Lesson 11: Percentage contexts

Goals

- Comprehend "interest," "markup," "markdown," and "commission" as other contexts that involve adding or subtracting a percentage of the initial amount.

- Determine the original pound amount before a markup, markdown, or commission.

- Explain (orally) how to calculate the new pound amount after a markup, markdown, or commission.

Learning Targets

- I understand and can solve problems about commission, interest, markups, and discounts.

Lesson Narrative

In this lesson students are introduced to contexts involving markups, discounts, and commissions, and they continue to study contexts involving tax and tips.

Questions about rounding may naturally come up in this lesson. This lesson primarily involves pound amounts, so it is sensible to round to the nearest penny (the nearest hundredth of a pound). Percentages may be rounded to the nearest whole percentage or fraction of a percentage, depending on the situation.

Building On

- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.

- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percentage.

Addressing

- Use proportional relationships to solve multistep ratio and percentage problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Building Towards

- Use proportional relationships to solve multistep ratio and percentage problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Instructional Routines

- Clarify, Critique, Correct

- Three Reads

- Discussion Supports

- Take Turns

- Think Pair Share

Required Materials
Four-function calculators
Pre-printed slips, cut from copies of the blackline master

Sales Tax	Markup	Interest
Gratuity (Tip)	Markdown (Discount)	Commission

Card Sort: Percentage Situations

Kiran ate breakfast at a café.
The total bill was £5. He left £5.75.

Why did he leave £5.75?

Card Sort: Percentage Situations

Tyler bought a shirt from the
25% off rack.
It normally costs £16. He paid £12.

Why did he pay £12?

Card Sort: Percentage Situations

Clare's aunt used a credit card to pay £40 for an outfit. Six weeks later, the credit card company added £0.60 to her bill.

Why does she owe £40.60 for the outfit now?

Card Sort: Percentage Situations

Lin bought 1 pretzel from a snack cart.
She paid £4.83.

Salted pretzel	£4.50
Cheesy pretzel	£4.50
Cinnamon pretzel	£4.50

Why did she pay £4.83?

Card Sort: Percentage Situations

A car dealership paid £9 000 for a car.
Here is the sticker price for a customer to buy the car.

Wholesale price	£9 000
Suggested retail price	£10 395

Card Sort: Percentage Situations

Andre is saving money for college.
He had £1 500 in his account. At the end of the year, the bank adds 3% of the balance to his account.

current balance	£1 500
dividend (3%)	£1 545

Why are they charging £10 395?	Why does his account have £1 545 now?
Card Sort: Percentage Situations Priya used this coupon to buy a game that normally costs £24.50. **SATURDAY ONLY!** **10% OFF** your store purchase of one single item. Exclusions apply. Why did she pay £22.05?	Card Sort: Percentage Situations Diego's uncle sells computers. In addition to his wages, he gets paid 20% of the amount he sells. *(see table below)* Why did he earn the extra £600?

income	quantity	rate	current total
wage	40	£14.7	£590
sales	£3 000	0.2	£600

Required Preparation

Print and cut up slips from the Card Sort: Percentage Situations blackline master. Prepare 1 copy for every 2 students. These may be re-used if you have multiple classes.

It is recommended that students be provided access to four-function calculators so that they can focus on reasoning about how numbers are related to each other, representing those relationships, and deciding which operations are appropriate (rather than focusing on calculation.)

Student Learning Goals

Let's learn about more situations that involve percentages.

11.1 Leaving a Tip

Warm Up: 5 minutes

The purpose of this warm-up is to help students connect their current work with percentage contexts to their prior work on percentage increase and efficient ways of finding percentage increase.

Launch

Consider telling students that these questions may have more than one correct answer. Students to work in groups of 2. 2 minutes of quiet think time followed by partner and then whole-class discussion.

Student Task Statement

Which of these expressions represent a 15% tip on a £20 meal? Which represent the total bill?

A. 15×20

B. $20 + 0.15 \times 20$

C. 1.15×20

D. $\frac{15}{100} \times 20$

Student Response

B and C represent the total bill while the D represents the tip.

Activity Synthesis

For each expression, ask a few students to explain whether they think it represents: the total bill, the tip, or neither. For each expression, select a student to explain their reasoning.

11.2 A Car Dealership

10 minutes

The purpose of this activity is to introduce students to a context involving markups and markdowns or discounts, and to connect this to the work on percentage increase and percentage decrease they did earlier. The first question helps set the stage for students to see the connection to markups and percentage increase. Look for students who solve the second question by finding 90% of the retail price, and highlight this approach in the discussion.

Instructional Routines
* Three Reads

* Think Pair Share

Launch

Tell students that a mark-up is a percentage that businesses often add to the price of an item they sell, and a mark-down is a percentage they take off of a given price. If helpful, review the meaning of wholesale (the price the dealership pays for the car) and retail price (the price the dealership charges to sell the car). Sometimes people call mark-downs discounts.

Provide access to calculators. Students in groups of 2. Give students 5 minutes of quiet work time, followed by partner then whole-class discussion.

Reading, Writing: Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., A car dealership bought a car. The dealership wants to make a profit. They need to decide what price the car should be.). If needed, discuss the meaning of unfamiliar terms at this time (e.g., profit, wholesale, retail

price, commission, etc.). Use the second read to identify the important quantities by asking students what can be counted or measured (e.g., wholesale price, profit or mark-up, and retail price). In the third read, ask students to brainstorm possible mathematical solution strategies to complete the task. This will help students connect the language in the word problem and the reasoning needed to solve the problem while keeping the intended level of cognitive demand in the task.

Design Principle(s): Support sense-making

Anticipated Misconceptions

It is important throughout that students attend to the meanings of particular words and remain clear on the meaning of the different values they find. For example, "wholesale price," "retail price," and "sale price" all refer to specific pound amounts. Help students organise their work by labelling the different quantities they find or creating a graphic organiser.

Student Task Statement

A car dealership pays a wholesale price of £12 000 to purchase a vehicle.

1. The car dealership wants to make a 32% profit.

    a. By how much will they mark up the price of the vehicle?

    b. After the markup, what is the retail price of the vehicle?

2. During a special sales event, the dealership offers a 10% discount off of the retail price. After the discount, how much will a customer pay for this vehicle?

Student Response
1. a. £3 840, because $0.32 \times 12\,000 = 3\,840$.
   b. £15 840. Possible explanations: $12\,000 + 3\,840 = 15\,840$ or $1.32 \times 12\,000 = 15\,840$.
2. £14 256. Possible explanations: because $0.1 \times 15\,840 = 1\,584$, and $15\,840 - 1,584 = 14\,256$ or $0.9 \times 15\,840 = 14\,256$.

Are You Ready for More?

This car dealership pays the salesperson a bonus for selling the car equal to 6.5% of the sale price. How much commission did the salesperson lose when they decided to offer a 10% discount on the price of the car?

Student Response

£102.96. Before the discount, the salesperson would have earned a bonus of £1 029.60 ($15\,840 \times 0.065 = 1\,029.60$). After the discount, the salesperson only earned £926.64 ($14\,256 \times 0.065 = 926.64$), so the salesperson lost £102.96 ($1029.6 - 926.64 = 102.96$).

Activity Synthesis

For the first question, help students connect markups to percentage increase.

Select students to share solutions to the second question. Highlight finding 90% of the retail price, and reinforce that a 10% discount is a 10% decrease.

Ask them to *describe how they would find* (but not actually find) . . .

- "The retail price after a 12% markup?" (Multiply the retail price by 0.12, then add that answer to the retail price. Alternatively, multiply the retail price by 1.12.)

- "The price after a 24% discount?" (Multiply the retail price by 0.24, then subtract that answer from the retail price. Alternatively, multiply the retail price by 0.76.)

11.3 Commission at a Gym

10 minutes

The purpose of this activity is to introduce students to the concept of a commission and to solve percentage problems in that context. Students continue to practice finding percentages of total prices in a new context of commission.

Monitor for students who use equations like $c = r \times p$ where c is the commission, r represents the percentage of the total that goes to the employee, and p is the total price of the membership.

Instructional Routines
- Clarify, Critique, Correct

- Think Pair Share

Launch

Tell students that a commission is the money a salesperson gets when they sell an item. It is usually used as an incentive for employees to try to sell more or higher priced items than they usually would. The commission is usually a percentage of the price of the item they sell.

Provide access to calculators. Students in groups of 2. Give students 2 minutes of quiet work time. Partner then whole-class discussion.

Anticipated Misconceptions

Students may find the percentage of an incorrect quantity. Ask them to state, in words, what they are finding a percentage of.

Students may not understand the first question. Tell them that a membership is sold for a certain price and the money is split with £42 going to the gym and £8 going to the employee.

Student Task Statement
1. For each gym membership sold, the gym keeps £42 and the employee who sold it gets £8. What is the commission the employee earned as a percentage of the total cost of the gym membership?

2. If an employee sells a family pass for £135, what is the amount of the commission they get to keep?

Student Response
1. 16%, because $42 + 8 = 50$ and $8 \div 50 = 0.16$.

2. £21.60, because $0.16 \times 135 = 21.6$.

Activity Synthesis

Select students to share how they answered the questions.

During the discussion, draw attention to strategies for figuring out which operations to do with which numbers. In particular, strategies involving equations like $c = r \times p$ where c is the commission, r represents the percentage of the total that goes to the employee, and p is the total price of the membership.

Reading, Writing: Clarify, Critique, Correct. Present an incorrect response to the second question that reflects a possible misunderstanding from the class. For example, "For the family membership of £135, the employee would keep £8." Prompt students to identify the error (e.g., ask, "Do you agree with the author's reasoning? Why or why not?"), and then write a correct version. This will help students to understand that the employee's commission is always a rate of 16% and not a flat amount of £8.
Design Principle(s): Maximise meta-awareness

11.4 Card Sort: Percentage Situations

Optional: 10 minutes

This activity gives students an opportunity to practice various vocabulary terms that come along with percentages. Students are asked to sort scenarios to different descriptors using the images, sentences or questions found on the scenario cards. The questions found on the scenario cards are intended to help students figure out which descriptor the scenario card belongs under.

As students work on the task, identify students that are using the vocabulary: tip, tax, gratuity, commission, markup/down, and discount. These students should be asked to share during the discussion.

Instructional Routines

- Discussion Supports

- Take Turns

Launch

Arrange students in groups of 2. Distribute the sorting cards, and explain that students will sort 8 scenarios into one of 6 categories. Demonstrate how students can take turns placing a scenario under a category and productive ways to disagree. Here are some questions they might find useful:

- Which category would you sort this under?

- What do you think this word means?

- What words can we use as clues about where to sort this card?

Representation: Internalise Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual processing; Organisation Speaking: Discussion Supports. Show central concepts multi-modally by using different types of sensory inputs: acting out scenarios or inviting students to do so, showing videos or images, using gestures, and talking about the context of what is happening. This will help students to produce and make sense of the language needed to communicate their own ideas.
Design Principle(s): Optimise output (for explanation)

Anticipated Misconceptions

Students should use the question at the bottom of the card to help them if they get stuck sorting the scenarios.

Student Task Statement

Your teacher will give you a set of cards. Take turns with your partner matching a situation with a descriptor. For each match, explain your reasoning to your partner. If you disagree, work to reach an agreement.

Student Response

Gratuity/Tip with "Kiran ate . . ."
Commission with "Diego's uncle . . ."
Interest with "Andre is saving . . ." and "Clare's aunt . . ."

Discount/Markdown with "Tyler bought . . ." and "Priya used . . ."
Sales Tax with "Lin bought . . ."
Markup with "A car dealership . . ."

Activity Synthesis

Ask identified students to share which situations they sorted under each word. Ask them:

- "What made you decide to put these situations under this descriptor?"

- "Were there any situations that you were really unsure of? What made you decide on where to sort them?"

Consider asking some groups to order the situations from least to greatest in terms of the pound amount of the increase of decrease and asking other groups to order them in terms of the percentage. Then, have them compare their results with a group that did the other ordering.

Answer students' remaining questions about any of these contexts. Tell students there is a copy of this chart at the end of the lesson that they can use as a reference tool during future lessons. Allow them a space to take notes on their own to remember it or details from one of the activity examples.

	paid to:	how it works:
sales tax	the government	added to the price of the item
gratuity (tip)	the server	added to the cost of the meal
interest	the lender (or account holder)	added to the balance of the loan, credit card, or bank account
markup	the seller	added to the price of an item so the seller can make a profit
markdown (discount)	the customer	subtracted from the price of an item to encourage the customer to buy it
commission	the salesperson	subtracted from the payment the store collects

Lesson Synthesis

In this lesson, we studied lots of different situations where people use percentages.

- "What are some situations in life in which people encounter percentages?"

- "Give examples of situations where you would encounter tax, tip, markup, markdown, commission." (Lots of possible answers.)

- "When an item is marked down 10%, why does it make sense to multiply the price by 0.9?" (Since there is 10% off of the price, the new cost is 90% of the original.)

- "When an item is marked up 25%, why does it make sense to multiply the price by 1.25?" (Since the item now costs 100% plus an extra 25%, the new item costs 1.25 times the original.)

11.5 The Cost of a Bike

Cool Down: 5 minutes

The purpose of this activity is to check whether students can solve a problem involving a mark-up and a discount.

Student Task Statement

The bike store marks up the wholesale cost of all of the bikes they sell by 30%.

1. Andre wants to buy a bike that has a price tag of £125. What was the wholesale cost of this bike?

2. If the bike is discounted by 20%, how much will Andre pay (before tax)?

Student Response
1. £96.15, because $125 \div 1.3 = 96.15$.

2. £100, because $125 \times 0.8 = 100$.

Student Lesson Summary

There are many everyday situations where a percentage of an amount of money is added to or subtracted from that amount, in order to be paid to some other person or organisation:

	goes to	how it works
sales tax	the government	added to the price of the item
gratuity (tip)	the server	added to the cost of the meal
interest	the lender (or account holder)	added to the balance of the loan, credit card, or bank account
markup	the seller	added to the price of an item so the seller can make a profit
markdown (discount)	the customer	subtracted from the price of an item to encourage the customer to buy it
commission	the salesperson	subtracted from the payment that is collected

For example,

- If a restaurant bill is £34 and the customer pays £40, they left £6 pounds as a tip for the server. That is 18% of £34, so they left an 18% tip. From the customer's perspective, we can think of this as an 18% increase of the restaurant bill.

- If an estate agent helps a family sell their home for £200 000 and earns a 3% commission, then the estate agent makes £6 000, because $(0.03) \times 200\,000 = 6\,000$, and the family gets £194 000, because $200\,000 - 6\,000 = 194\,000$. From the family's perspective, we can think of this as a 3% decrease on the sale price of the home.

Lesson 11 Practice Problems

Problem 1 Statement

A car dealership pays £8 350 for a car. They mark up the price by 17.4% to get the retail price. What is the retail price of the car at this dealership?

Solution

£9 802.90, although most dealerships round to the nearest 5 or 10.

Problem 2 Statement

A store has a 20% off sale on trousers. With this discount, the price of one pair of trousers before tax is £15.20. What was the original price of the trousers?

    a.   £3.04

    b.   £12.16

    c.   £18.24

    d.   £19.00

Solution D

Problem 3 Statement

Lin is shopping for a sofa with her dad and hears him ask the salesperson, "How much is your commission?" The salesperson says that her commission is $5\frac{1}{2}\%$ of the selling price.

    a.   How much commission will the salesperson earn by selling a sofa for £495?

    b.   How much money will the store get from the sale of the sofa?

Solution

    a.   £27.23. 5.5% of 495 is 27.225.

    b.   £467.77

Problem 4 Statement

A university student takes out a £7 500 loan from a bank. What will the balance of the loan be after one year (assuming the student has not made any payments yet):

a. if the bank charges 3.8% interest each year?

b. if the bank charges 5.3% interest each year?

Solution

a. £7 785.00

b. £7 897.50

Problem 5 Statement

Match the situations with the equations.

a. Mai slept for x hours, and Kiran slept for $\frac{1}{10}$ less than that.

b. Kiran practised the piano for x hours, and Mai practised for $\frac{2}{5}$ less than that.

c. Mai drank x oz of juice and Kiran drank $\frac{4}{3}$ more than that.

d. Kiran spent x pounds and Mai spent $\frac{1}{4}$ less than that.

e. Mai ate x grams of almonds and Kiran ate 1.5 times more than that.

f. Kiran collected x pounds of recycling and Mai collected $\frac{3}{10}$ less than that.

g. Mai walked x kilometres and Kiran walked $\frac{3}{8}$ more than that.

h. Kiran completed x puzzles and Mai completed $\frac{3}{5}$ more than that.

$y = 2.33x$

$y = 1.375x$

$y = 0.6x$

$y = 0.9x$

$y = 0.75x$

$y = 1.6x$

$y = 0.7x$

$y = 2.5x$

Solution

a. $y = 0.9x$

b. $y = 0.6x$

c. $y = 2.33x$

d. $y = 0.75x$

e. $y = 2.5x$

f. $y = 0.7x$

g. $y = 1.375x$

h. $y = 1.6x$