Lesson 14: Solving equivalent ratio problems

Goals

- Determine what information is needed to solve a problem involving equivalent ratios. Ask questions to elicit that information.
- Understand the structure of a what-why info gap activity.

Learning Targets

- I can decide what information I need to know to be able to solve problems about situations happening at the same rate.
- I can explain my reasoning using diagrams that I choose.

Lesson Narrative

The purpose of this lesson is to give students further practice in solving equivalent ratio problems and introduce them to the info gap activity structure. The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Addressing

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, bar models, double number line diagrams, or equations.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Information Gap Cards
- Co-Craft Questions
- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the blackline master

Info Gap: Hot Chocolate and Potatoes Problem Card 1 Jada mixes milk and cocoa powder to make hot chocolate. She wants to use all of the cocoa powder she has left. How much milk should Jada use?	Info Gap: Hot Chocolate and Potatoes Data Card 1 • Jada's recipe calls for 3 cups of milk. • Jada's recipe calls for 2 tablespoons of cocoa powder. • Jada has 2 gallons of milk. • Jada has 9 tablespoons of cocoa powder • There are 16 cups in 1 gallon.
Info Gap: Hot Chocolate and Potatoes Problem Card 2 Noah needs to peel a lot of potatoes before a large dinner. He has already peeled some potatoes. If Noah keeps peeling at the same rate, will he finish all the potatoes in time?	Info Gap: Hot Chocolate and Potatoes Data Card 2 • Noah has already peeled 8 potatoes. • Noah has been peeling for 10 minutes. • Noah needs to peel 60 more potatoes. • Noah needs to be finished peeling in 1 hour and 10 minutes. • There are 60 minutes in 1 hour.

Required Preparation

You will need the Hot Chocolate and Potatoes Info Gap blackline master for this lesson. Make 1 copy for every 4 students, and cut them up ahead of time.

Student Learning Goals

Let's practise getting information from our partner.

14.1 What Do You Want to Know?

Warm Up: 5 minutes

The warm-up prepares students for the next info gap activity by first asking them to brainstorm what information they would need to know to solve an equivalent ratio problem. Next, the teacher demonstrate asking students to share what they want to know and why they want to know it before giving them the information.

Launch

Give students 2 minutes of quiet think time.

Student Task Statement

Consider the problem: A red car and a blue car enter the road at the same time and travel at a constant speed. How far apart are they after 4 hours?

What information would you need to be able to solve the problem?

Student Response

Answers vary. Sample responses:

- How fast is each car travelling?
- Are the cars going the same direction?
- Did the cars enter the road at the same location?
- What is the difference between the speeds of the two cars?

Activity Synthesis

Demonstrate asking students the questions they will use in the Info Gap in the next activity. Ask them, "What specific information do you need?" As students pose questions, write them down and ask, "Why do you need that information?"

When students explain why they need the information, provide it to them. After sharing each piece of information, ask the class whether they have enough information to solve the problem. When they think they do, give them 2 minutes to solve the problem and then have them share their strategies.

- The red car is travelling faster than the blue car.
- One car is travelling 5 miles per hour faster than the other car.
- The slower car is travelling at 60 miles per hour.
- The blue car is travelling at 60 miles per hour.
- The faster car is travelling at 65 miles per hour.
- The red car is travelling as 65 miles per hour.
- Both cars entered the road at the same location.
- Both cars are travelling in the same direction.

14.2 Info Gap: Hot Chocolate and Potatoes

30 minutes

In this info gap activity, students solve problems involving equivalent ratios. If students use a table, it may take different forms. Some students may produce a table that has many rows that require repeated multiplication. Others may create a more abbreviated table and use more efficient multipliers. Though some approaches may be more direct or efficient than others, it is important for students to choose their own method for solving them, and to explain their method so that their partner can understand.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Instructional Routines

- Information Gap Cards

Launch

Arrange students in groups of 2. In each group, distribute a problem card to one student and a data card to the other student.

Action and Expression: Internalise Executive Functions. Begin with a small-group or whole-class demonstration and think aloud of a sample situation to remind students how to use the info gap structure. Keep the worked-out table and double number lines on display for students to reference as they work.
Supports accessibility for: Memory; Conceptual processing Conversing: This activity gives students a purpose for discussing information necessary to solve problems involving equivalent ratios. Display questions or question starters for students who need a starting point such as: "Can you tell me . . . (specific piece of information)", and "Why do you need to know . . . (that piece of information)?"
Design Principle(s): Cultivate Conversation

Anticipated Misconceptions

Students may misinterpret the meaning of the numbers or associate quantities incorrectly and multiply 8 by 6 (because 10×6 is 60). Encourage them to organise the given information in a table or a double number line.

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response

Jada should use 13.5 cups of milk. Possible strategies:

- Finding a multiplier that relates 2 to 9 tablespoons of cocoa. They may ask "2 times what is 9?" and use 4.5 as the factor to multiply by 3.
- Multiplying the number of cups of milk by 9 to correspond to 18 tablespoons of cocoa, and then dividing it by 2 for 9 tablespoons of cocoa.
- Multiplying by $\frac{1}{2}$ (or dividing by 2) to find the number of cups of milk that correspond to 1 tablespoon of cocoa, and then multiplying that number by 9 for 9 tablespoons of cocoa, as shown in the table below.

milk (cups)	cocoa (tablespoons)
3	2
$\frac{3}{2}$ or 1.5	1
$13\frac{1}{2}$ or 13.5	9

$\times \frac{1}{2}$, $\times 9$, $\times \frac{1}{2}$, $\times 9$

No, Noah does not have enough time. It will take him 75 minutes to finish peeling all the potatoes. Possible strategies:

elapsed time (minutes)	potatoes peeled
10	8
$\frac{10}{8}$, $\frac{5}{4}$, or 1.25	1
75	60
1	$\frac{8}{10}$, $\frac{4}{5}$, or 0.8
25	20
70	56

- It will take 75 minutes to peel 60 potatoes (1.25 minutes per potato).
- He could peel 56 potatoes in 70 minutes (0.8 potatoes per minute).

Activity Synthesis

Select one student to explain each distinct approach. Highlight how multiplicative reasoning and using the table are similar or different in each case.

When all approaches have been discussed, ask students: "When might it be helpful to first find the amount that corresponds to 1 unit of one quantity and scale that amount up to any value we want?" Encourage students to refer to all examples seen in this lesson so far.

14.3 Comparing Reading Rates

Optional: 10 minutes

This activity provides an opportunity for additional practice in solving equivalent ratio problems. Monitor for students solving the problems in different ways.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Co-Craft Questions
- Think Pair Share

Launch

Give students 4 minutes of quiet work time and then have them discuss their solutions with a partner.

Representation: Internalise Comprehension. Activate or supply background knowledge about equivalent ratios describing situations that happen at the same rate which can be displayed on double number lines. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing Writing: Co-Craft Questions. Before asking students to solve the given question, display only Lin's, Diego's and Elena's reading rates. Invite students think of possible mathematical questions about this situation.

Keep in mind students do not need to answer their created questions. Students share and revise their questions with a partner and then with the whole class. Record questions shared with the class in a public space. This helps students connect mathematical language to get into the context of the problem as they reason about the three different quantities (i.e., number of pages read, number of pages in the book, and number of days reading).
Design Principle(s): Maximise meta-awareness

Student Task Statement

- Lin read the first 54 pages from a 270-page book in the last 3 days.
- Diego read the first 100 pages from a 320-page book in the last 4 days.
- Elena read the first 160 pages from a 480-page book in the last 5 days.

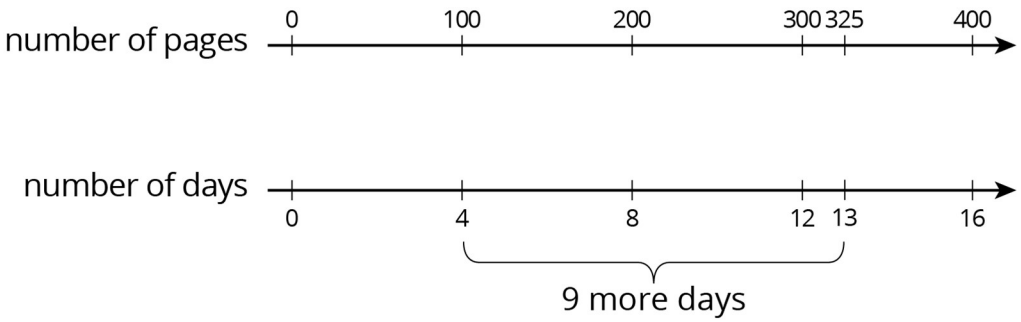
If they continue to read every day at these rates, who will finish first, second, and third? Explain or show your reasoning.

Student Response

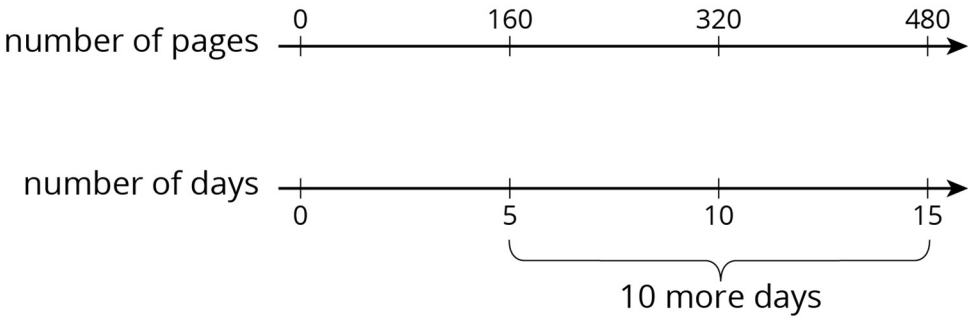
First: Diego, Second: Elena, Third: Lin

Possible Strategy:

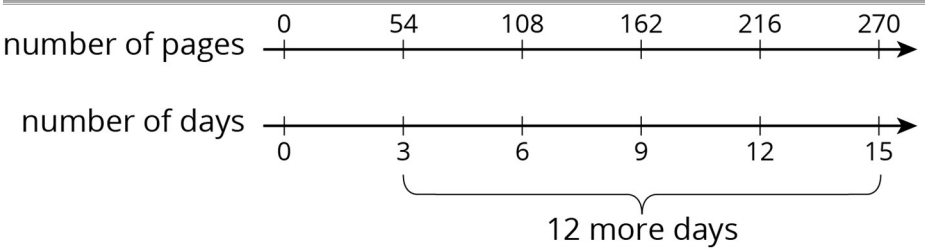
- Diego



- Elena



- Lin



Are You Ready for More?

The ratio of cats to dogs in a room is 2 : 3. Five more cats enter the room, and then the ratio of cats to dogs is 9 : 11. How many cats and dogs were in the room to begin with?

Student Response

22 cats and 33 dogs

Activity Synthesis

Select students to present their solutions. Sequence solutions with diagrams first and then tables. Make sure students see connections between the different representations and ways of solving the problems.

Lesson Synthesis

When solving problems involving equivalent ratios, we often have three pieces of information and need to find a fourth. For example:

- If you eat 12 strawberries in 3 minutes, how long will it take to eat 8 strawberries at that rate?

We can use a table to solve this problem very quickly. For example:

number of strawberries	number of minutes
12	3
1	$\frac{1}{4}$
8	2

- If you jump 8 times in 10 seconds, how many jumps can you make in 45 seconds at that rate?

Where would you put the one in this table? What is the answer to the question?

number of jumps	number of seconds
8	10

14.4 Water Tap

Cool Down: 5 minutes

Student Task Statement

Jada wants to know how fast the water comes out of her tap. What information would she need to know to be able to determine that?

Student Response

Answers vary. Sample response: She would need to know how much water comes out in some amount of time.

For example, she could time how long it takes to fill up some container that she knows the size of.

Student Lesson Summary

To solve problems about something happening at the same rate, we often need:

- Two pieces of information that allow us to write a ratio that describes the situation.
- A third piece of information that gives us one number of an equivalent ratio. Solving the problem often involves finding the other number in the equivalent ratio.

Suppose we are making a large batch of fizzy juice and the recipe says, "Mix 5 cups of cranberry juice with 2 cups of sparkling water." We know that the ratio of cranberry juice to sparkling water is 5 : 2, and that we need 2.5 cups of cranberry juice per cup of sparkling water.

We still need to know something about the size of the large batch. If we use 16 cups of sparkling water, what number goes with 16 to make a ratio that is equivalent to 5: 2?

To make this large batch taste the same as the original recipe, we would need to use 40 cups of cranberry juice.

cranberry juice (cups)	sparkling water (cups)
5	2
2.5	1
40	16

Lesson 14 Practice Problems

Problem 1 Statement

A chef is making pickles. He needs 15 gallons of vinegar. The store sells 2 gallons of vinegar for £3.00 and allows customers to buy any amount of vinegar. Decide whether each of the following ratios correctly represents the price of vinegar.

a. 4 gallons to £3.00

b. 1 gallon to £1.50

c. 30 gallons to £45.00

d. £2.00 to 30 gallons

e. £1.00 to $\frac{2}{3}$ gallon

Solution

a. No. (The ratio is not equivalent; 4 gallons of vinegar would cost £6).

b. Yes.

c. Yes.

d. No. (The ratio is not equivalent; 2 gallons of vinegar cost £3, and £30 would buy 20 gallons).

e. Yes.

Problem 2 Statement

A caterer needs to buy 21 pounds of pasta to cater a wedding. At a local store, 8 pounds of pasta cost £12. How much will the caterer pay for the pasta there?

a. Write a ratio for the given information about the cost of pasta.

b. Would it be more helpful to write an equivalent ratio with 1 pound of pasta as one of the numbers, or with £1 as one of the numbers? Explain your reasoning, and then write that equivalent ratio.

c. Find the answer and explain or show your reasoning.

Solution

a. Answers vary. Sample responses: £12 for every 8 pounds; £12 to 8 pounds; 8 pounds to £12.

b. Answers vary. Sample response: Finding 1 pound would be easier and more helpful. The cost of 1 pound can be easily found by dividing £12 by 8 and the result (the unit rate) can be multiplied by 21. The ratio is £1.50 to 1 pound.

c. £31.50. Possible reasonings: $21 \times (1.50) = 31.50$.

pasta (pounds)	cost (pounds)
8	12
1	1.50

21	31.50

Problem 3 Statement

Lin is reading a 47-page book. She read the first 20 pages in 35 minutes.

a. If she continues to read at the same rate, will she be able to complete this book in under 1 hour?

b. If so, how much time will she have left? If not, how much more time is needed? Explain or show your reasoning.

Solution

No, it will take Lin 82.25 minutes to finish her book. Possible strategies:

a. Using a table:

number of pages	times in minutes
20	35
1	1.75 (or equivalent)
47	82.25 (or equivalent)

Additional 22.25 or $22\frac{1}{4}$ minutes (or 22 minutes and 15 seconds) are needed.

b. 40 pages will take 70 minutes, which is already more than an hour, so Lin cannot finish the 47-page book in an hour.

Problem 4 Statement

Diego can type 140 words in 4 minutes.

a. At this rate, how long will it take him to type 385 words?

b. How many words can he type in 15 minutes?

If you get stuck, consider creating a table.

Solution

Answers vary. Sample response:

number of words	number of minutes
140	4
1	$\frac{1}{35}$
385	11
35	1
525	15

a. It will take 11 minutes to type 385 words.

b. He can type 525 words in 15 minutes.

Problem 5 Statement

A train that travels 30 miles in $\frac{1}{3}$ hour at a constant speed is going faster than a train that travels 20 miles in $\frac{1}{2}$ hour at a constant speed. Explain or show why.

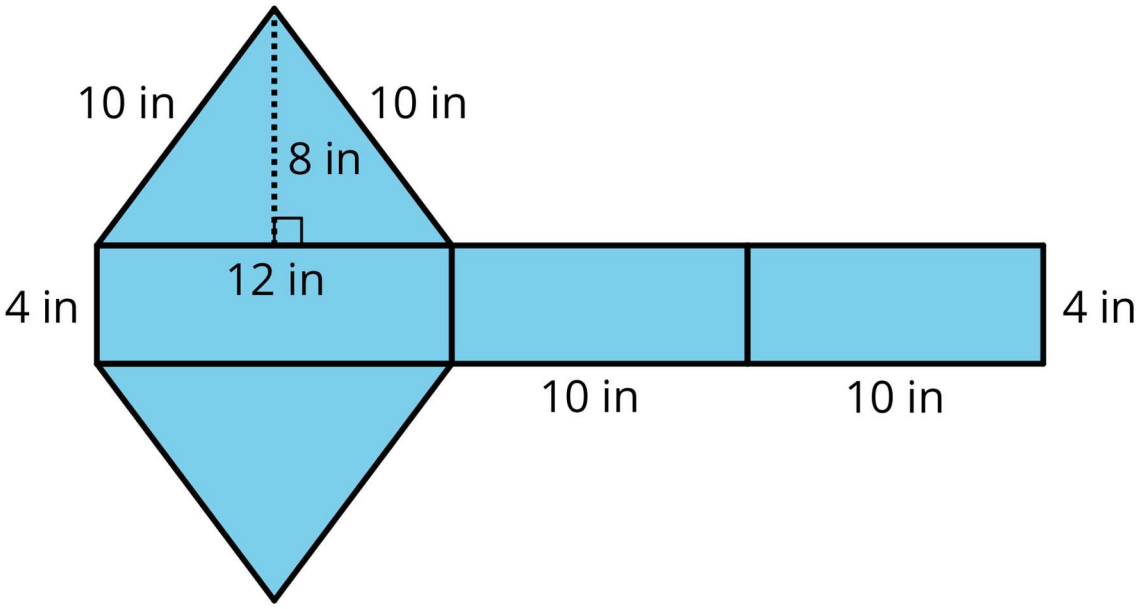
Solution

Answers vary. Sample responses:

- In 1 hour, the first train will travel 90 miles, while the second train only travels 40 miles. The first train is going faster.
- The train travelling 30 miles in $\frac{1}{3}$ of an hour takes $\frac{1}{9}$ of an hour to go 10 miles. The train travelling 20 miles in $\frac{1}{2}$ of an hour takes $\frac{1}{4}$ of an hour to go 10 miles. This means that the first train is travelling faster.

Problem 6 Statement

Find the surface area of the polyhedron that can be assembled from this net. Show your reasoning.



Solution

224 square inches. Reasoning varies. Sample reasoning: The three rectangular faces have areas 48, 40, and 40 square inches. Each triangle has a base of 12 inches and a

height of 8 inches, so each triangle has an area of 48 square inches. $48 + 40 + 40 + 2(48) = 224$.