Lesson 5: Say it with decimals

Goals

- Comprehend and use the term “repeating” (in spoken language) and the notation $1.\dot{3}$ (in written language) to refer to a decimal expansion that keeps having the same number over and over forever.
- Coordinate fraction and decimal representations of situations involving adding or subtracting a fraction of the initial value.
- Use long division to generate a decimal representation of a fraction, and describe (in writing) the decimal that results.

Learning Targets

- I can use the distributive property to rewrite an equation like $x + 0.5x = 1.5x$.
- I can write fractions as decimals.
- I understand that “half as much again” and “multiply by 1.5” mean the same thing.

Lesson Narrative

In this lesson students continue to study situations of fractional increase and decrease. They start to use decimal notation to express the situations. For example, they see that "one quarter less than x" can be expressed as $\frac{3}{4}x$ or as $0.75x$.

Addressing

- Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- Recognise and represent proportional relationships between quantities.

Building Towards

- Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- Collect and Display
- Clarify, Critique, Correct
- Discussion Supports
- Notice and Wonder

Required Preparation

Card Sort: More Representations Elena cycled x miles, and Noah cycled $\frac{2}{3}$ more than that.	Card Sort: More Representations $y = 1.\dot{6}x$
Card Sort: More Representations Noah saved x pounds, and Elena saved $\frac{1}{10}$ less than that.	Card Sort: More Representations $y = 0.9x$
Card Sort: More Representations Noah bought x pounds of apples, and Elena bought $\frac{1}{4}$ less than that.	Card Sort: More Representations $y = 0.75x$
Card Sort: More Representations Elena swam x lengths in the pool, and Noah swam $\frac{2}{5}$ less than that.	Card Sort: More Representations $y = 0.6x$
Card Sort: More Representations Elena ate x pretzels, and Noah ate $\frac{1}{3}$ more than that.	Card Sort: More Representations $y = 1.\dot{3}x$
Card Sort: More Representations Noah ran x kilometres, and Elena ran $\frac{3}{10}$ more than that.	Card Sort: More Representations $y = 1.3x$

Print and cut up slips from the More Representations Card Sort blackline master. Prepare 1 copy for every 2 students. These can be re-used if you have more than one class. Consider making a few extra copies that are not cut up to serve as an answer key.

Student Learning Goals

Let's use decimals to describe increases and decreases.

5.1 Notice and Wonder: Fractions to Decimals

Warm Up: 5 minutes

The purpose of this warm-up is to get students ready to think about decimal expansions of fractions. This will be useful in the next activity when students use long division to find decimal expansions of different fractions and find out why some repeat and others don't. In this activity, they are given calculator answers for different unit fractions (don't have to be unit fractions, could be any fractions) and they are starting to notice and verbalise different patterns. By the end of this activity they should be curious about why the decimal expansions of different fractions behave so differently.

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at a collection of decimals, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

A calculator gives the following decimal representations for some unit fractions:

$\frac{1}{2} = 0.5$

$\frac{1}{3} = 0.3333333$

$\frac{1}{4} = 0.25$

$\frac{1}{5} = 0.2$

$\frac{1}{6} = 0.1666667$

$\frac{1}{7} = 0.142857143$

$\frac{1}{8} = 0.125$

$\frac{1}{9} = 0.1111111$

$\frac{1}{10} = 0.1$

$\frac{1}{11} = 0.0909091$

What do you notice? What do you wonder?

Student Response

Things students may notice:

- Different fractions have different numbers of digits
- Some decimals repeat
- Some decimals don't repeat (not really, but that's what it looks like)
- Some decimals almost repeat, except for the last digit (0.0909091)

Things students may wonder:

- Do some decimals really finish after 7 digits or do they keep going?
- Why are some decimals shorter than others?
- Are there other fractions that only have one digit after the decimal?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If rounding does not come up during the conversation, ask students to discuss this idea.

5.2 Repeating Decimals

15 minutes

This activity introduces students to the term *repeating* for describing the decimal they get after using long division to convert a fraction. If desired, the term *terminating* can also be introduced. You may wish to use the term *recurring* instead/as well as repeating.

Instructional Routines

- Discussion Supports

Launch

Explain to students that we can use long division to calculate the decimal representation of a fraction. For example, $\frac{7}{8}$ is equal to $7 \div 8$.

Using long division, we can see that $\frac{7}{8}$ is equal to 0.875. This process works for any fraction.

Here is another example: $\frac{7}{12}$ is equal to $7 \div 12$.

In this case, the division will never result in a remainder of 0. Because we keep getting 3 over and over again, this is called a **repeating/recurring decimal** and can be written as $0.58\dot{3}$.

Arrange students in groups of 2. Give students 3–5 minutes of quiet think time on the first problem and 1–2 minutes to compare their responses and discuss the second question with their partner. Then give students 2–3 minutes of quiet think time on the remaining question. Follow with whole-class discussion.

Anticipated Misconceptions

Some students may set up their long division with the divisor and dividend in the wrong places. They will get $2.\dot{7}$, $2.\dot{7}\dot{2}$, and 2.75 as their answers. Prompt them to think about what is being divided and what it is being divided by.

Student Task Statement

1. Use **long division** to express each fraction as a decimal.

   $\frac{9}{25}$

   $\frac{11}{30}$

   $\frac{4}{11}$

2. What is similar about your answers to the previous question? What is different?

3. Use the decimal representations to decide which of these fractions has the greatest value. Explain your reasoning.

Student Response

1. $\frac{9}{25} = 0.36, \frac{11}{30} = 0.3\dot{6}, \frac{4}{11} = 0.\dot{3}\dot{6}$

2. Answers vary. Sample response: All 3 of these decimals have the same two numbers in the tenths and hundredths places, but $\frac{11}{30}$ and $\frac{4}{11}$ are both repeating decimals while $\frac{9}{25}$ is not.

3. $\frac{11}{30}$ is the largest because it has a 6 in the thousandths place when written as a decimal. $\frac{4}{11}$ has a 3 in the thousandths place and $\frac{9}{25}$ would have a 0 in the thousandths place.

Are You Ready for More?

One common approximation for π is $\frac{22}{7}$. Express this fraction as a decimal. How does this approximation compare to 3.14?

Student Response

3. $1.\dot{4}2857\dot{1}$ is closer to π than 3.14 is.

Activity Synthesis

The purpose of this discussion is to help students make sense of the value of repeating decimals. Ask students to explain the strategies they used to answer question 3. If no students mention place value, ask them to use this language to describe their strategies.

Speaking: Discussion Supports. To support students in explaining their reasoning for the last question, provide sentence frames for students to use when they are comparing fractions. For example, "___ is (larger/smaller) because ____." Revoice student ideas using mathematical language as necessary.
Design Principle(s): Support sense-making; Optimise output for (comparison)

5.3 More and Less with Decimals

15 minutes

This activity continues work done in the previous lesson that connected various representations of proportional relationships including images, equations, and descriptions. In this activity students match diagrams, descriptions, and equations that represent a proportional relationship, involving variables x and y. Students then create their own diagram to represent an equation.

Identify students that create diagrams representing each of the unmatched equations, so they can share their work during the discussion.

Instructional Routines

- Clarify, Critique, Correct

Launch

Give students 7–10 minutes of quiet think time on the questions. Follow with whole-class discussion.

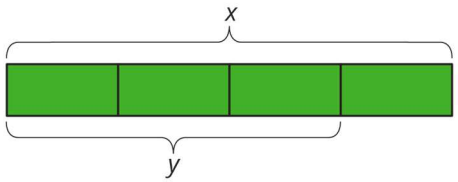
Representation: Internalise Comprehension. Begin the activity with concrete or familiar contexts. For example, display a bar model with concrete values for x and y such as 20 and 15 respectively. Ask students "How much did 20 decrease by to get to 15?" Highlight the connections between the concrete and abstract bar models. In the concrete bar model, 20 decreased by $\frac{1}{4}$ to get 15. In the abstract bar model, x also decreased by $\frac{1}{4}$ to get y.
Supports accessibility for: Conceptual processing; Memory

Student Task Statement

1. Match each diagram with a description and an equation.

   Diagrams:

A



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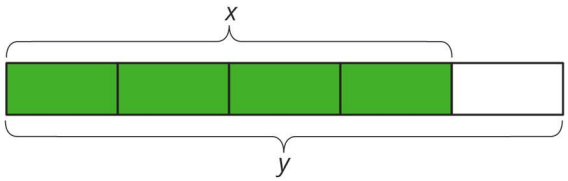


Descriptions:

An increase by $\frac{1}{4}$

An increase by $\frac{1}{3}$

An increase by $\frac{2}{3}$

A decrease by $\frac{1}{5}$

A decrease by $\frac{1}{4}$

Equations:

$y = 1.\dot{6}x$

$y = 1.\dot{3}x$

$y = 0.75x$

$y = 0.4x$

$y = 1.25x$

2. Draw a diagram for one of the unmatched equations.

Student Response

1. Diagram A can match to an increase by $\frac{1}{3}$ or a decrease by $\frac{1}{4}$. Diagram A matches to $y = 0.75x$. Diagram B can match to an increase by $\frac{1}{4}$ or a decrease by $\frac{1}{5}$. Diagram B matches to $y = 1.25x$.
2. Answers vary.

Activity Synthesis

Ask previously identified students to share their diagrams and explain how they represent the equation they chose. Then, ask students that did not share their diagrams to explain whether x (or y) is increased or decreased by some amount in the discussed equations.

Representing, Writing: Clarify, Critique, Correct. Present an incorrect diagram to represent one of the unused equations that reflects a possible misunderstanding from the class. For example, for the equation $y = 0.75x$, draw a diagram where y is greater than x. Prompt students to identify the error, and then write a correct diagram to represent the equation. This will support students to understand the relationship between equations and diagrams.
Design Principle(s): Maximise meta-awareness

5.4 Card Sort: More Representations

Optional: 10 minutes

This activity continues the card sort from the previous lesson by including a fourth representation for each relationship, an equation with a decimal instead of a fraction.

Instructional Routines

- Collect and Display

Launch

Remind students of card sort activity of the earlier lesson. Demonstrate how to set up and do the matching activity. Choose a student to be your partner. Mix up the rest of the cards and place them face-up. Point out that each card contains a description, table, or equation (with fractions or decimals). Select one of each style of card and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree (e.g., by explaining your mathematical thinking, asking clarifying questions, etc.).

Arrange students in groups of 2. Give each group pre-printed cut-up slips for matching. Place two copies of uncut blackline masters in envelopes to serve as answer keys.

Representation: Internalise Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual processing; Organisation Representing, Conversing: Collect and Display. As students work, circulate and listen to the language students use to justify their matches. Write down common or important phrases you hear students say as they match each representation, and display for all to see. Remind students to borrow language from the display as needed, and continue to update the display during the whole-class discussion.
Design Principle(s): Support sense-making; Optimise output (for justification)

Student Task Statement

Your teacher will give you a set of cards that have proportional relationships represented 2 different ways: as descriptions and equations. Mix up the cards and place them all face-up.

Take turns with a partner to match a description with an equation.

1. For each match you find, explain to your partner how you know it's a match.
2. For each match your partner finds, listen carefully to their explanation, and if you disagree, explain your thinking.
3. When you have agreed on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.

Student Response

The blackline master shows the correct matches.

Activity Synthesis

After students complete the task, ask students to share a few of their arguments for the matches they came up with (not all need to be shared). Ask students to compare the equations with decimals and fractions.

Lesson Synthesis

This lesson was similar to the last one, except we focused on writing things with decimals.

"Give examples of how we can use the distributive property to create equivalent expressions that make it easier for us to calculate an amount plus (or minus) a fraction of that amount, but written with decimals" (e.g. $100x + 12x = 112x$)

5.5 Reading More

Cool Down: 5 minutes

Student Task Statement

Kiran read for x minutes, and Andre read for $\frac{5}{8}$ more than that. Write an equation that relates the number of minutes Kiran read with y, the number of minutes that Andre read. Use decimals in your equation.

Student Response

$y = 1.625x$ or equivalent. Andre read $\frac{5}{8}x = 0.625x$ more minutes than Kiran read. $x + 0.625x = 1.625x$. $y = 1.625x$

Student Lesson Summary

Long division gives us a way of finding decimal representations for fractions.

For example, to find a decimal representation for $\frac{9}{8}$, we can divide 9 by 8.

So $\frac{9}{8} = 1.125$.

Sometimes it is easier to work with the decimal representation of a number, and sometimes it is easier to work with its fraction representation. It is important to be able to work with both. For example, consider the following pair of problems:

- Priya earned x pounds doing chores, and Kiran earned $\frac{6}{5}$ as much as Priya. How much did Kiran earn?
- Priya earned x pounds doing chores, and Kiran earned 1.2 times as much as Priya. How much did Kiran earn?

Since $\frac{6}{5} = 1.2$, these are both exactly the same problem, and the answer is $\frac{6}{5}x$ or $1.2x$. When we work with percentages in later lessons, the decimal representation will come in especially handy.

Glossary

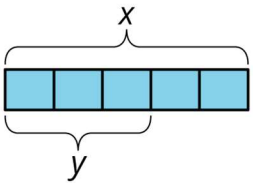
- long division
- repeating decimal

Lesson 5 Practice Problems

Problem 1 Statement

a. Match each diagram with a description and an equation.

A



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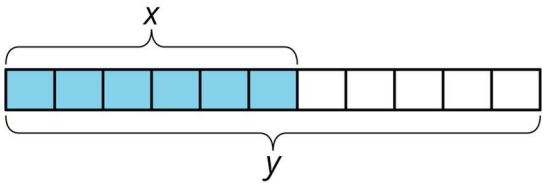

Descriptions:

An increase by $\frac{2}{3}$

An increase by $\frac{5}{6}$

A decrease by $\frac{2}{5}$

A decrease by $\frac{5}{11}$

Equations:

$y = 1.8\dot{3}x$

$y = 1.\dot{6}x$

$y = 0.6x$

$y = 0.4x$

b. Draw a diagram for one of the unmatched equations.

Solution

a. Diagram A: A decrease by $\frac{2}{5}$ and $y = 0.6x$
Diagram B: An increase by $\frac{5}{6}$ and $y = 1.8\dot{3}x$

b. Answers vary.

Problem 2 Statement

At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Since then, the family ate 0.3 of the peanut butter. How many ounces of peanut butter are in the pantry now?

a. 0.7×80

b. 0.3×80

c. $80 - 0.3$

d. $(1 + 0.3) \times 80$

Solution A

Problem 3 Statement

a. On a hot day, a football team drank an entire 50-gallon cooler of water and half as much again. How much water did they drink?

b. Jada has 12 library books checked out and Han has $\frac{1}{3}$ less than that. How many books does Han have checked out?

Solution

a. 75 gallons

b. 8 books

Problem 4 Statement

If x represents a positive number, select **all** expressions whose value is greater than x.

a. $\left(1-\frac{1}{4}\right)x$

b. $\left(1+\frac{1}{4}\right)x$

c. $\frac{7}{8}x$

d. $\frac{9}{8}x$

Solution ["B", "D"]

Problem 5 Statement

A person's resting heart rate is typically between 60 and 100 beats per minute. Noah looks at his watch, and counts 8 heartbeats in 10 seconds.

a. Is his heart rate typical? Explain how you know.

b. Write an equation for h, the number of times Noah's heart beats (at this rate) in m minutes.

Solution

a. No. Noah's heart rate is 48 beats per minute, because $10 \times 6 = 60$, and $8 \times 6 = 48$.

b. $h = 48m$