| Grades 9-12 (AS)

Duration: 20 min

Tools: one Logifaces Set / group

Group work

Keywords: Transformations, Rotation around a line, Reflection in a plane, Translation | 532 - Rotating Blocks in GeoGebra

◿◺ △

MATHS / TRANSFORMATIONS | **TEACHER**
Logifaces

Erasmus+

2019-1-HU01-KA201-0612722019-1 |

DESCRIPTION

Students' task is to start with a Logifaces block of truncated prism shape drawn in GeoGebra and move it by transformations to another given target position. See exercises 526 - Calculate the Coordinates for the particular coordinates and 527 - Coordinates in GeoGebra for the drawings in GeoGebra.

This table shows the coordinates of the top vertices when the base vertices have the coordinates $(0, 0, 0)$, $(4, 0, 0)$, $(2, 2\sqrt{3}, 0)$ in each case.

Block			
112	$(0, 0, 1),\ (4, 0, 1), (2, 2\sqrt{3}, 2)$	$(0, 0, 1),\ (4, 0, 2), (2, 2\sqrt{3}, 1)$	$(0, 0, 2),\ (4, 0, 1), (2, 2\sqrt{3}, 1)$
122	$(0, 0, 1), (4, 0, 2),\ (2, 2\sqrt{3}, 2)$	$(0, 0, 2),\ (4, 0, 2), (2, 2\sqrt{3}, 1)$	$(0, 0, 2),\ (4, 0, 1), (2, 2\sqrt{3}, 2)$
223	$(0, 0, 2),\ (4, 0, 2), (2, 2\sqrt{3}, 3)$	$(0, 0, 2),\ (4, 0, 3), (2, 2\sqrt{3}, 2)$	$(0, 0, 3),\ (4, 0, 2), (2, 2\sqrt{3}, 2)$
233	$(0, 0, 2),\ (4, 0, 3), (2, 2\sqrt{3}, 3)$	$(0, 0, 3),\ (4, 0, 3), (2, 2\sqrt{3}, 2)$	$(0, 0, 3),\ (4, 0, 2), (2, 2\sqrt{3}, 3)$
113	$(0, 0, 1),\ (4, 0, 1), (2, 2\sqrt{3}, 3)$	$(0, 0, 1),\ (4, 0, 3), (2, 2\sqrt{3}, 1)$	$(0, 0, 3),\ (4, 0, 1), (2, 2\sqrt{3}, 1)$
133	$(0, 0, 1),\ (4, 0, 3), (2, 2\sqrt{3}, 3)$	$(0, 0, 3),\ (4, 0, 3), (2, 2\sqrt{3}, 1)$	$(0, 0, 3),\ (4, 0, 1), (2, 2\sqrt{3}, 3)$
123	$(0, 0, 1),\ (4, 0, 2), (2, 2\sqrt{3}, 3)$	$(0, 0, 2),\ (4, 0, 3), (2, 2\sqrt{3}, 1)$	$(0, 0, 3),\ (4, 0, 1), (2, 2\sqrt{3}, 2)$
132	$(0, 0, 1),\ (4, 0, 3), (2, 2\sqrt{3}, 2)$	$(0, 0, 3),\ (4, 0, 2), (2, 2\sqrt{3}, 1)$	$(0, 0, 2),\ (4, 0, 1), (2, 2\sqrt{3}, 3)$

LEVEL 1   Start: any given coordinates in 526 - Calculate the Coordinates (see the table above), target position: any other given coordinates of the same block.

LEVEL 2   Start: any given coordinates of block 123 (or 132) in 526 - Calculate the Coordinates (see the table above), target position: any other given coordinates of the block 132 (or 123).

HINT It is enough to find transformations in the plane that transform an equilateral triangle with labelled vertices into a congruent triangle at the same position, but with permuted labels at the vertices. Then the 3 dimensional equivalents of the transformations give the solution.

SOLUTIONS / EXAMPLES

We present one example for a possible solution for one block. The solutions for the other blocks are similar.

Note on the use of GeoGebra:
In GeoGebra, it is impossible yet to rotate or translate a truncated prism as a whole. Instead, one can divide a truncated prism into a regular prism and a pyramid (see exercise 527 - Coordinates in GeoGebra) and perform the transformations on these two polyhedra separately. In this exercise only the pyramid needs to be transformed, because the base face is the same in the start and target location.
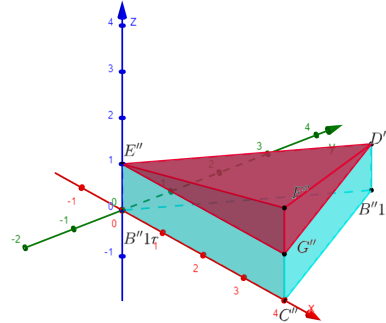
LEVEL 1

Start: block 112 with top vertices:
$(0, 0, 1),\ (4, 0, 1), (2, 2\sqrt{3}, 2)$
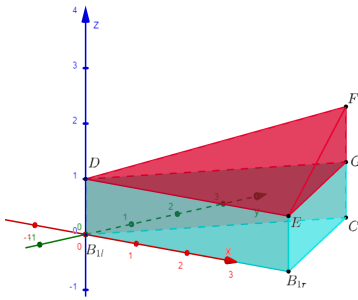
Target: block 112 with top vertices:
$(0, 0, 1),\ (4, 0, 2), (2, 2\sqrt{3}, 1)$

FIRST SOLUTION

1. Rotation around the z-axis by -120°.

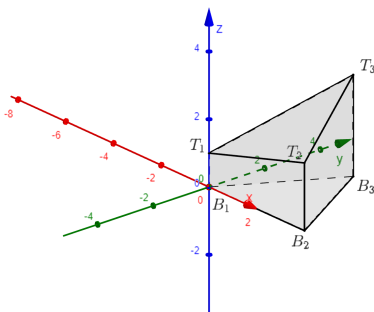2. Translation by the vector $\overrightarrow{B'_{1r}B'_{1l}}$.

SECOND SOLUTION

1. Draw the centre M of the base triangle by connecting each vertex with the midpoint of the opposite edge, and taking the intersection of these lines.

2. Draw a line perpendicular to the plane xy through the point M.
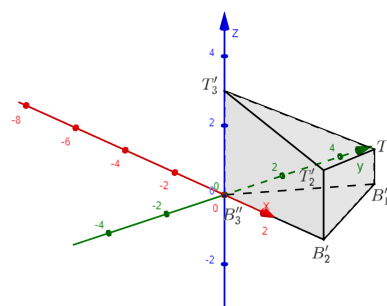
3. Rotation around the line by 120°.

LEVEL 2

Start: block 123 with top vertices:
$(0, 0, 1),\ (4, 0, 2), (2, 2\sqrt{3}, 3)$

Target: block 132 with top vertices:
$(0, 0, 3),\ (4, 0, 2), (2, 2\sqrt{3}, 1)$

SOLUTION

1. Draw a plane through the points B_2 and T_2 and the midpoint of the segment B_1B_3.
2. Reflection of the truncated prism (or just the pyramid part of the truncated prism) in this plane.

PRIOR KNOWLEDGE

Reflection in a plane, Rotation around an axis, Transformations in GeoGebra

RECOMMENDATIONS / COMMENTS

Exercises 526 - Calculate the Coordinates and 527 - Coordinates in GeoGebra are recommended before this exercise.