm^2
- b. cm^3
- c. cm
- d. cm^2
- e. cm^3

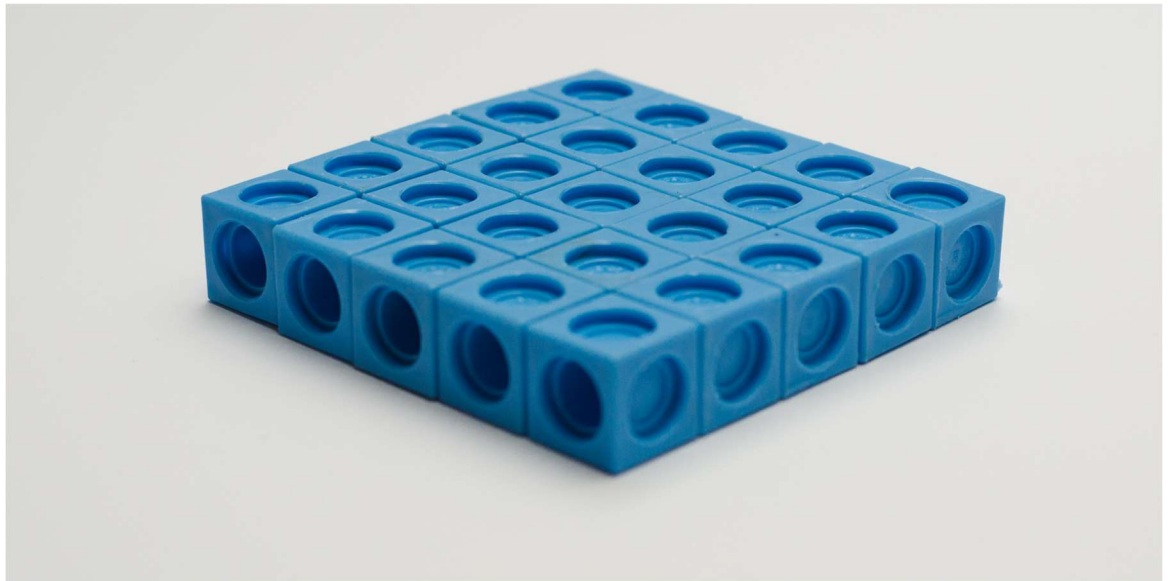
5. Problem 5 Statement

Find the volume and surface area of each cuboid.

- a. Cuboid A: 3 cm by 3 cm by 3 cm
-



- b. Cuboid B: 5 cm by 5 cm by 1 cm



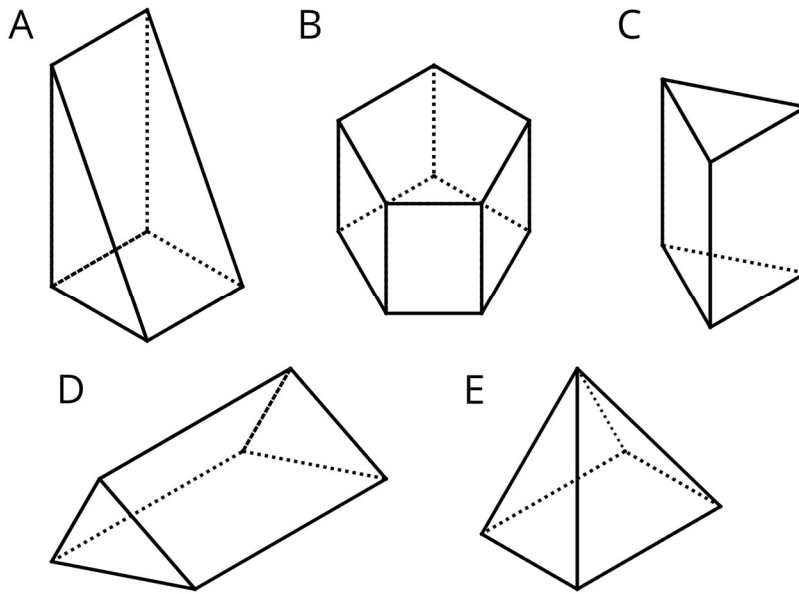
- c. Compare the volumes of the cuboids and then their surface areas. Does the cuboid with the greater volume also have the greater surface area?

Solution

- a. Volume: 27 cm^3 , surface area: 54 cm^2
b. Volume: 25 cm^3 , surface area: 70 cm^2
c. Cuboid A has a greater volume, but cuboid B has a greater surface area.

6. Problem 6 Statement

Which shape is a triangular prism? Select **all** that apply.



- a. A
- b. B
- c. C
- d. D
- e. E

Solution ["A", "C", "D"]



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