Lesson 9: Solving problems about proportional relationships

Goals

- Decide whether it makes sense to represent a situation with a proportional relationship, and explain (orally) the reasoning.
- Determine what information is needed to solve a problem involving proportional relationships. Ask questions to elicit that information.
- Write an equation to represent a proportional relationship, and use the equation to solve problems about the situation.

Learning Targets

- I can ask questions about a situation to determine whether two quantities are in a proportional relationship.
- I can solve all kinds of problems involving proportional relationships.

Lesson Narrative

In the previous two lessons students have been comparing proportional and nonproportional relationships using tables and equations. In this lesson they learn to recognise proportional relationships from descriptions of the context. This lesson includes the first Info Gap activity students encounter. In the Info Gap activity, students keep asking questions until they get all the information needed to solve the problem. In this case, that means they must determine if there are quantities in a proportional relationship they can work with. They see the connection between the constant of proportionality and a constant rate in the situation, and they compare different proportional relationships using the constant of proportionality. They reason quantitatively about situations and connect their reasoning with the equation for a proportional relationship.

Addressing

- Analyse proportional relationships and use them to solve real-world and mathematical problems.
- Recognise and represent proportional relationships between quantities.

Instructional Routines

- Information Gap Cards
- Compare and Connect

Required Materials

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Make 1 copy of the Info Gap blackline master for every 2 students, and cut them up ahead of time.

Student Learning Goals

Let's solve problems about proportional relationships.

9.1 What Do You Want to Know?

Warm Up: 10 minutes

This warm-up prepares students for the Info Gap activity that follows. First, students brainstorm what information they would need to know to solve a problem that involves constant speed. Then, the teacher demonstrates the process of asking a student why they need a specific piece of information before sharing it with them, in preparation for students following this procedure with their partner in the next activity.

Launch

Give students 1 minutes of quiet think time to brainstorm what information they would need to know to solve the problem, followed by a whole-class discussion demonstrating the procedure for the Info Gap activity.

Student Task Statement

Consider the problem: A person is running a distance race at a constant rate. What time will they finish the race?

What information would you need to be able to solve the problem?

Student Response

Answers vary. Sample responses:

- How long is the race?
- How fast is the person running?
- How far did the person run in 1 minute?
- What time did they start the race?

Activity Synthesis

Ask students:

- "What specific information do you need?"
- "Why do you need that information?"

Share each piece of information with the class after a student specifically asks for it (and explains why they need to know it).

- The race is 10 000 metres long.
- The race started at 9:15 a.m.
- In 1 minute, the person ran $156\frac{1}{4}$ metres.
- An equation relating distance and time is given by $d = 156\frac{1}{4} \times t$ where d represents distance in metres and t represents time in minutes.
- It takes 32 minutes for the person to run 5 000 metres.
- The person runs at a pace of 6.4 minutes per kilometre (or 1 000 metres).

After you share each piece of information, ask the class whether they have enough information to be able to solve the problem. When they think they do, give them 2 minutes to solve the problem and then have them share their strategies. (The person should finish the race at 10:19 a.m.)

Tell students that they will be working in groups of two in the next activity and that they will be using the same procedure that you just demonstrated to solve a problem.

9.2 Info Gap: Biking and Rain

30 minutes

In this info gap activity, students write equations for several proportional relationships given in the contexts of a bike ride and steady rainfall. They use the equations to make predictions.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Here is the text of the cards for reference and planning:

Info Gap: Biking and Rain **Problem Card 1** Mai and Noah each leave their houses at the same time and ride their bikes to the park. 1. For each person, write an equation that relates the distance they travel and the time. 2. Who will arrive at the park first?	Info Gap: Biking and Rain **Data Card 1** • Noah lives 1 kilometre farther away from the park than Mai does. • Mai lives 8 000 metres from the park. • Noah lives 9 000 metres from the park. • Mai and Noah each bike at a constant speed. • Mai bikes 250 metres per minute. • Noah bikes 300 metres per minute.
Info Gap: Biking and Rain **Problem Card 2** A slow, steady rainstorm lasted all day. The rain was falling at a constant rate. 1. Write an equation that relates how much rain has fallen and how long it has been raining. 2. How long will it take for 5 cm of rain to fall?	Info Gap: Biking and Rain **Data Card 2** • The rainstorm lasted for 24 hours. • 9.6 centimetres of rain fell during the storm. • The rate of the rainfall was 2 millimetres of rain every 30 minutes. • There are 10 millimetres in 1 centimetre. • There are 60 minutes in an hour.

Instructional Routines

- Information Gap Cards

Launch

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.
Supports accessibility for: Memory; Organisation Conversing: This activity uses *Information Gap* to give students a purpose for discussing information necessary to write equations for proportional relationships. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?"
Design Principle(s): Cultivate Conversation

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.

2. Ask your partner for the specific information that you need.

3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.

5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.

2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.

3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.

4. Read the *problem card* and solve the problem independently.

5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response

Card 1:

1. For Mai, $d = 250t$ where d represents distance in metres and t represents the amount of time in minutes. For Noah, $d = 300\ t$.

2. Noah will arrive first, since it will only take him 30 minutes ($9\,000 \div 300 = 30$) while Mai takes 32 minutes ($8\,000 \div 250 = 32$).

Card 2:

1. Answers vary. Possible responses:

   - $r = 0.4t$, where r represents the amount of rain that has fallen in centimetres and t represents the amount of time in hours.

- $t = 2.5r$, where r represents the amount of rain that has fallen in centimetres and t represents the amount of time in hours.
- $r = \frac{1}{150}t$, where r represents the amount of rain that has fallen in centimetres and t represents the amount of time in minutes.
- $t = 150r$, where r represents the amount of rain that has fallen in centimetres and t represents the amount of time in minutes.

2. 12.5 hours

Activity Synthesis

Invite students to share their equations and predictions. Record their equations displayed for all to see. Ask them to explain how they determined the constant of proportionality as well as why it made sense to represent the situation with a proportional relationship.

9.3 Moderating Comments

Optional: 10 minutes

In this activity students compute rates to decide which job applicant is working the fastest checking online comments. They compare rates and total number of comments checked, then see that using rates is the more useful information in this situation.

Instructional Routines

- Compare and Connect

Launch

Keep students in the same groups of 2.

Representation: Internalise Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

A company is hiring people to read through all the comments posted on their website to make sure they are appropriate. Four people applied for the job and were given one day to show how quickly they could check comments.

- Person 1 worked for 210 minutes and checked a total of 50 000 comments.
- Person 2 worked for 200 minutes and checked 1 325 comments every 5 minutes.
- Person 3 worked for 120 minutes, at a rate represented by $c = 331t$, where c is the number of comments checked and t is the time in minutes.

- Person 4 worked for 150 minutes, at a rate represented by $t = \left(\frac{3}{800}\right)c$.

1. Order the people from greatest to least in terms of total number of comments checked.

2. Order the people from greatest to least in terms of how fast they checked the comments.

Student Response

- Person 2 checked a total of 53 000 comments, because $\frac{1\,325}{5} \times 200 = 53\,000$
- Person 1 checked a total of 50 000 comments.
- Person 4 checked a total of 40 000 comments, because $\frac{800}{3} \times 150 = 40\,000$.
- Person 3 checked a total of 39 720 comments, because $331 \times 120 = 39\,720$.
- Person 3 checked 331 comments per minute.
- Person 4 checked about 267 comments per minute, because $800 \div 3 = 266.\dot{6}$.
- Person 2 checked 265 comments per minute, because $1\,325 \div 5 = 265$.
- Person 1checked about 238 comments per minute, because $50\,000 \div 210 \approx 238$.

Are You Ready for More?

1. Write equations for each job applicant that allow you to easily decide who is working the fastest.

2. Make a table that allows you to easily compare how many comments the four job applicants can check.

Student Response

1. Person 1: $c = \frac{50\,000}{210}t \approx 238.1t$. Person 2: $c = \frac{1\,325}{5}t = 265t$. Person 3: $c = 331t$. Person 4: $c = \frac{800}{3}t \approx 266.7t$

person	time taken in minutes	comments per minute	total comments
1	210	$\frac{50\,000}{210} \approx 238.1$	50 000
2	200	$\frac{1\,325}{5} = 265$	53 000
3	120	331	39 720
4	150	$\frac{800}{3} \approx 226.7$	40 000

Activity Synthesis

Ask students which job applicant should get the job and why.

Representing, Conversing: Compare and Connect. Use this routine to prepare students for the whole-class discussion. At the appropriate time, invite students to create a visual display showing which job applicant should get the job and why. Allow students time to quietly circulate and analyse the selections and justifications in at least 2 other displays in the room. Give students quiet think time to consider what is the same and what is different and whether or not they agree. Next, ask students to find a partner to discuss what they noticed. Listen for and amplify observations that highlight advantages and disadvantages to each method of determining the top job applicant. This will help students identify when comparing rates is more effective than the sum of quantities.
Design Principle(s): Optimise output (for justification); Cultivate conversation

Lesson Synthesis

Whenever we have a situation involving constant rates, we are likely to have a proportional relationship between the quantities of interest.

- "What are some situations that we have seen where quantities were proportional to each other?"
- "When we are in a situation where we have a proportional relationship between two quantities, what information do we need to find an equation?"
- "How can we decide if a proportional relationship is a good representation of a particular situation?"
- "Equations are good tools to make predictions or decisions. When and how did we use an equation to make a prediction or a decision today?"

9.4 Steel Beams

Cool Down: 5 minutes

Student Task Statement

A steel beam can be cut to different lengths for a project. Assuming the weight of a steel beam is proportional to its length, what information would you need to know to write an equation that represents this relationship?

Student Response

Answers vary. Sample responses:

- Are the length of the steel beam and its weight in a proportional relationship?
- The weight of steel per 1 metre length of beam.

- The length of a steel beam that weighs 1 kilogram.
- The weight and length of a certain steel beam.

Student Lesson Summary

Whenever we have a situation involving constant rates, we are likely to have a proportional relationship between quantities of interest.

- When a bird is flying at a constant speed, then there is a proportional relationship between the flying time and distance flown.
- If water is filling a tub at a constant rate, then there is a proportional relationship between the amount of water in the tub and the time the tub has been filling up.
- If an aardvark is eating termites at a constant rate, then there is proportional relationship between the number of termites the aardvark has eaten and the time since it started eating.

Sometimes we are presented with a situation, and it is not so clear whether a proportional relationship is a good model. How can we decide if a proportional relationship is a good representation of a particular situation?

- If you aren't sure where to start, look at the quotients of corresponding values. If they are not always the same, then the relationship is definitely not a proportional relationship.
- If you can see that there is a single value that we always multiply one quantity by to get the other quantity, it is definitely a proportional relationship.

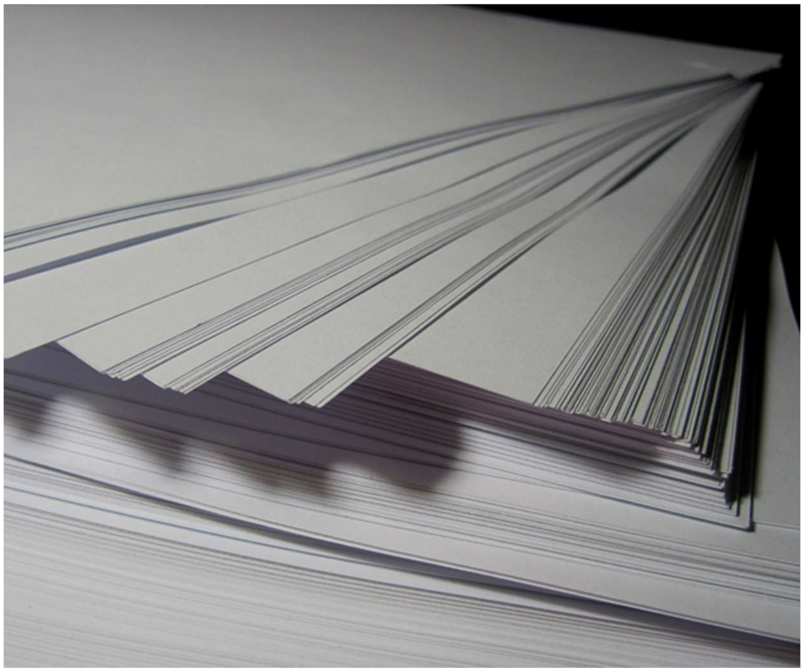
After establishing that it is a proportional relationship, setting up an equation is often the most efficient way to solve problems related to the situation.

Lesson 9 Practice Problems

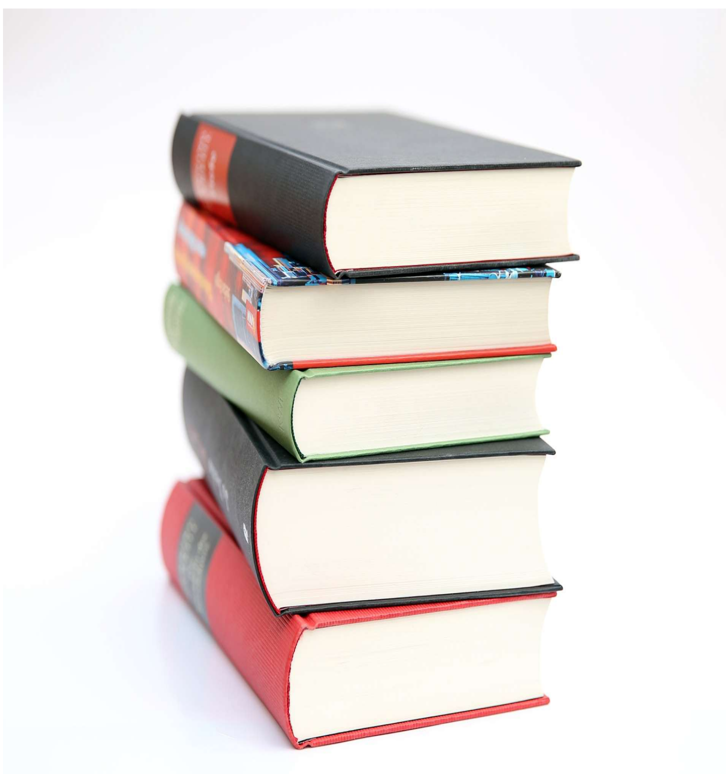
1. **Problem 1 Statement**

   For each situation, explain whether you think the relationship is proportional or not. Explain your reasoning.

   a. The weight of a stack of standard 8.5x11 copier paper vs. number of sheets of paper.



b. The weight of a stack of different-sized books vs. the number of books in the stack.



Solution

a. There is a proportional relationship between weight and number of sheets of paper. Each piece of paper has the same weight. To find the weight of a stack, multiply the number of sheets of paper by the weight of a single sheet of paper. (In case it comes up: We're assuming for this question that each piece of paper is the same weight. Manufacturing being what it is, though, we acknowledge that's not true.)

b. The relationship between the number of books and the weight of the stack is not proportional. Each book has a different weight, the weight of the stack can't be determined by multiplying the number of books by the weight of one book.

2. **Problem 2 Statement**

Every package of a certain toy also includes 2 batteries.

a. Are the number of toys and number of batteries in a proportional relationship? If so, what are the two constants of proportionality? If not, explain your reasoning.

b. Use t for the number of toys and b for the number of batteries to write two equations relating the two variables.

$b =$

$t =$

Solution

a. Yes. 2 and $\frac{1}{2}$ are the constants of proportionality

b. $b = 2t$ and $t = \frac{1}{2}b$

3. **Problem 3 Statement**

Lin and her brother were born on the same date in different years. Lin was 5 years old when her brother was 2.

a. Find their ages in different years by filling in the table.

Lin's age	Her brother's age
5	2
6	
15	
	25

b. Is there a proportional relationship between Lin's age and her brother's age? Explain your reasoning.

Solution

Lin's age	Her brother's age
5	2
6	3
15	12
28	25

a. There is no proportional relationship. Every year, they each age by one year, so the ratio of their ages changes every year.

4. **Problem 4 Statement**

A student argues that $y = \frac{x}{9}$ does not represent a proportional relationship between x and y because we need to multiply one variable by the same constant to get the other one and not divide it by a constant. Do you agree or disagree with this student?

Solution

Disagree. Dividing by 9 is the same as multiplying by $\frac{1}{9}$. We can look at the equation $y = \frac{x}{9}$ as $y = \frac{1}{9}x$. Also, $\frac{y}{x} = \frac{1}{9}$ is constant for all corresponding values of x and y.

5. **Problem 5 Statement**

Quadrilateral A has side lengths 3, 4, 5, and 6. Quadrilateral B is a scaled copy of Quadrilateral A with a scale factor of 2. Select **all** of the following that are side lengths of Quadrilateral B.

a. 5

b. 6

c. 7

d. 8

e. 9

Solution ["B", "D"]