Lesson 7: More expressions and equations

Goals

- Create expressions and equations to represent a linear relationship with two or more related quantities in context.
- Determine the solution to an equation of the form $px + q = r$ and explain (orally) the solution method.
- Interpret expressions, equations, and solutions that represent a linear relationship with two or more related quantities.

Lesson Narrative

This lesson is optional. The activities in this lesson can all be solved using year 8 mathematics, but are more sophisticated than earlier activities and are often left for future years when students have access to a wider variety of algebraic tools.

Each of the situations described in the activities involve two or more unknown quantities and multiple relationships or actions. Initially students are walked through the steps of writing an expression to describe a situation, using properties to rewrite the expression with fewer terms, writing an equation to represent the situation, solving the equation, and considering the reasonableness of solutions. As they proceed through the lesson, the supports are slowly removed and students work on their own to reason through the problem.

As with all lessons in this unit, all related topics have been addressed in prior units. This lesson provides an *optional* opportunity to go more deeply into working with expressions and equations.

Addressing

- Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- Compare and Connect
- Discussion Supports
- Think Pair Share

Student Learning Goals

Let's solve harder problems by writing equivalent expressions.

7.1 Tickets for the School Play

Optional: 15 minutes

This activity walks students through the process of defining a variable, writing an expression, writing the expression with fewer terms, estimating a reasonable solution, computing a solution, and finally checking that the solution makes sense and is correct. Note that there are two unknown quantities (prices for student and adult tickets) and students are guided to express one in terms of the other.

Monitor for students who write the expressions different ways in the first three questions and invite them to share during the discussion. There are many possible correct answers, but some forms will lead to equations that are easier to solve (i.e. $px + q$) and some will be harder to solve.

Launch

If your school or a nearby school has recently performed a play, consider asking if any students went to see it and have them briefly describe the experience. Alternatively, display photos from any school play, including images of the tickets or ticket booth. Invite students to share what they notice and what they wonder.

Give students 5 minutes of quiet work time followed by a whole-class discussion.

Anticipated Misconceptions

Students might write the expression for the cost of a student ticket as $2 - a$ instead of $a - 2$ because they confuse how to represent "2 less than" a quantity. Provide these students with values for the price of an adult ticket, and have them find the price of a student ticket. Help them see the structure of subtracting 2 from the cost of an adult ticket and not the other way around.

For students struggling with the last two questions, remind them that they know how to write expressions with fewer terms, and that they know how to solve equations of the forms $p(x + q) = r$ and $px + q = r$. Support them in grappling with how these concepts and skills can come together to help them solve problems.

Student Task Statement

Student tickets for the school play cost £2 less than adult tickets.

1. If a represents the price of one adult ticket, write an expression for the price of a student ticket.

2. Write an expression that represents the amount of money they collected each night:

   a. The first night, the school sold 60 adult tickets and 94 student tickets.

   b. The second night, the school sold 83 adult tickets and 127 student tickets.

3. Write an expression that represents the total amount of money collected from ticket sales on both nights.

4. Over these two nights, they collected a total of £1 651 in ticket sales.

   a. Write an equation that represents this situation.

   b. What was the cost of each type of ticket?

5. Is your solution reasonable? Explain how you know.

Student Response

1. $a - 2$

2.

   a. $60a + 94(a - 2)$ (or $154a - 188$)

   b. $83a + 127(a - 2)$ (or $210a - 254$)

3. $364a - 442$ or equivalent

4.

   a. $364a - 442 = 1651$

   b. £5.75 adult, £3.75 student

5. Answers vary. Sample response: Yes, it's reasonable that tickets to the school play would cost somewhere between £0 and £10 and also that the adult tickets would cost most than the students tickets. I can show it is correct, because $5.75(60 + 83) + 3.75(94 + 127) = 822.25 + 828.75 = 1651$.

Activity Synthesis

The purpose of the discussion is for students to reflect on the problem solving process. Consider asking the following questions:

- "Why was the price of an adult ticket chosen as the variable?" (This was an arbitrary choice.)
- "Could the problem be worked by choosing the price of a student ticket as the variable? How would the expressions be different? The equation? The solution?" (Yes, instead of -2 we would have +2 in the expression. The solution to the equation would be 3.75 instead of 5.75.)
- "What does each term in the expressions you wrote represent?"

- "How did you find the price of a student ticket?"
- "How did you check that your solution is correct?"

7.2 A Souvenir Stand

Optional: 15 minutes

In this activity students continue to be guided through a solution process, but with fewer supports than in the previous activity. There are now three unknown quantities and students are asked to write an expression for their total, but students are not specifically guided to think about expressions for each individual quantity.

Monitor for students who compute the profit in different ways, such as:

- finding the total income and total cost, and then subtracting them
- finding the profit for each item, and then adding them together

Instructional Routines

- Discussion Supports

Launch

Give students 6–7 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

The souvenir stand sells hats, postcards, and magnets. They have twice as many postcards as hats, and 100 more magnets than post cards.

1. Let h represent the total number of hats. Write an expression in terms of h for the total number of items they have to sell.
2. The owner of the stand pays £8 for each hat, £0.10 for each post card, and £0.50 for each magnet. Write an expression for the total cost of the items.
3. The souvenir stand sells the hats for £11.75 each, the postcards for £0.25 each, and the magnets for £3.50 each. Write an expression for the total amount of money they would take in if they sold all the items.
4. Profits are calculated by subtracting costs from income. Write an expression for the profits of the souvenir stand if they sell all the items they have. Use properties to write an equivalent expression with fewer terms.
5. The souvenir stand sells all these items and makes a total profit of £953.25.
    a. Write an equation that represents this situation.
    b. How many of each item does the souvenir stand sell? Explain or show your reasoning.

Student Response

1. $h + 2h + (2h + 100)$
2. $8h + 0.10(2h) + 0.50(2h + 100)$ (or $9.2h + 50$)
3. $11.75h + 0.25(2h) + 3.50(2h + 100)$ (or $19.25h + 350$)
4. $10.05h + 300$
5.
    a. $10.05h + 300 = 953.25$
    b. $h = 65$. 65 hats, 130 postcards, 230 magnets. $10.05h + 300 = 953.25$, $10.05 = 653.25$, $h = 65$

Activity Synthesis

Invite students to share their answers to the last two questions (and others, if they wish). Consider asking questions like the following:

- "Is it a good idea to check your solution by substituting values into the expressions and equations you wrote?" (No, you might have written or simplified them incorrectly. The best option is to go back to the original problem)
- "Why was the number of hats chosen as the unknown quantity to represent with a variable? Would you have chosen differently? How would that change the solution process?" (This is just one choice. We could make a different one.)
- "How did you calculate the profit?" (Calculate the total income minus total cost for all items, or add item by item.)
- "I saw that many of you wrote this expression $8h + 0.10(2h) + 0.50(2h + 100)$. What is the meaning of the different parts of the expressions in this situation?" ($8h$ is the cost for the hats, $0.10(2h)$ is the cost for the postcards, and $0.50(2h + 100)$ is the cost for the magnets.)

Representation: Internalise Comprehension. Use colour and annotations to illustrate connections between representations. As students share their diagrams and reasoning, use colour and annotations to scribe their thinking on a display of each problem so that it is visible for all students.
Supports accessibility for: Visual-spatial processing; Conceptual processing Representing, Reading: Discussion Supports. When students explain how they wrote their expressions, provide sentence frames such as "___ represents ___ because . . . ." Encourage students to consider what details are important to share and to think about how they will explain their reasoning using mathematical language. This will help students to explicitly connect the language of the problem with the structure of the expressions that represent the context.
Design Principle(s): Maximise meta-awareness, Support sense-making

7.3 Jada Crochets a Scarf

Optional: 15 minutes

In this activity, most of the supports for the solution process are removed. Students need to think about how to choose which quantity to represent with a variable, how to represent the other two quantities with expressions in terms of the variable, and how to write an expression for a total. The only guidance offered is the reminder to write the expression with as few terms as possible.

While students are working, monitor for students who choose to have their variable represent different quantities (e.g. number of single stitches, double stitches, or triple stitches).

Instructional Routines

- Compare and Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 6–7 minutes of quiet work time followed by partner and whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts. After students have solved the first two problems, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.
Supports accessibility for: Organisation; Attention

Student Task Statement

Basic crochet stitches are called single, double, and triple. Jada measures her average stitch size and sees that a "double crochet" stitch is not really twice as long; it uses $\frac{1}{2}$ inch less than twice as much yarn as a single crochet stitch. Jada's "triple crochet" stitch uses 1 inch less than three times as much yarn as a single crochet stitch.

1. Write an expression that represents the amount of yarn Jada needs to crochet a scarf that includes 800 single crochet stitches, 400 double crochet stitches, and 200 triple crochet stitches.
2. Write an equivalent expression with as few terms as possible.
3. If Jada uses 5 540 inches of yarn for the entire scarf, what length of yarn does she use for a single crochet stitch?

Student Response

1. Answers vary. Sample response: Let x represent the length of yarn needed for a single crochet stitch. $800x + 400(2x - 12) + 200(3x - 1)$

2. $2\,200x - 400$

3. 2.7 inches of yarn for a single crochet stitch. $2\,200x - 400 = 5\,540$, $2\,200x = 5\,940$, $x = 2.7$

Activity Synthesis

Display the following questions for all to see and have students discuss them with their partner:

- "How did you define your variable? Why did you make this choice?"
- "How did you come up with your expression for the total amount of yarn? Explain what each part of your expression represents."
- "What decisions did you make while rewriting the expression with fewer terms?"
- "How did you, or could you, check your solution?"

After students have discussed with their partner, invite students to share with the class anything interesting they noticed while comparing with their partner. For example, did they both write the same exact expression, or did they write different but equivalent expressions? Does anyone think their partner had a particularly creative strategy for solving the problem?

If any students chose their variable to represent the number of double or triple crochet stitches, invite them to explain their solution strategy to the class and discuss how it is the same and different from having the variable represent the number of single crochet stitches.

Representing, Writing: Compare and Connect. As students share their solution strategies, invite pairs to look for similarities and differences between the approaches presented. Invite students to discuss "What is the same and what is different?" about how students defined their variable and wrote their expressions. Ask students to make connections between how the expressions relate to how you define the variable of the expression (e.g., "What does the variable mean in this expression compared to that expression?"). This will help students understand the importance of clearly defining the variable so their expressions accurately represent the situation.
Design Principle(s): Maximise meta-awareness; Support sense-making