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2) Eixos Trocados

$x^2+z^2=9$

$\hookrightarrow r^2\cos^2\theta + z^2 = 9 \quad \nexists$

$x = r\cos\theta$

$y = y \qquad \Rightarrow r^2\cos^2\theta + r^2 sen^2\theta = 9$

$z = r sen\theta \qquad \boxed{r^2 = 9}$

3) Denominadores

$\frac{x^2}{4} + \frac{y^2}{9} = 1 \rightarrow \frac{4r^2\cos^2\theta}{4} + \frac{9r^2 sen^2\theta}{9} = 1$

$x = 2r\cos\theta \qquad = \frac{x^2}{4} + \frac{y^2}{3} = r^2$

$y = 3r sen\theta$

Resolução Exercícios 14.6.

a) Use coordenadas cilíndricas para encontrar o volume do sólido.
O sólido compreendido pelo parabolóide $z = x^2+y^2$ e o plano $z = 9$.

$z = r^2 \qquad 0 \leq r \leq 3$ ← Pq não existe raio negativo

$z = 9 \qquad 0 \leq \theta \leq 2\pi$

$r^2 \leq z \leq 9$

$r^2 = 9$

$r = \pm 3 \quad V = \int_0^{2\pi}\int_0^3\int_{r^2}^9 r \, dz\, dr\, d\theta$

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$$\int_{r^2}^{9} r\,dz = rz\Big|_{r^2}^{9} = r(9-r^2)$$

$$\int_0^3 9r-r^3\,dr = \frac{9r^2}{2}-\frac{r^4}{4}\Big|_0^3 = \left(\frac{9.3^2}{2}-\frac{3^4}{4}\right)-0$$

$$\frac{81}{2}-\frac{81}{4} = \frac{81}{4}$$

$$\int_0^{2\pi} \frac{81}{4}\,d\theta = \frac{81\theta}{4}\Big|_0^{2\pi} = \boxed{\frac{81\pi}{2}}$$

coordenadas esféricas

13) O sólido limitado acima pela esfera $\rho=4$ e abaixo pelo cone $\phi=\frac{\pi}{3}$

$$\int_0^{2\pi}\int_0^{\pi/3}\int_0^4 \rho^2 \operatorname{sen}\phi \; d\rho\, d\phi\, d\theta$$

$0 \leq \rho \leq 4$
$0 \leq \phi \leq \pi/3$
$0 \leq \theta \leq 2\pi$

$$\int_0^4 \rho^2 \operatorname{sen}\phi\,d\rho = \frac{\rho^3}{3}\operatorname{sen}\phi\Big|_0^4 = \frac{4^3}{3}\operatorname{sen}\phi$$

$$\int_0^{\pi/3} \frac{64}{3}\operatorname{sen}\phi\,d\phi = \frac{64}{3}-\cos\phi\Big|_0^{\pi/3} = \frac{64}{3}\left(-\cos\frac{\pi}{3}-(-\cos 0)\right)$$

$$\frac{64}{3}\left(-\frac{1}{2}+1\right) = \frac{32}{3}$$

$$\int_0^{2\pi} \frac{32}{3}\,d\theta = \frac{32\theta}{3}\Big|_0^{2\pi} = \boxed{\frac{64\pi}{3}}$$

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1) a) A cunha cilíndrica $1 \leq r \leq 3, \pi/6 \leq \theta \leq \pi/2, 0 \leq z \leq 5$ tem volume $V = ?$

$$\int_{\pi/6}^{\pi/2}\int_{1}^{3}\int_{0}^{5} 1 \, dz\, dr\, d\theta$$

$$\int_0^5 1\, dz = 5 \rightarrow \int_1^3 5\, dr = 5r\Big|_1^3 = 5(3-1) = 10$$

$$\int_{\pi/6}^{\pi/2} 10\, d\theta = 10\theta\Big|_{\pi/6}^{\pi/2} \quad 10\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \boxed{\frac{20\pi}{3}}$$

2) Seja G a região sólida dentro da esfera de raio 2 centrada na origem e acima do plano $z = 1$. Em cada parte, preecha as lacunas com o integrando e os extremos de integração que faltam nas integrais iteradas em coordenadas cilíndricas

a) O volume de G é: ?

$x^2 + y^2 + z^2 = 4$

$z^2 = 4 - x^2 - y^2$ $\qquad 1 \leq z \leq \sqrt{4-r^2}$

$z = \sqrt{4 - (x^2+y^2)}$ $\qquad 0 \leq r \leq 3$

$z = \sqrt{4 - r^2}$ $\qquad 0 \leq \theta \leq 2\pi$

$1^2 = (\sqrt{4-r^2})^2$ $\qquad V = \int_0^{2\pi}\int_0^{3}\int_1^{\sqrt{4-r^2}} 1\, dz\, dr\, d\theta$

$1 = 4 - r^2$

$1 - 4 = -r^2$

$-3 = -r^2$

$r^2 = 3 \qquad r = 3$

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1) Encontre a massa e o centro de gravidade da lâmina.

1) Uma lâmina com densidade $\delta(x,y) = x+y$ limitada pelo eixo x das abcissas, a reta $x=1$ e a curva $y=\sqrt{x}$.

$$M = \iint_R \delta(x,y)\, dA = \int_0^1 \int_0^{\sqrt{x}} x+y \; dy\, dx$$

$$\int_0^{\sqrt{x}} x+y \; dy = xy + \frac{y^2}{2} \Big|_0^{\sqrt{x}} = x.\sqrt{x} + \frac{(\sqrt{x})^2}{2} - 0$$

$$\boxed{x\sqrt{x} + \frac{x}{2}}$$

$$\int_0^1 x^{3/2} + \frac{x}{2}\, dx = \int_0^1 x^{3/2}\, dx + \frac{1}{2}\int_0^1 x\, dx$$

$$\frac{2x^{5/2}}{5} + \frac{1}{2}\,\frac{x^2}{2} = \frac{2}{5}x^{5/2} + \frac{x^2}{4} \Big|_0^1 =$$

$$\left(\frac{2}{5}1^{5/2} + \frac{1^2}{4}\right) - 0 = \boxed{\frac{13}{20}}$$

$$M_x = \int_0^1 \int_0^{\sqrt{x}} (x+y)\, y\, dy\, dx$$

$$\int_0^{\sqrt{x}} xy + y^2\, dy = \frac{xy^2}{2} + \frac{y^3}{3} \Big|_0^{\sqrt{x}} = x\frac{(\sqrt{x})^2}{2} + \frac{(\sqrt{x})^3}{3} =$$

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$$\frac{x^2}{2} + \frac{(x^{1/2})^3}{3} = \frac{x^2}{2} + \frac{x^{3/2}}{3}$$

$$\int_0^1 \frac{x^2}{2} + \frac{x^{3/2}}{3}\, dx = \frac{1}{2}\int_0^1 x^2 dx + \frac{1}{3}\int_0^1 x^{3/2} dx$$

$$\frac{1}{2}\,\frac{x^3}{3}\Big|_0^1 = \frac{1}{6} - 0 = \frac{1}{6}$$

$$\frac{1}{3}\,\frac{x^{5/2}}{5/2} = \frac{2x^{5/2}}{15}\Big|_0^1 = \frac{2}{15}\,1^{5/2} - 0 = \frac{2}{15} = \boxed{\frac{3}{10}}$$

$$m_y = \int_0^1 \int_0^{\sqrt{x}} (x+y)x\, dy\, dx$$

$x^{1/2}$ x^2

$$\int_0^{\sqrt{x}} x^2 + yx\, dy = x^2 y + \frac{y^2}{2}x \Big|_0^{\sqrt{x}} = x^2 \cdot \sqrt{x} + \frac{(\sqrt{x})^2 \cdot x}{2} - 0$$

$$x^{5/2} + \frac{x^2}{2}$$

$$\int_0^1 x^{5/2} + \frac{x^2}{2}\, dx = \int_0^1 x^{5/2} dx + \frac{1}{2}\int_0^1 x^2 dx$$

$$\frac{2x^{7/2}}{7}\Big|_0^1 = \frac{2}{7} + \frac{1}{2}\,\frac{x^3}{3}\Big|_0^1 = \frac{1}{6} - 0 = \frac{1}{6}$$

$$\frac{2}{7} + \frac{1}{6} = \boxed{\frac{19}{42}} \qquad \bar{x} = \frac{m_y}{m} = \frac{\frac{19}{42}}{\frac{13}{20}} = \frac{19}{42} \cdot \frac{20}{13} = \frac{190}{273}$$

$$\bar{y} = \frac{m_x}{m} = \frac{3}{10} \cdot \frac{20}{13} = \frac{6}{13}$$

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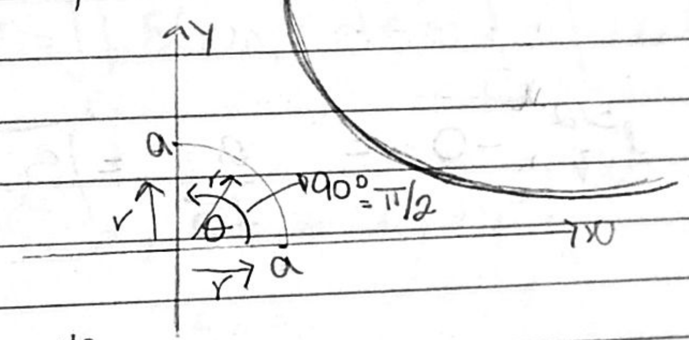
$m = \frac{13}{20}$, centro de grav. $\left(\frac{190}{273} ; \frac{6}{13} \right)$

3) Uma lâmina com densidade $\delta(x,y) = xy$ localizada no primeiro quadrante e limitada pelo círculo $x^2+y^2=a^2$ e os eixos de coordenadas.



$r = a$

$0 \leq \theta \leq \pi/2$

$x = r\cos\theta$

$y = r\,\text{sen}\,\theta$

$$m = \int_0^{\pi/2}\int_0^{a} r\cos\theta \cdot r\,\text{sen}\,\theta \cdot r\,dr\,d\theta$$

$$\int_0^{a} r^3\cos\theta\,\text{sen}\,\theta\,dr = \frac{r^4\cos\theta\,\text{sen}\,\theta}{4}\Big|_0^a = \frac{a^4\cos\theta\,\text{sen}\,\theta}{4}$$

$$\int_0^{\pi/2} \frac{a^4}{4}\cos\theta \cdot \text{sen}\,\theta\,d\theta = \frac{a^4}{4}\int_0^{\pi/2} \overbrace{\cos\theta\,\underbrace{\text{sen}\,\theta}_{u}\,d\theta}^{du}$$

$u = \text{sen}\,\theta$

$du = \cos\theta\,d\theta$

$$\frac{a^4}{4}\int_0^{\pi/2} u\,du = \frac{a^4}{4}\frac{u^2}{2} = \frac{a^4\,\text{sen}^2\theta}{8}\Big|_0^{\pi/2} \quad \frac{a^4}{8}\,\text{sen}^2\frac{\pi}{2} - \text{sen}^2 0$$

$\boxed{\frac{a^4}{8}}$

$\bar{x} = \bar{y}$ simetria de densidade e região

$$m_y = \int_0^{\pi/2}\int_0^{a} r^3\cos\theta\,\text{sen}\,\theta \cdot r\cos\theta\,dr\,d\theta$$

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$$\int_0^a r^4 \cos^2\theta \,\text{sen}\,\theta \, dr$$

$$\frac{r^5}{5}\cos^2\theta \,\text{sen}\,\theta \Big|_0^a = \frac{a^5}{5}\cos^2\theta \,\text{sen}\,\theta$$

$$\int_0^{\pi/2} \frac{a^5}{5}\cos^2\theta \,\text{sen}\,\theta \, d\theta = \frac{a^5}{5}\int_0^{\pi/2} \underbrace{\cos^2\theta}_{u} \overbrace{\text{sen}\,\theta \, d\theta}^{-du}$$

$u = \cos\theta$

$du = -\text{sen}\,\theta \, d\theta$

$-du = \text{sen}\,\theta \, d\theta$

$$\frac{a^5}{5}\int_0^{\pi/2} -u^2 . \, du = -\frac{a^5}{5} \frac{u^3}{3}\Big|_0^{\pi/2}$$

$$= -\frac{a^5}{15} \cos^3\theta \Big|_0^{\pi/2} = -\frac{a^5}{15} . \cos^3\left(\frac{\pi}{2}\right) + \frac{a^5}{15}\cos^3(0)$$

$$= \frac{a^5}{15}$$

$$\bar{x} = \frac{M_y}{M} = \frac{a^5}{15} . \frac{8}{a^4} = \frac{8a^5}{15a^4} = \boxed{\frac{8a}{15}}$$

$$\bar{y} = \frac{8a}{15}$$

$M = \frac{a^4}{8}$ centro $g = \left(\frac{8a}{15} ; \frac{8a}{15}\right)$

7) Faça uma conjectura sobre as coordenadas do centroide e confirme-a por integração.

$V = 1$ $\therefore$ centroide $\left(\frac{1}{2} ; \frac{1}{2} ; \frac{1}{2}\right)$

$$\bar{x} = \int_0^1\int_0^1\int_0^1 x \, dx\, dy\, dz$$

$$\int_0^1 x \, dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$$, similar $\bar{y} = \bar{z} = \frac{1}{2}$

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